AMPLITUDES OF SOLAR G-MODES FROM NUMERICAL SIMULATIONS

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ABSTRACT

The current study is an extension of two previous investigations into the excitation and damping of internal solar gravity waves. The basic assumption is that convective overshoot into the convectively stable interior generates gravity waves. Numerical simulations indicate that the efficiency in this process is ≤0.1% (Ref. 1). Assuming that this is a stationary process it is possible to derive a value for the amplitude of the g-modes in the interior. Another set of numerical simulations give the amount of the transmission of gravity waves through the convection zone as function of frequency and horizontal wave numbers. These calculations are carried out for 5 different frequencies and horizontal extents equivalent to *l*-values approximately in the range 6-18. By combining the results of the excitation and transmission simulations the horizontal velocity amplitude of solar g-modes near the solar surface is estimated to be in the range 0.01-1 $\mathrm{mms^{-1}}$.

Keywords: Helioseismology, gravity waves, convection, g-modes.

1. INTRODUCTION

The identification and classification of internal gravity modes of the Sun is of fundamental importance as a diagnostic tool to determine the physical and dynamical structure of the solar core and its vicinity. The GOLF (Ref. 2) and VIRGO (Ref. 3) investigations on the SOHO mission are specifically designed to undertake these tasks.

Several attempts have been made to identify solar g-modes from ground and space based observations (Ref. 4 and references therein). Despite these attempts and results no repetitive unambiguous mode identification and classification has been made. The most persistent results apply to the observations of the 160 minutes periodicity. Several indications support that this mode is of solar origin (Ref. 5), but no classification has been made of the mode which is consistent with the physical conditions of the solar interior derived from p-mode inversions.

The negative results may be caused by a combination of several factors:

- The small surface amplitude of the modes. This may be caused by an unexpected small amplitude in the interior and/or excessive damping in the convection zone or atmosphere.
- The frequency interval where they most probably exist (20-150 μHz) is difficult to access by ground based observations due to diurnal effects on the observations.
- The presence of a large solar background signal caused by the evolution and rotation of solar surface structures.

Some of these problems will be overcome with the advent of the specially designed helioseismic investigations on SOHO.

There is no exact apriori knowledge on how large amplitude solar global g-modes should be. It is clear, from stability calculations that no more than a few, if any, of the modes are overstable in the current Sun (Ref. 6 and references therein). This implies that other mechanisms must be responsible for the excitation if the internal g-modes exist.

The idea that the penetration of convective matter from the solar convection zone into the convectively stable solar interior may generate internal gravity waves has been discussed by several authors (Refs. 1, 7, 8).

The gravity waves or g-modes are trapped in the stable region beneath the solar convection zone. In order to be observable at the solar surface the signatures of the g-modes must penetrate the convection zone where these modes or waves are evanescent. Several studies have looked into the transmission of g-modes or gravity waves through the solar convection zone using different methods and strategies (Refs. 9, 10, 11, 12). The studies in Ref. 10 use a mixing length approach for the convection zone and include the effects of the solar atmosphere. The preliminary results in Ref. 11 treats the hydrodynamical interaction between the gravity waves and convection consistently, but does not include the effects of the regions close to and in the atmosphere. In this study we present new extended numerical simulations describing the *l*- and frequency dependence of the transmission of gravity waves through the convection zone, but still without a proper treatment of the atmosphere. These simulations are combined with the results of the convective excitation efficiency of internal gravity waves (Ref. 1).

2. METHOD

We have used a numerical code for solving the appropriate hydrodynamical equations with physical parameters close to those describing the solar interior. The code is two-dimensional and use pseudospectral methods on the fully compressible problem. The strength of the code is that it treats the wave mechanics in a consistent way. The upper and lower boundaries are open, thus transmitting the wave energy created in the model. The turbulent cascade in the simulations is balanced by the use of a numerical spectral viscosity. This combined with the treatment of the radiative flux maintains the thermal driving of the convection zone and thus allows the simulations to be carried out for a long time.

