Astron. Astrophys. 312, 610–614 (1996)

ASTRONOMY
AND
ASTROPHYSICS

Theoretical amplitudes of solar g-modes

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Received 6 November 1995 / Accepted 30 January 1996

Abstract. This work presents predictions on the amplitudes of solar g-modes. It is based on the assumption that convective overshoot into the convectively stable interior generates gravity waves. Based on this assumption it is possible to estimate values for the amplitude of the g-modes below the solar convection zone. Numerical simulations, giving the transmission of gravity waves through the convection zone have been carried out for 5 different frequencies and horizontal extents equivalent to ℓ-values approximately in the range 6-18. The horizontal velocity amplitude of solar g-modes near the solar surface is estimated to be in the range $0.01 - 5$ mms^{-1}. The largest value assumes that all the available energy from the excitation goes into a single mode. Using assumptions on the ratio between intensity and velocity oscillations the amplitude in intensity of a $\ell = 6$ mode is estimated to be $\leq 5 \cdot 10^{-7}$.

Key words: helioseismology – gravity waves – convection – g-modes

1. Introduction

Several attempts have been made to directly identify solar global g-modes from ground and space based observations (Hill et al., 1991 and references therein). Despite these attempts no repetitive unambiguous mode identification and classification has been made.

The negative results may be caused by the small surface amplitude of the modes, by the noise caused by the terrestrial atmosphere, by the diurnal effects caused by gaps in ground based observations and by the presence of a large solar background signal caused by the evolution and rotation of solar surface structures.

Stability calculations indicate that no more than a few, if any, of the modes are overstable in the current Sun (Cox et al., 1991 and references therein). This implies that other mechanisms must be responsible for the excitation if the internal g-modes exist. The idea that the penetration of convective matter from the solar convection zone into the convectively stable solar interior may generate internal gravity waves has been discussed by several authors (Press, 1981; Gough, 1985, Hurlburt et al., 1986; Andersen, 1994).

The presence of gravity waves or g-modes trapped in the stable region beneath the solar convection zone does not necessarily imply that these are observable at the solar surface. The signatures of the g-modes must penetrate the convection zone where these modes or waves are evanescent. Several studies have looked into the transmission of g-modes or gravity waves through the solar convection zone using different methods and strategies (Christensen-Dalsgaard, 1980; Berthomieu & Provost, 1990; Andreassen et al., 1992; Andersen et al., 1993; Toutain et al., 1995). The studies in Berthomieu & Provost (1990) used a mixing length approach for the convection zone and included the effects of the solar atmosphere. The preliminary results in Andreassen et al. (1992) treated the hydrodynamical interaction between the gravity waves and convection consistently, but did not include the effects of the regions close to and in the atmosphere.

This study is an extension of the work presented by Andreassen et al. (1992). Here we present new extended numerical simulations describing the ℓ-and frequency dependence of the transmission of gravity waves through the convection zone, but still without a proper treatment of the atmosphere. These simulations are combined with the results of the convective excitation efficiency of internal gravity waves (Andersen, 1994) to give an estimate of the surface amplitude of solar g-modes.

2. Numerical method

The numerical code used in this study is two- dimensional and use pseudospectral methods on the fully compressible problem. The strength of the code is that it treats the wave mechanics in a consistent way. The upper and lower boundaries are open, thus transmitting the wave energy created in the model.

The turbulent cascade in the simulations is balanced by the use of a numerical spectral viscosity. Combined with the current treatment of the radiative flux this maintains the thermal driving of the convection zone and thus allows the simulations to be carried out for a long time. The details and developments of the method and code are described earlier (Andreassen et al., 1992; Andersen et al., 1993; Andersen, 1994).

The equations of mass, momentum and energy may be expressed as:

$$\rho_t + \rho(u_x + v_z) + u\rho_x + v\rho_z \quad = 0 \tag{1}$$

$$u_t + uu_x + vu_z(1/\rho)p_x \quad = \Pi_{xx} + \Pi_{xz} \tag{2}$$

$$v_t + uv_x + vv_z(1/\rho)p_z + \rho g \quad = \Pi_{zx} + \Pi_{zz} \tag{3}$$

$$p_t + up_x + vp_z + \gamma p(u_x + v_z) = \Psi \tag{4}$$

Here the t, x and z indices indicate derivation in time and the horizontal and vertical directions, respectively. The horizontal and vertical velocity components are given by u and v. The right hand terms are diffusion terms including a term describing the background heat flux. The functional forms of the diffusion terms used in the current calculations are

