ROTATION OF THE SOLAR CORE INFERRED FROM GONG DATA

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ABSTRACT

We present inferences of the solar core rotation derived from GONG frequency splittings. The GONG data for the low-degree modes ($l \leq 6$) are for the one-year period Aug. 1995-1996; these splittings were determined using a new method of fitting helioseismic spectra (Rabello-Soares & Appourchaux 1998). These are combined with GONG data for higher degree modes, as obtained by the GONG team (Hill et al. 1996). We have employed a new two-dimensional implementation of the optimally localized averages (OLA) inversion method (Chaplin et al. 1998). As in one dimension, the OLA produces better-localized averaging kernels than do least-squares techniques. This is particularly beneficial in the deep interior, where it is in any case difficult to obtain localized information.

Key words: solar core: rotation; helioseismology: MOLA inversion; GONG data

1. INTRODUCTION

Inferences about the solar core rotation are complicated by the fact that only the very low-degree p modes penetrate to the core. Furthermore, for modes of very low l it is much more difficult to measure the splitting than at higher degree: there are fewer components in the multiplets to measure, and the measured lifetimes of the modes are in general comparable to or smaller than the rotation period, so that adjacent rotationally split peaks partly overlap. These difficulties to a large extent explain the non-concordant results for the low-degree splittings measured in the past by several researchers, giving rise to very different results for the solar core rotation rate (see also Eff-Darwich & Korzennik 1998).

Recently, independent measurements by the IRIS and BiSON networks have given apparently conflicting results. The IRIS data indicate that the core rotates at, or perhaps slightly above, the surface rate (e.g. Lazrek et al. 1996; Corbard et al. 1998), whereas the BiSON data have shown evidence for core rotation below the mean rate of the solar envelope (Elsworth et al. 1995; Chaplin et al. 1998). The latter result is in agreement with the rotation rate found by the LOI/SOHO prototype, LOI-T, data (Rabello-Soares et al. 1997).

In this work, we have redetermined the low-degree frequency splittings by analyzing GONG data using a new method of fitting helioseismic spectra (Rabello-Soares & Appourchaux 1998; see also Section 4). Combining these splittings with other GONG data we have then carried out inversions targeted specifically at obtaining averages of the rotation rate localized as well as possible to the solar core.

2. LINEAR INVERSION TECHNIQUES

The inverse problem for the solar internal angular velocity $\Omega(r,\theta)$ is

$$d_i = \int_0^R \int_0^{\pi/2} K_i(r,\theta)\Omega(r,\theta) r \mathrm{d}r \mathrm{d}\theta + \epsilon_i \ , \qquad i = 1, ..., M \ , \tag{1}$$

where r is distance to the centre and θ is colatitude. Here the index i numbers the data d_i. These may be either individual splittings $m^{-1}\Delta\omega_{nlm}$ in angular frequency ω, for a mode of radial order n, degree l and azimuthal order m, or expansion coefficients $2\pi a_k(n,l)$ resulting from suitable polynomial expansions of $\Delta\omega_{nlm}$. We assume that the kernels K_i, which obviously depend on the type of data considered, are known. The observational errors are given by ϵ_i and are assumed to be uncorrelated.

The observed data are acoustic modes of degree $l = 1$ to 150. To infer the rotation of the deep solar interior we require data for deeply penetrating modes. The behaviour of the kernels in the core of the Sun depends on two properties. The modes have lower turning points at $r = r_\mathrm{t}$, determined by $c(r_\mathrm{t})/r_\mathrm{t} \simeq \omega/(l + 1/2)$, where c is the adiabatic sound speed. Thus modes of low degree and/or high frequency penetrate most deeply. Particularly, as already pointed out, only the modes with very low degree penetrate to the core. Secondly, the dominant contributions to the frequency splittings come from the outer parts of the Sun. Near the centre, the kernels decrease with decreasing r as $K_i \propto r^{2l}$. However, for $l = 1$, the leading-order terms cancel, so that $K_i \propto r^4$. Hence splittings for $l = 1$ are less sensitive to the core rotation than might otherwise have been expected.