The solar model is the same as used in Ref. 1 and is adapted from Model C in (Ref. 13). We use a plane parallel approximation extending from 0.5 to 1 solar radii with a width of 0.3 solar radius. The upper part of the model (from 0.9 to 1.0) has been modified to be subadiabatic in order to study the transmission of gravity waves through the major part of the convection zone.

The details and developments of the method and code are described earlier (Refs. 1, 11, 12) and the main limitations of the method and numerical simulations are:

- As for other large scale solar numerical simulations the Reynolds numbers in the calculation is much lower than for the Sun.
- The simulations are not carried out globally and the results concerns the properties of gravity waves and not trapped internal g-modes.
- The code is two-dimensional and it is clear that this constrains both the dissipation and excitation in a way that is not representative for the hydrodynamic instabilities in the solar interior.
- The plane-parallel approximation does not allow for an accurate description of the solar conditions.
- The code is unable to treat ionization effects and radiative transfer close to the surface. The model used is thus not descriptive of the Sun in the outer parts of the solar envelope.
- The simulations do not include the effects of magnetic fields, this may be of importance in the boundary layer between the convection zone and the radiative interior.

Initially the static model is randomly perturbed in density (<0.1 %) within in the region corresponding to the solar convection zone. In Ref. 1 the simulation was carried out for 1100 timesteps, each timestep being the sound travel time of one scale height at the bottom of the model. transformed to solar conditions this is equivalent to about 240 seconds. The initial velocities large velocities (≈ 5% of sound speed) are damped and a quasi-stationary convective like pattern is generated with typical velocities of ≤0.5 % of the sound speed after about 200 timesteps. Transferred to the solar values for the sound speed in the lower part of the convection zone this is equivalent to about 800 ms^{-1}. The total kinetic energy in convection zone $\approx 5*10^{38}$ erg (Ref. 6) which implies an average velocity of about 40 ms^{-1}. This is typically a factor of 20 lower than the values obtained in our simulations. However, the results in Ref. 1 indicate that the major part of the kinetic energy is carried in relatively few downward directed plumes spanning several scaleheights. This is in accordance with other simulation results (Ref. 8) and studies based on the entropy balance (Ref. 14). The effective filling factor of the downward plumes in our two dimensional simulations is about 0.1 (Refs. 1, 11, 12). However, the velocities in the downward directed plumes in our simulations may be overestimated compared to the solar case due to the plane parallel approximation used in the simulations.

The calculations in this study start with the convection zone developed in Ref. 1 at timestep 400 and continue to 750. Below the convection zone the model is equal to the initial static model. For each simulation a single gravity wave of known amplitude and horizontal extent are numerically generated at a depth equivalent of 0.55 solar radii. The horizontal extents of the generated waves are equivalent to l values of approximately 6, 12 and 18, respectively, at the bottom of the convection zone. For the lowest l-value waves of frequencies 53, 77, 104, 166 and 277 μHz were calculated. For the two other l-values only the three highest frequencies were calculated.

It should also be noted that we have made no attempts to determine specific radial wave functions for the "g-modes" discussed here. This implies that the explicit effects of the radial nodes of g-modes have not been studied. G-modes with l=6 and the frequencies calculated here would have radial order n-values of about 7, 11, 16, 27 and 48 respectively. This is assuming that P_0=35.5 minutes.

3. GRAVITY WAVE TRANSMISSION

The spatial and temporal distributions at different depths of horizontal component of the kinetic energy density ($\approx \rho v^2$) are used as tracers for the gravity waves. The cross spectra of this horizontal kinetic energy distribution at depths equivalent of 0.6 and 0.96 solar radii are calculated. The value of the energy transmission is calculated as the gain between the spectra. By using both the temporal and spatial variation it is ensured that the waves observed in the upper part of the model have the same properties as the modes induced below the convection zone.