$$\Pi_{ij} = \nu_0 \varpi_i(z) \frac{\partial}{\partial x_j} \left(\nu(x_j) * \frac{\partial v_i}{\partial x_j} \right) \tag{5}$$

for the momentum equations and

$$\Psi = \epsilon_0 \varpi_i(z) \frac{\partial}{\partial x_j} \left(\nu(x_j) * \frac{\partial T}{\partial x_j} \right) + \kappa(z) \frac{\partial^2 T_0}{\partial z^2} \tag{6}$$

for the heat equation, where the indices i and j indicate the x and z directions in spectral space; here $*$ denote convolution and ϖ is the Chebyshev norm which gives uniform truncations across the nonuniform vertical mesh. The parameters ν_0 and ϵ_0 depend on the values of the spectral viscosity parameters and $\nu(x_j)$ gives the functional form of spectral viscosity. The parameter κ gives the total thermal conductivity as function of depth.

The solar model is the same as used in Andersen (1994) and is adapted from Christensen-Dalsgaard (1991). We use a plane parallel approximation extending from 0.5 to 1 solar radii with a width equivalent of 0.3 solar radii. The upper part of the model (from 0.9 to 1.0) has been modified to be subadiabatic in order to study the transmission of gravity waves through the major part of the convection zone.

Initially the static model is randomly perturbed in density ($\leq$ 0.1%) within the region corresponding to the solar convection zone. In Andersen (1994) the simulation was carried out for 1100 timesteps, each timestep being the sound travel time of one scale height at the bottom of the model. Transformed to solar conditions this is equivalent to about 240 seconds. After about 200 timesteps a quasi-stationary convective like pattern is generated with typical velocities of $\leq 0.5\%$ of the sound speed. The results in Andersen (1994) indicate that the major part of the kinetic energy is carried in a few downward directed plumes spanning several scaleheights. This is in accordance with other simulation results (Hurlburt et al., 1986) and studies based on the entropy balance (Ginet, 1994). The effective filling factor of the downward plumes in our two dimensional simulations is about 0.1.

The calculations in this study start at timestep 400 in the convection zone model developed in the simulations of Andersen (1994). Below the convection zone the model is equal to the initial static model. Each simulation run has a length of 350 timesteps and a gravity wave of known amplitude and horizontal wavenumber is numerically generated below the convection zone. To excite this gravity wave a force f is applied to the vertical component of the equation of motion, this force has the form described below and is similar to the one given in Andreassen et al. (1992):

$$f(x, z, t) = \epsilon \cdot e^{-(z-\delta)^2/\sigma^2} \cdot e^{i(\omega t - \kappa x)} \tag{7}$$

Here the parameter ϵ gives the amplitude of the force, in the current calculations $\epsilon = 0.02$. The parameters δ and σ give the position and the width of the exciting force, respectively. The position parameter gives a peak excitation at a depth equivalent of 0.55 solar radii and the width is half of the scaleheight at 0.5 solar radii. The values of ω and κ determine the period and horizontal extent of the forcing function. In the start-up of this generation a spectrum of waves are generated, travelling with speeds ranging from the gravity wave propagation up to the sound speed. After a few periods the characteristics of a clean single gravity wave is manifested above the generating region.

The horizontal extents of the generated waves are equivalent to ℓ-values of approximately 6, 12 and 18, respectively, at the bottom of the convection zone. For the lowest ℓ-value waves of frequencies 53, 77, 104, 166 and 277 μHz were calculated. For the two other ℓ-values only the three highest frequencies were calculated.

The calculations were carried out with a very efficient parallel code using four processors on the Cray YMP supercomputer in Trondheim, Norway. For the 11 simulations carried out in connection with this work a total of about 150 hours of computing time were needed.

It must be noted that we have made no attempts to determine specific radial wave functions for the "g-modes" discussed here. This implies that the explicit effects of the radial nodes of g-modes have not been studied. In a standard solar model like the one used in this work g-modes with $\ell = 6$ and the selected frequencies would have radial order n-values of about 7, 11, 16, 27 and 48, respectively.

