THE ART OF FITTING P-MODE SPECTRA

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1. Maximum Likelihood Estimators and p-mode spectra

For deriving p-mode parameters from m, ν diagrammes, one has to treat correctly the statistics of the observation. The correct statistical treatment of these diagrammes was first achieved by Schou (1992) (PhD thesis, Aarhus University). Fitting p-mode spectra requires 4 major steps:

1. Compute the mode leakage matrices
2. Compute mode covariance matrices from the previous matrices
3. Compute the noise covariance matrices
4. Compute and maximize the likelihood of the observation

The leakage matrices link the observed m Fourier spectrum to the modes m'. Most often, the matrices are not diagonal and leads to correlation between the observed m. Nevertheless, we have shown that even if there are correlation between the $2l+1$ Fourier spectra, taking into account correctly the correlation leads implicitly to the removal of these correlations. Unfortunately, what is true for the modes does not apply to the noise mainly because they have different correlation characteristics.

Here we must point out that using only power spectra of each m signal to derive p-mode parameters is an approximation that will lead to underestimating the splitting at low degrees. This is the case of the official GONG splittings that are well known to be severely underestimated. In any case, after having taken into account all the correlations, it can be shown, using for example Monte-Carlo simulations, that the maximization of the likelihood leads to unbiased p-mode parameters. The result we obtained on the GONG data (Appourchaux, Rabello-Soares and Gizon, these proceedings) show that the artificial bias of the GONG splitting can be removed using the steps mentioned above.

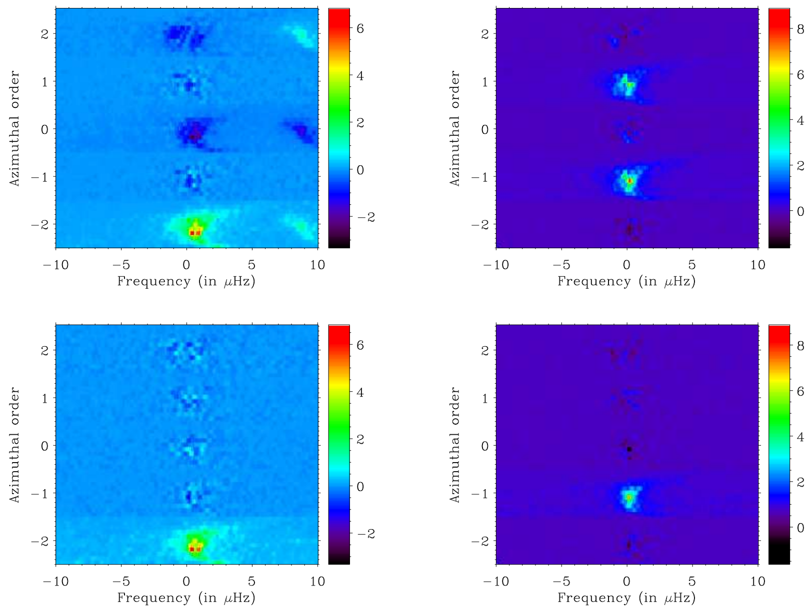

Figure 1. Correlations between the $2l+1$ Fourier spectra of $l=2$ for 1-year of LOI/SOHO data. Two top diagrammes are for spherical harmonic filters: (Top, left) For $l=2, m=2$. (Top, right) For $l=2, m=1$. Two lower diagrammes are for inverted leakage matrix: same as for the previous diagrammes: (Bottom, left) For $l=2, m=2$. (Bottom, right) For $l=2, m=1$. Each panel is composed of 5 $(2l+1)$ echelle diagramme of the correlation between the Fourier spectra, i.e. $\mathrm{Re}(y_m(\nu) y^*_{m'}(\nu))$ or real part of the cross echelle diagrammes. We can note the absence of correlation between the m's for which Δm is odd. We should also point out that the $l=0$ modes have been removed by applying the inverse of the combined leakage matrix of $l=0$ and $l=2$ (Left lower panel).

2. Visualization of correlations

Here we show that even if you do not know anything about the instruments that obtained the observations, it is still possible to measure on the data the leakage and mode covariance matrices. Figure 1 shows a way to visualize the correlations between the $2l+1$ Fourier spectra. These diagrammes are useful for investigating the knowledge of the leakage matrix. For instance, one can make similar diagrammes not only with the original data ($\vec{y_l}$ but with also with $\vec{x_l} = C_l^{-1}\vec{y_l}$, where C_l is the leakage matrix for the degree l. In this case the leakage matrix for $\vec{x_l}$ is purely diagonal for the given l; for other degrees this does not apply. If the assumed leakage matrix is correct then the cross echelle diagrammes will display power only for $m = m'$.

Here we would like to stress that what really matters is the knowledge of the leakage matrices. The main source of bias for the splitting lies in the imperfect knowledge of the leakage matrix elements. We have also shown using Monte-Carlo simulations that underestimating or overestimating leakage matrices elements will result in underestimating the splitting. These effects have also be found using real data such as the LOI or GONG (Appourchaux, Rabello-Soares and Gizon, these proceedings).