The results from these calculations are shown in Fig. 1. A clear dependence on both frequency and l-value can be seen. These results are somewhat smaller, but comparable with the initial results obtained with a simpler code (Ref. 11). The results indicate an attenuation in energy that is about an order of magnitude higher than other results (Ref. 10) for the horizontal extent corresponding to $l \approx 6$. In these earlier results the amplitude (A_u) above the evanecent convection zone relate to the amplitude (A_l) by the relation: $A_u/A_l{}^{a}(x_l/x_u)^{\div l(l+1)}$ (Ref. 10, 15, 16). Here the values x_l and x_u give the radial boundaries of the convection zone. The dependence on l-value in our simulations is somewhat weaker than earlier results.

The increase of the transmission at the lowest frequency is either a general property of the wave-convection interaction or a resonance effect between the temporal and spatial structures of the convection zone and the gravity waves.

The frequency dependence shown in Fig. 1 is different from the results in Ref. 10. In that context it should be noted that since our transmission values are calculated from the horizontal kinetic energy density of the gravity waves the results are not directly comparable to the results calculated from equipartion in energy (Ref. 10). It is also not clear that the extrapolation from the gravity waves to the global g-modes can be made as simple as we have done.

Figure 1. The relative energy transmission through the convection zone (to 0.96 solar radii) for gravity waves of the given frequencies and horizontal wavenumbers equivalent to the indicated *l*-values.

4. G-MODE AMPLITUDES

The results from Ref. 1 indicate an overall energy efficiency of ≤0.1% in the generation of internal gravity waves from the motions in the convection zone. This value has been calculated as the ratio between the horizontal kinetic energy density in the gravity waves at a depth equivalent of approximately 0.6 solar radii and the total kinetic energy density at approximately 0.75 solar radii in the convection zone.

The energy transmission through the convection zone of gravity waves is of the order of 10^{-4} as shown in Fig. 1. This equivalent to an amplitude decrease to about 1% for a mode passing through the convection zone.

In order to combine the results in Ref. 1 and the transmission results presented here to predict the amplitudes of g-modes it is necessary to make some assumptions:

1. The internal gravity waves excited from the convective overshoot are converted into g-modes in stationary way, that is the wave energy generated is balanced by internal damping and leakage through the convection zone. The excitation does not have to be periodic as it is assumed that from the randomly excited gravity waves only those coincide with global modes will survive.
2. The filling factor of the downward directed plumes which dominate the excitation of gravity waves is in the two-dimensional case ≈0.1. This should convert to about 0.01 in the three dimensional case. The gravity wave excitation is caused by motions of typically 10-100 ms^{-1}, a value of 50 ms^{-1} has been used in the calculations.
3. The number of excited internal g-modes is in the region 1-1000.

Making these assumptions and using an expression for the gravity wave energy density (Ref. 7), we get the results for the horizontal velocity amplitude at a level of 0.96 solar radii as shown in Fig. 2. Depending on the assumed number of modes the horizontal velocities are in the region 0.01 to 1 mms^{-1}. It should be noted that these results depend on the excitation velocity (v) linearly and with the square root of the number of modes (N). This implies that relations between these parameters that satisfy $v*N^{-0.5}$=constant, will produce the same velocities above the convection zone.

Figure 2. The frequency dependence for modes of approximately *l*=6-7 of the horizontal velocity component at 0.96 solar radii. The upper curve assumes that only one mode is excited and the lower curve assumes that 1000 modes are excited.

The calculations presented here are for the horizontal velocities. Similar results are obtained for the vertical component, but with amplitudes about a factor of 3-5 smaller than the horizontal. This implies that for the level of accuracy discussed here the results on the horizontal wave velocity amplitude are representative of the total velocity amplitudes.

Using the same assumptions we can estimate the total amount of gravity wave energy below the convection zone to about 10^{34} ergs.

Our calculations have not at all taken into account the surface effects. In order to be directly comparable with observations this should be done at least to the level of detail done in Ref. 10. It is crucial to remember that solar global oscillations are never observed directly at the solar surface, it is always the response of the solar atmosphere to the pertubations caused by the oscillations that is filtered through the observing instrument. In the following discussion it is assumed that the velocities resulting from our calculations and shown in Fig. 2 are representative of the surface velocity amplitudes of the modes.