3. Gravity wave transmission

The spatial and temporal distributions of the horizontal component of the kinetic energy density at different depths are used as tracers for the gravity waves. To calculate the transmission through the convection zone the value of this kinetic energy density is compared at depths equivalent to 0.6 and 0.96 solar radii. For each of the two depths a two-dimensional array is generated containing the values of the kinetic energy density as function of horizontal scale and time. The time interval used is from 400 to 750 timesteps. For the mode frequencies used in this study this implies that about five periods of the lowest frequency (53 μHz) are measured. For the highest frequency (277 μHz) this number is about 23.

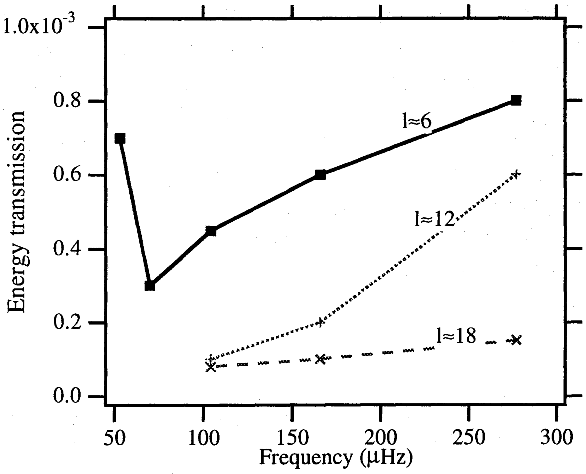

Fig. 1. The relative energy transmission through the convection zone (to 0.96 solar radii) for gravity waves of the given frequencies and equivalent values of the spherical harmonics.

The two-dimensional cross-spectra of these two arrays in time and horizontal extent are calculated using standard FFT techniques. The value of the energy transmission is calculated as the gain between the spectra at these depths. The averaging of the Fourier components required to calculate the gain is done over 5 points. Tests with 3 and 7 points average did not give significantly different results. Only at the specific frequency and horizontal extent of the induced gravity wave are both the gain and the cross-correlation values significantly above the noise. For the 53 μHz and the 277 μHz gravity waves the signal to noise ratios are about 3 and 8, respectively. This method is a modification of the method used by Andreassen et al. (1992). It is an improvement because by using both the temporal and spatial variations simultaneously it is ensured that the waves observed in the upper part of the model have the same properties as the waves numerically induced below the convection zone.

The transmission results from these calculations are shown in Fig. 1. A clear dependence on both frequency and ℓ-value can be seen. For a horizontal extent corresponding to $\ell \approx 6$ these results indicate an attenuation in energy that is nearly an order of magnitude higher than previous results. Both the simple estimation made by Christensen-Dalsgaard (1980) and the results for an adiabatically stratified convection zone (Berthomieu and Provost, 1990) follow a form:

$$log(A_l/A_s) \sim \sqrt{\ell(\ell+1)} \qquad (8)$$

Where A_l and A_s are the values below the convection zone and at the surface, respectively. This implies a significantly stronger dependence on ℓ than the values we have calculated here.

The increased attenuation observed in our calculations are most probably caused by the interaction between the gravity waves and structures in the convection zone having comparable horizontal spatial scales. The much weaker dependence on ℓ for high ℓ-values found in our simulations may have the same cause or may be caused by our incorrect treatment of the surface boundary layer.

The increase of the transmission at the lowest frequency is either a general property of the wave- convection interaction or a resonance effect between the temporal and spatial structures of the convection zone and the gravity waves. This effect cannot be further studied in detail with the available set of simulations.

However, it should be noted that the data for the lowest frequency have the lowest signal to noise ratio in the cross-correlations, making this data point the least reliable one. The uncertainty is not large enough to remove the significance of the increased transmission.

4. G-mode amplitudes

The results from Andersen (1994) indicate an overall energy efficiency of $\leq 10^{-3}$ in the generation of internal gravity waves from the motions in the convection zone. This value was calculated as the ratio between the horizontal kinetic energy density in the gravity waves at a depth equivalent of approximately 0.6 solar radii and the total kinetic energy density at approximately 0.75 solar radii in the convection zone. The rationale for this choice was somewhat arbitrary. However, since there is a clear correlation between the kinetic energy in the different parts of the lower convection zone the choice of reference point only influences the scaling factor. The energy transmission through the convection zone of gravity waves is of the order of 10^{-4} to 10^{-3}, depending on degree and frequency, as shown in Fig. 1. We have attempted to combine these results to give an estimate of the surface amplitude of g-modes. In order to do this a number of assumptions have to be made, some of which clearly are uncertain.

