AN UNBIASED AVERAGE ROTATIONAL SPLITTING FROM VIRGO/SPM ?

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ABSTRACT

Some methods for fitting Lorentz profiles to oscillation power spectra for the purposes of measuring degeneracy splitting are systematically biased. To be specific, rotational splitting is likely to be overestimated. The reason is that the maximum-likelihood methods that are sometimes employed incorporate an inappropriate statistical model. We present arguments for a preferred procedure, and confirm using artificial data that the outcome of using it gives a better representation of the actual splitting. We apply the analysis to SPM data from VIRGO, and report on the value of the mean rotational splitting, averaged over p modes of low degree.

Keywords: solar rotation - p modes - maximum likelihood

1. Introduction

The knowledge of the internal rotation of the Sun is of prime importance for understanding the Sun's internal dynamics, and how, in particular, the magnetohydrodynamical phenomena regulate the solar cycle. The internal rotation has been measured down to $0.3R_\odot$ (e.g. Tomczyk *et al.*, 1996) from the splitting of solar p modes. Unfortunately, below $0.3R_\odot$ the sensitivity of even the low-degree p modes to the rotation of the core is very weak. For example, increasing by 100 nHz the rotation below $0.3R_\odot$ would increase the splitting of an $l = 1$ mode by no more than about 10 nHz. With such weak sensitivity, the effect of systematic errors on the measurement of the low-degree splittings becomes important.

Sources of systematic error are the following:

- stochastic nature of the excitation of the modes,
- mode blending,
- error in the statistical distribution of the p modes.

The first phenomenon can lead to misidentification of the components of a multiplet. The second prevents one from distinguishing components of a multiplet at frequencies higher than about 2000 μHz. Both affect more the integrated sunlight instruments than the imaging instruments (the Luminosity Oscillations Imager (LOI) on VIRGO and SOI/MDI are capable of detecting the individual components of a multiplet). Of course, at frequencies lower than 2000 μHz, the integrated solar velocity instruments can do as well as the imaging instruments. The third effect is the result of incorrect statistical assumptions, or inaccurate modeling of the physics of the mode. An example is the neglect of the correlation between the modes (Chang *et al.*, 1997, in these proceedings). In this paper, we report on a preliminary study of these effects for the $l = 1$ modes measured by the sunphotometers (SPM) of VIRGO; in addition, we also use data from the LOI to estimate the magnitude of the bias. In the first section, we describe how the modes were fitted. In the second section, we discuss the results of a Monte-Carlo simulation for interpreting how the bias builds up. We conclude by giving some advice on a better way to fit the data.

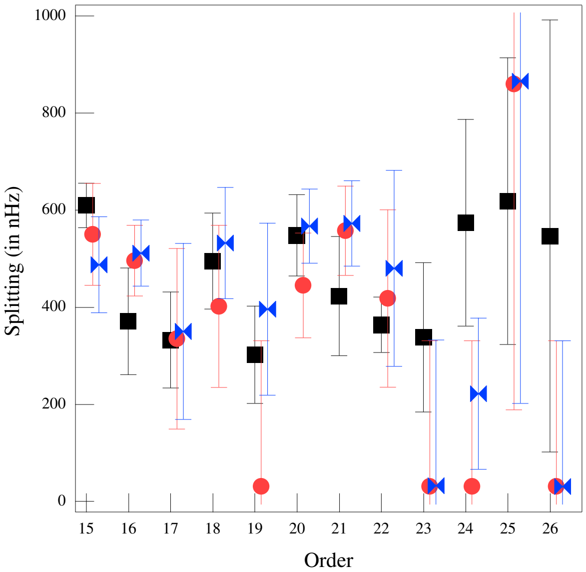

Figure 1: $l = 1$ sidereal splitting for the SPM, assuming the same amplitude (full circles), different amplitudes (bow ties); and for the LOI (full squares). The weighted averages are respectively: 466 ± 39 nHz, 486 ± 33 nHz and 470 ± 25 nHz.

2. Fitting of VIRGO/SPM spectra

The SPM and LOI instruments of VIRGO, aboard SOHO, have been described by Fröhlich *et al.*, 1996. The former measures the spectral irradiance of the Sun as a star at 3 wavelengths (402 nm, 500 nm, 862 nm). It is sensitive mainly to low-degree modes ($l \leq 3$). The latter measures the radiance in 12 pixels at 500 nm. It is also sensitive to low-degree modes ($l \leq 10$), but unlike SPM it can separate the $2l + 1$ components of a (l, n) multiplet. The data analysis is described by Fröhlich *et al.*, 1997. The time series analyzed here extends from 27 March 1996 to 10 August 1996 (137 days). The SPM power spectra have been analyzed by fitting Lorentzian profiles using maximum-likelihood estimates. For the $l = 1$ modes, we used two strategies for fitting the SPM spectra:

- The $m = -1$ and $m = +1$ modes are assumed to have the same amplitude,
- The $m = -1$ and $m = +1$ modes are assumed to have different amplitudes.