The surface velocity and irradiance amplitudes of g-modes have previously been calculated for assumed mode energies of about 10^{37} erg (Ref. 10). For this mode energy the lowest order g-modes should have full disc velocity amplitudes of the order of 0.5 ms^{-1} in the 100 μHz region. However, ground based measurements indicate an upper limit of ≤1 mms^{-1} (Refs. 17, 18) for l=1-3. Combining these results implies an upper limit of about 10^{33} ergs for the mode energy of a mode around 100 μHz.

The results from our study concerns higher l-values and cannot be directly compared to the observations in Ref. 17. From the results described in Ref. 10 the amplitudes of the l=1-3 modes should be 8-20 times larger than the l=6-7 mode simulated in our calculations. As mentioned previously we find a somewhat smaller l dependance for higher l-values. If this can be extrapolated to the lowest l-values we would expect a lower relative amplitude of these modes compared to our higher l calculations than in Ref. 10.

Additionally, in order to compare with observations we have to make the spatial integration over the solar surface. For a full disc spatial filtering like that used in Ref. 17 and the predominantly.horizontal amplitudes of the g-modes will emphasize the l=2, 3 and 4 more than for the p-modes. Typically the surface mode amplitudes will be reduced by factors of 2-10 for these modes when observed with full disc instruments. This is still within the uncertainties of our amplitude predictions.

For the observational comparison for higher l-values it is necessary to calculate the sensitivity functions for the relevant detector configuration (Ref. 19). This comparison has not been done in the framework of this study.

Combining the limits set by the observations (Ref. 17) with our results we find that a minimum of the order of 100 g-modes have to be excited. Otherwise the amplitude of the modes should be large enough to be seen in Ref. 17.

Our simulations do not take into account the response of the solar atmosphere on the pertubations from the oscilations, it is thus not possible to directly calculate the mode amplitude in irradiance or radiance. However, the predictions of the ratios of velocity and radiance amplitudes at the surface for integrated light measurements and l≤3 (Ref. 10) are about 10^{-5} with the velocity measured in ms^{-1}. More recent calculations from the same group (Ref. 20) indicate a ratio for an l=6 mode at 100 μHz to be about 10^{-4}. From these predicitions we estimate the surface amplitude of an l=6 mode around to be in the range $0.001\text{-}0.1*10^{-6}$. For lower l-values these amplitudes may be up to an order of magnitude higher.

5. CONCLUSIONS

We have presented new numerical simulations of gravity wave transmission through the convection zone on horizontal scales equivalent of l-values in the region 6 to 18. These results indicate an attenuation of the energy through the convection zone of the order of 10^{-4} to 10^{-3}, dependent on the l-values and frequencies of the modes.

Combined with earlier studies (Ref. 1) we have found that the energy input to generate internal gravity waves is sufficient to maintain gravity modes with at total energy below the convection zone of about 10^{34} ergs. Earlier results (Ref. 21) indicate a maximum mode energy from convective excitation of the order of 10^{26} to 10^{27} ergs. If these earlier results are correct a total number of gravity modes in the Sun would have to be more than 10^8 and each mode would have an unobservable amplitude at the surface. This discrepancy may cast doubts on the assumptions made in our study. However, it would seem very difficult to change the parameters of the simulations and other assumptions to the extent which is needed reduce the excitation to a level which will bring the results in accordance with Ref. 20.

Our results indicate that the velocity amplitude of g-modes near the surface for l-values about 6 round 100 μHz should be in the range 0.01-1 mms^{-1}. This result depends mainly on the energy input and the number of modes. The lower l-values should have amplitudes somewhat larger (factor 3-5).

We make a crude estimate the surface amplitude of an l=6 mode a to be in the range $0.001\text{-}0.1*10^{-6}$. For lower l-values these amplitudes may be an order of magnitude higher. This level will clearly be difficult to observe directly by the VIRGO investigation due to the background solar noise signal (Ref. 22).

These results are not very encouraging for the detection and classification of g-modes from the data which will be available from the helioseismic experiments on the SOHO mission. It is clear that the eventual detection will require sofisticated reduction techniques and a collaborative effort from the GOLF, MDI and VIRGO investigations.

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