Two simple and slightly different approaches to the estimation of the surface amplitudes can be made, either by overall energy considerations or by assuming values for the velocity of the elements in the convection zone producing the excitation of the internal gravity waves. Both approaches are related and have the following common assumptions:

a) The internal gravity waves excited from the convective overshoot are converted into g-modes in a stationary manner, implying that the wave energy generated is balanced by internal damping and leakage through the convection zone.

b) The filling factor of the downward directed plumes which dominate the excitation of gravity waves is about 0.1 in the two-dimensional case. This implies a value of about 0.01 in the three dimensional case.

c) The number of excited internal g-modes is in the region 1–1000.

For the simple energy considerations it is assumed that the total kinetic energy content in the convection zone is in the region $10^{38} - 10^{39}$ erg (Cox et al, 1991). Using an excitation efficiency of 10^{-3} and the plume filling factor of 0.01 gives a total upper limit of energy in the internal g-modes of $10^{33} - 10^{34}$ erg.

Scaling the numerical calculations presented here and in Andersen (1994) to solar values indicates that the motions causing the wave excitation typically have values of 50 ms^{-1}. This value together with the parameters of the solar model and using an expression for the gravity wave energy density ($\approx \rho(N/\omega)^2 v^2$) (Press, 1981), we also get an upper limit of the energy in the g-modes of about 10^{34} erg.

Combining these results for the energy available to the internal g-modes with the mode attenuation results shown in Fig. 1 we can estimate the horizontal velocity amplitude at a level of 0.96 solar radii. The results for ℓ=6 modes are shown in Fig. 2. Depending on the assumed number of modes the horizontal velocities are in the region 0.01 to 5 mms^{-1}. The upper curve assumes that only a single g-mode is excited and the lower curve assumes that 1000 modes are excited.

The calculations presented here are for the horizontal velocities. Similar results are obtained for the vertical component, but with amplitudes about a factor of 3-5 smaller than the horizontal for the lowest frequencies. For the higher frequencies the horizontal and vertical velocities are comparable. This implies that for the level of accuracy discussed here the results on the horizontal wave velocity amplitude are representative of the total velocity amplitudes within a factor of two.

Our calculations have not at all taken into account the surface effects. In order to be directly comparable with observations this should be done at least to the level of detail done in Berthomieu & Provost (1990) and Toutain et al. (1995). In the following discussion it is assumed that the velocities resulting from our calculations and shown in Fig. 2 are representative of the surface velocity amplitudes of the modes.

Ground based measurements indicate an upper limit of about 1 mms^{-1} at 100 μHz (Jimenéz et al., 1988; Pallé et al., 1995, Elsworth et al., 1994) for $\ell = 1-3$. The results from our study concerns higher ℓ-values and cannot be directly compared to these observations.

From the results described in Berthomieu and Provost (1990) the amplitudes of the $\ell = 1-3$ modes should be 8-20 times larger than the $\ell = 6-7$ mode simulated in our calculations. However, as described in the previous section we find a smaller ℓ dependence for these higher ℓ-values than Berthomieu and Provost (1990). If this ℓ dependency can be extrapolated to the lowest $\ell = 1-3$ values we would expect a lower relative amplitude of these modes compared to our higher ℓ calculations than in Berthomieu & Provost (1990) or Toutain et al. (1995). This conclusion would imply that for a given mode energy the $\ell = 1-3$ modes should have a surface amplitude more than a factor of 10 smaller than predicted by Berthomieu & Provost (1990) or Toutain et al. (1995). The reason for this possible discrepancy is not understood and should be studied further.

If the limits set by the velocity observations are descriptive for the ℓ-values we have calculated they should imply that a minimum of 10-100 g-modes have to be excited to keep the amplitudes below the detection limit set by the observations.

Our simulations do not take into account the response of the solar atmosphere on the perturbations from the oscillations, it is thus not possible to directly calculate the mode amplitude in irradiance or radiance. However, the predictions of the ratios of relative radiance and velocity (in mms^{-1}) amplitudes at the surface for an $\ell = 6$ mode at 100μHz to be about 10^{-7} (Toutain et al., 1995). From these predictions we estimate, depending on the number of modes excited that the surface amplitude in intensity of an $\ell = 6$ mode around 100μHz is in the range $10^{-9} - 0.5 \cdot 10^{-6}$.