For linear inversion methods, the solution at a given point (r_0, θ_0) is determined by a set of inversion coefficients $c_i(r_0, \theta_0)$, such that the inferred value of Ω is:

$$\left\langle \Omega(r_0,\theta_0) \right\rangle = \sum_i c_i(r_0,\theta_0) d_i \ . \qquad (2)$$

As a result, the solution is characterized by the 'averaging kernel', obtained as the corresponding linear combination of the mode kernels:

$$\mathcal{K}(r_0,\theta_0,r,\theta) = \sum_i c_i(r_0,\theta_0) K_i(r,\theta) \ . \qquad (3)$$

It follows from Equations (1) and (2) that

$$\left\langle \Omega(r_0,\theta_0) \right\rangle \simeq \int_0^R \int_0^{\pi/2} \mathcal{K}(r_0,\theta_0,r,\theta)\Omega(r,\theta) r \mathrm{d}r \mathrm{d}\theta \ . \qquad (4)$$

It is generally required that $\mathcal{K}$ has unit integral, so that the inferred value is a proper average of the angular velocity. The standard deviation of the solution is obtained as:

$$\sigma^2 \left[\left\langle \Omega(r_0,\theta_0) \right\rangle \right] = \sum_i c_i^2(r_0,\theta_0)\ \epsilon_i^2 \ . \qquad (5)$$

3. MOLA TECHNIQUE

A number of different inversion techniques can be used for inverting the constraints given in Equation (1). We have used Multiplicative Optimally Localized Averages (MOLA), where the coefficients are found by minimizing:

$$\int_0^R \int_0^{\pi/2} \mathcal{K}^2(r_0,\theta_0,r,\theta) J(r_0,\theta_0,r,\theta) r \mathrm{d}r \mathrm{d}\theta + \lambda \sum_i c_i^2(r_0,\theta_0)\epsilon_i^2 \ , \qquad (6)$$

where J is a weight function that is small near (r_0, θ_0) and large elsewhere (e.g. Backus & Gilbert 1968). This, together with the normalization constraint, forces $\mathcal{K}$ to be large near (r_0, θ_0) and small elsewhere, as desired.

As we have already mentioned, it is a hard task to determine the rotation in the core mainly because of the small number of modes and their large errors due to the difficulty in measuring the low-degree mode splittings. Hence the goal is to concentrate the averaging kernel, as far as possible, to the central region; the details of the kernel within that region are less important. Following the idea developed by Chaplin et al. (1998), we are going to use the following choice of the weight function:

$$J(r_0,\theta_0,r,\theta) = \left\{ \begin{array}{cc} 0 & \text{if} \quad \xi^2+\eta^2 \le 1 \\ \xi^4+\eta^4 & \text{otherwise} \ , \end{array} \right. \qquad (7)$$

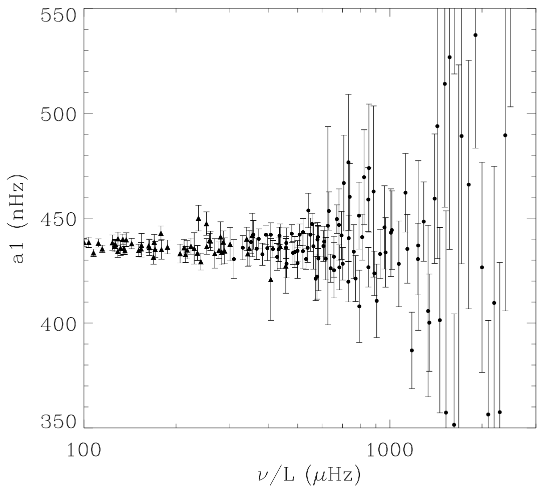

Figure 1. a_1 splitting coefficients for low-degree modes $l \le 6$ (circles) and high-degree modes $7 \le l \le 20$ (triangles), plotted as a function of ν/L, where $L^2 = l(l+1)$.

where

$$\xi = \frac{r - r_0}{\Delta_r} \ , \qquad \eta = \frac{r(\theta-\theta_0)}{\Delta_\theta} \ ; \qquad (8)$$

in all cases considered, we have taken $\theta_0 = \pi/2$ and $\Delta_\theta = 3R$, hence essentially applying no restriction to the latitudinal variation of the averaging kernels.