The first strategy was motivated by the fact that the expectations of the amplitudes to which the two components are excited are identical; the second stems from noticing that in any single real spectrum the amplitudes are actually different. Only if one were to average many spectra would one expect to see both components with the same amplitudes. For the LOI data, we fitted the full (m, ν) diagram of the $l = 1$ power spectra also using maximum-likelihood estimates as described by Appourchaux et al, 1997; the amplitudes of the 3 components were assumed to be different. Fig. 1 shows the splitting as a function of order n for the 3 fits. One can see that the splittings measured by the LOI and the SPM are qualitatively in agreement. One can note that the splittings obtained by fitting Lorentzians with the same amplitude are systematically lower than those with different amplitudes. This bias shows up in the weighted averages, and results in a 20 nHz difference for the SPM data. It should be noted that the LOI value is closer to the SPM value with equal amplitudes. Since the LOI values are intrinsically less biased, we infer that the fitting of the modes to Lorentzians having different amplitudes is inappropriate.

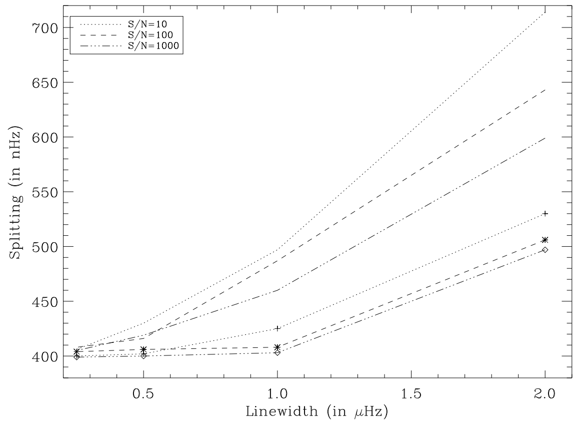

Figure 2: Monte-Carlo simulation of an $l = 1$ splitting as a function of the linewidth and for different signal-to-noise ratios. The observation time is 165 days. The curves with symbols and without symbols are respectively from the fits assuming the same amplitude and different amplitudes.

3. Monte-Carlo simulation

In order to confirm that bias does actually arise, we performed Monte-Carlo simulations. The power spectrum of an $l = 1$ mode was generated assuming that the probability distribution was χ^2 with 2 degrees of freedom, and that both doublets have the same amplitude. We performed simulations with 3 signal-to-noise ratios (1000.,100.,10.), 4 linewidths (2 μHz, 1 μHz, 0.5 μHz, 0.25 μHz), one splitting of 0.4 μHz, and 2 observation times (165 days and 660 days). For each set of parameters, we performed 300 simulations, and studied the probability distribution of the log of the amplitude, linewidth and noise, and of the mean frequency and splitting. For each spectrum, we fitted the doublet over a window of 28 μHz assuming both equal and unequal amplitudes. Fig. 2 shows the splitting inferred from the simulations. It is quite clear that fitting with the same amplitude biases less than when we fit with different amplitudes. For an observation time of 660 days, the biases are reduced by a factor of about 4, i.e. as the ratio of the observation times.

We must point out that the bias does not arise from bias in the statistical fluctuations of parameters, but more likely from the influence of fluctuations in the power. Error estimates increase by about a factor 2 to 3 when fitting different amplitudes. This higher dispersion appears to increase artificially the distance between the 2 components, thereby increasing the apparent splitting. Indeed, we note that the splitting is also overestimated when the linewidth is large, even when fitting with the same amplitude; this is a mode blending effect. All these results are based on empirical observation rather than on pure analysis; but evidently they result from the nonlinearity of the maximum-likelihood estimates.

4. Discussion and conclusion

Use of an inappropriate statistical model is likely to result in bias. We should point out that there is a statistical test allowing one to check for the significance of additional parameters in the fit (Appourchaux *et al.*, 1995); this test should be used more frequently for testing the reliability of the inferences. That test reveals, even under circumstances in which a parameter of the model should best be changed, when the data cannot be used to determine how that change should be effected. For example, we carried out additional numerical experiments in which the mean amplitudes of the two modes differed in actuality by a factor 2. When the linewidth was as large as the doublet separation, the test of significance of the additional parameter assigning the ratio of the amplitudes was negative. Under such circumstances, one can only resort to a theoretical assignment of the parameter. For this reason it is extremely important that the statistical model be as realistic as is practicable, and that the physics of that model be investigated as thoroughly as possible to confirm its validity.

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