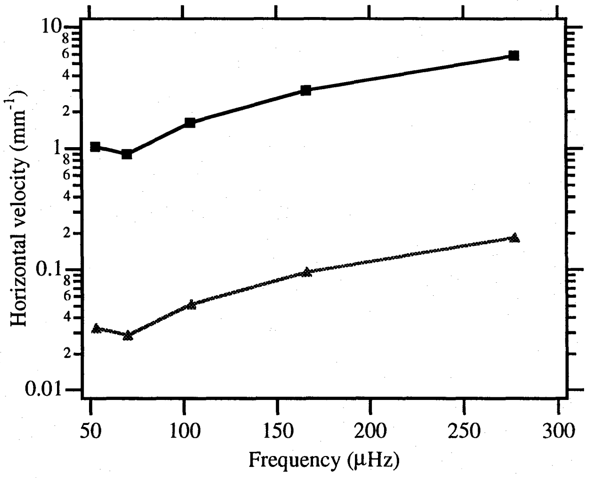

Fig. 2. The frequency dependence for modes of degree 6 of the horizontal velocity component at 0.96 solar radii. The upper and lower curves assume that 1 and 1000 modes are excited, respectively.

5. Conclusions

We have presented new numerical simulations of gravity wave transmission through the convection zone on horizontal scales equivalent of ℓ-values in the region 6 to 18. These results indicate an attenuation of the energy through the convection zone of the order of 10^{-4} to 10^{-3}, depending on both the ℓ-values and frequencies of the modes. Combined with earlier studies (Andersen, 1994) we have found that the energy input to generate internal gravity waves is sufficient to maintain gravity modes with at total energy below the convection zone of about 10^{34} ergs.

Our results indicate that the velocity amplitude of g-modes near the surface for ℓ-values about 6 at about 100 μHz should be in the range 0.01-5 mms^{-1}. This result depends mainly on the energy input and the number of modes. The lower ℓ-values should have amplitudes somewhat larger (factor $3-10$). These results are comparable to the estimates of a few mms^{-1} made by Gough (1985) for low degree and low order g-modes. He also assumed that convection was the excitation source but used a completely different approach. However, he did not carry out the full hydrodynamical wave simulations as we have done.

We estimate the intensity surface amplitude of an $\ell = 6$ mode a to be in the range $10^{-9} - 0.5 \cdot 10^{-6}$. For lower ℓ-values the ratio of the intensity to velocity signal is nearly two orders of magnitude smaller (Toutain et al. 1995). This implies that, even if the overall perturbation in velocity is larger at lower ℓ-values the amplitudes in intensity may be an order of magnitude smaller than for the $\ell = 6$ mode presented here.

Our results also indicate that the highest frequency g-modes should have the highest amplitude. But the low amplitude level indicated from our results, even for these high frequency modes, will make them difficult to observe either with velocity or irradiance and radiance measurements due to the background solar noise signal.

The measurements to be done by the VIRGO (Fröhlich and Andersen, 1995), GOLF (Gabriel, 1995) and SOI (Scherrer, 1995) investigations on the imminent ESA/NASA SOHO mission will all attempt to observe g-modes.

Both the GOLF and SOI investigations will primarily observe velocity oscillations. SOI will also observe an intensity signal, but this may be slightly contaminated by velocity effects. If the g-modes have surface velocities near the maximum level of what we have estimated (5 mms^{-1}), the modes with $\ell \leq 3$ should marginally be observed with GOLF after about one year. The SOI has sensitivity to higher ℓ-values, but it is not known if the instrument stability is good enough to observe these low frequency modes.

We have carried out tests with coherent $\ell = 1$ g-modes with amplitudes of $4 \cdot 10^{-7}$. The noise background has been estimated using the simulations from Andersen et al. (1994). These modes are marginally detectable with VIRGO, without prior knowledge of the frequencies, with an uninterrupted observation series of 1.5 years.

From these results it should be clear that the probability of finding the lowest degree g-modes with the VIRGO experiment may be small. If g-modes with ℓ up to about 6 are excited the probability is larger for detecting them with the imaging LOI instrument than in velocity measurements.

These results are not very encouraging for the detection and classification of g-modes from the data which will be available from the helioseismic experiments on the SOHO mission. It is clear that the eventual detection will require sophisticated reduction techniques using all the available data from SOHO.

Acknowledgements. The author would like to thank Ø. Andreassen and C.E. Wassberg for the development of the basic code on which all of the current simulations are based. The computing time was provided from a grant of "Tungregneutvalget" in the Research Council of Norway. In addition the referee should be thanked for commenting unclear points in the presentation.

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