4. DATA

The GONG data for the low-degree modes ($l \le 6$) are for the one-year period Aug. 1995-1996. The fitting was done using a method developed by Schou (1992), where the Fourier spectra are fitted instead of the power spectra. The main difference is that the $2l+1$ spectra in a multiplet are assumed to be statistically dependent of each other, in contrast to the power fitting. This means that the leakage between the modes is taken into account in a more complete way (see Rabello-Soares & Appourchaux 1998). **[I suggest expanding this paragraph, to provide a little more detail on how this analysis method differs from the one previously applied. Not needed, obviously, if a short paper on the analysis is included in the proceedings]**.

These are combined with GONG data for higher degree modes ($7 \le l \le 150$), as obtained by the GONG team fitting the power spectra (Hill et al. 1996). For the results presented here, we have used splittings obtained from a three-month series starting on June 6, 1996, which is essentially contained within the period on which the low-degree data are based. Due to the rapid increase in linewidth with frequency, only modes with frequency smaller than 3500 μHz have been used for the high-degree modes.

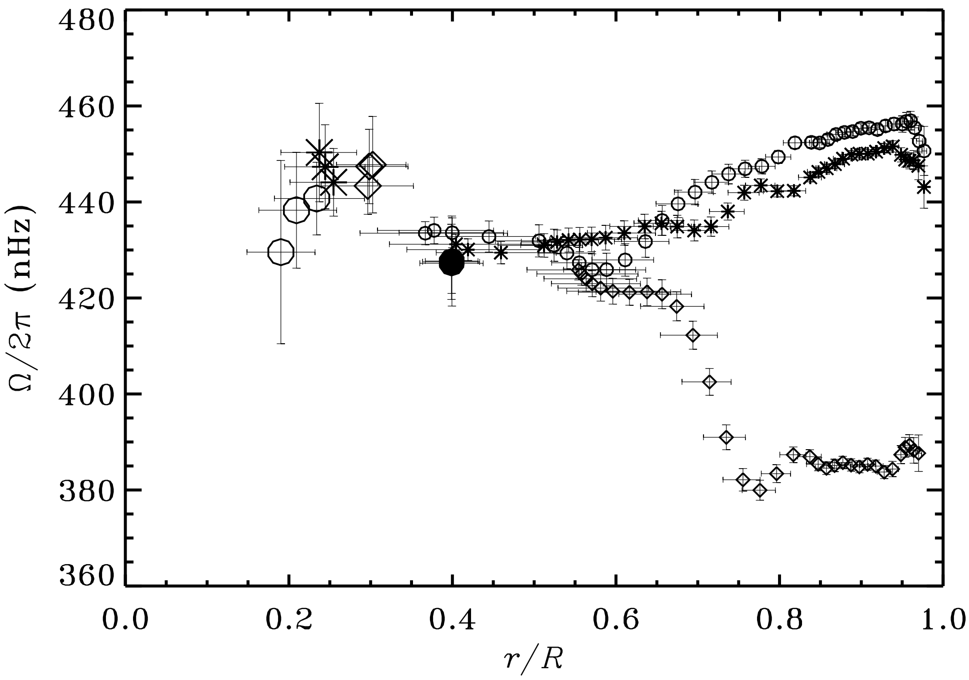

Figure 2. Results of SOLA and MOLA inversion of the GONG data. The vertical error bars correspond to 1σ errors in the inferred rotation, based on the errors in the observed splittings, whereas the horizontal bars, extending from the first to the third quartile point in the radial variation of the averaging kernels, provide a measure of resolution. The small symbols were obtained from SOLA inversion at target latitudes $0°$ (circles), $30°$ (stars), and $60°$ (diamonds). The large symbols were obtained with the MOLA technique where no localization in latitude was attempted. Different symbols denote different target locations: $r_0 = 0$ (open circles), $r_0 = 0.2R$ (crosses), $r_0 = 0.3R$ (diamonds), $r_0 = 0.4R$ (closed circles); the results at a given target location are distinguished by the trade-off between resolution and error, as determined by the parameter λ in Equation (6).

Figure 1 illustrates the data for modes of degree 1 to 20 plotted against ν/L, i.e., the turning-point position. There is no obvious inconsistency between the low-degree data (circles) and the high-degree data (triangles). It is evident that the splittings are more scattered and the errors increase strongly, unfortunately, for the more deeply penetrating modes, making the determination of the core rotation very difficult.

5. RESULTS

Figure 2 shows the results of the inversion of the GONG data. For completeness, we have also included results based on SOLA inversion for $r \geq 0.35R$, at three different latitudes (Pijpers & Thompson 1992; Larsen & Hansen 1997; see also Larsen et al. 1998). The MOLA results were obtained at target radii $r_0 = 0$, $0.2R$, $0.3R$, and $0.4R$, with $\Delta_r = 0.25R$, $0.15R$ $0.1R$, and $0.07R$, respectively. To illustrate the trade-off between error and locatization, at each target radius r_0 we include results for three different values of the trade-off parameter λ. Figure 3 shows the radial variation of several averaging kernels, including the most deeply penetrating inversion. As in the case of Chaplin et al. (1998) we have been able to localize the kernel quite efficiently to the core of the Sun, albeit at the expense of a fairly substantial error; furthermore, the figures clearly show the trade-off between the error and the degree to which the averaging kernel is confined to the core, for $r_0 = 0$.

The behaviour in the outer parts of the Sun show the now familiar pattern: there is, at least at low latitudes, a substantial shear in rotation just beneath the surface; at the base of the convection zone there is a transition to nearly constant rotation in the interior.

A striking, although barely significant, feature of the present results is the apparent jump in rotation, at around $r \simeq 0.3R$. This appears to be associated with a corresponding variation in the splittings at $l = 6$, which may be seen in the averages shown in Figure 4 (on the other hand, it is essentially invisible in the individual splittings shown in Figure 1). Indeed, it is probably significant that the transition from the use of existing GONG frequency splittings to the new fits takes place at $l = 6$. Thus it is likely that the jump is causes by systematic differences caused by the anal-

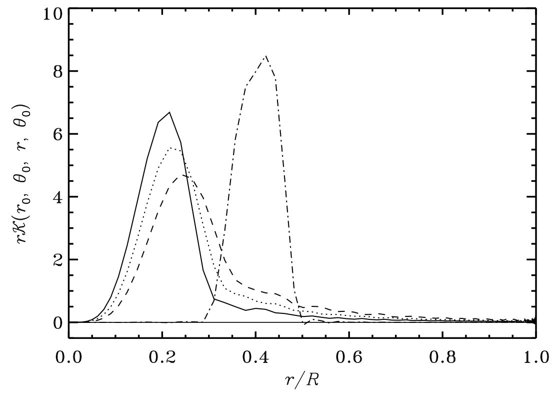

Figure 3. Radial cuts through averaging kernels for some of the MOLA inversions shown in Figure 2. The cuts have been normalized to have unit integral with respect to r/R. The solid, dotted and dashed lines show kernels for $r_0 = 0$, $\Delta_r = 0.25R$ and increasing λ; the corresponding errors in $\Omega/2\pi$ are 7.6, 12.1 and 19.1 nHz, respectively. The dot-dashed line shows a kernel for $r_0 = 0.4R$, $\Delta_r = 0.07R$ and with error 10.8 nHz.

ysis methods or by the use of different data sets. In particular, it would be very valuable to apply the analysis technique of Rabello-Soares & Appourchaux (1998) over a larger range in degree.

Finally we note that, unlike Chaplin et al. (1998), we find no indication of a slowly rotating core. Thus the uncertainty about the rotation of the very deep solar interior unfortunately persists.

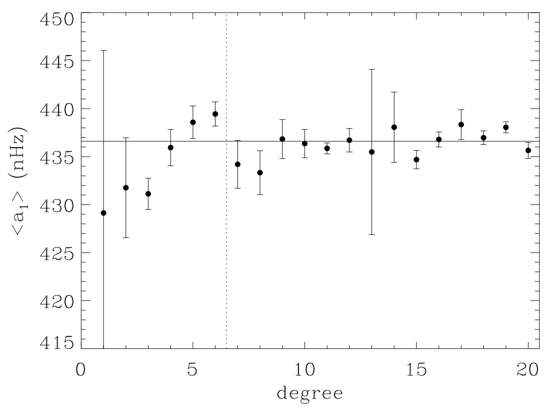

Figure 4. Weighted average of a_1 splitting coefficients for each degree l as a function of l. The errors bars are the standard error of the weighted mean. The dotted line divides the data obtained fitting the Fourier spectra (left-hand side) and the power spectra (right-hand side).

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