Report of the Hare-and-Hound exercise #3 (HH3) The case of HD57006

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Abstract

In this report we present the analysis done to the synthetic spectra of H57006. For this evolved star, we could not identify the modes and extract their frequencies and splittings with their formal errors in the same way as for a solar like star. For this reason, this star revealed to be very difficult from the data analysis point of view.

The Echelle diagram only permitted us to identify the $l = 0$ modes, but the fitting could reliably be made only below 500μHz. The understanding of the high-order-mode behaviour was rather impossible; we were restricted to the low order analysis.

For the study of long-lived modes, we had to devise a technique that could distinguish narrow peaks ('g-mode like') among the wider modes ('p-mode like').

1 Introduction

In the context of the Hare-and-Hound exercises, HD57006 is one of the targets chosen to be studied in terms of its seismology. It is a post main sequence star with about 1.6 $M_\odot$, an evolved star with a contracting core. Because of that, it is very complex in terms of its frequencies, and rise some doubts about our ability to identify the modes in such stars.

The oscillation frequencies are influenced by the sound speed, c, and the Brunt-Väisäla frequency, N (Tassoul, 1980; Audard and Provost,1994). As both depend on the mean molecular weight, μ, (hence on the central hydrogen abundance, X_c) they don't change very much during the main sequence. However, when the main sequence phase of stellar evolution ends, X_c is exhausted and the star settles in a state in which the hydrogen burns in a shell surrounding a helium core. The mass of the helium core is increased by the hydrogen burning in the shell leading to an expansion of the envelope of the star. Such stellar interior transformation strongly affects μ, consequently the oscillation frequencies.

The modes are trapped in propagation zones, whose extension and location depends on variation of N and on the Lamb frequency $S_l = Lc/r$, with $L^2 = l(l+1)$. The propagation of p modes happens when the mode frequency is greater than N and S_l, while the propagation of g modes happens when the frequency of the mode is smaller than N and S_l. At the beginning of the evolution N is relatively small in stellar interior and varies smoothly with radius, therefore there is a well-defined separation between the modes whose eigenfunctions have nodes in acoustic propagation zone and those whose eigenfunctions have nodes in the gravity propagation zone. As the evolution proceeds a strong gradient of chemical composition develops at the convective core frontier, giving rise to a large peak in N close to the frontier changing the structure of the mode propagation zones. As a result the eigenfunctions of g modes of low radial order develop nodes in the acoustic propagation zone, while eigenfunctions of the low order p modes develop nodes inside the zone of the peak of N. These modes have a mixed character, they behave as g modes close to the core frontier and as a p modes in outer regions of the star (Dziembowski and Pamyatnykh, 1991; Audard, Provost and Christensen-Dalsgaard, 1995).

In this report, we describe an attempt of identification of the modes and extraction of the frequencies of a star that has the terrifying scenario of such mixed modes.

2 Data Analysis

One fundamental quantity in seismology is the large separation, $\Delta\nu_0$. It represents nearly the uniform separation between the two modes of same degree and successive order ($\Delta\nu_0 \approx \nu_{n,l} - \nu_{n-1,l}$) for p modes of high radial order and low degree. The frequency of these modes can be described using an analytical asymptotic way (Tassoul, 1980). The $\Delta\nu_0$ is proportional to the characteristic frequency $\Omega_g = \sqrt{GM/R^3}$ therefore it can be a measure of the mean density of the star, and hence of its mass and radius if g is known. The $\Delta\nu_0$ is also proportional to the inverse of the acoustic radius expressed in seconds.

Useful information can also be obtained from the small frequency separation that measures the departures from the uniformity. It corresponds to the difference between two modes with same $n + \frac{l}{2}$ and is defined as $d_{n,l} = \nu_{n,l} - \nu_{n-1,l}$. This quantity is sensitive to conditions in the structure of the core and to chemical composition and therefore can be an age indicator (Audard and Provost, 1994).

Another quantity that can be derived from the oscillation frequencies is the second frequency difference $\delta_2\nu = \nu_{n,l} + 2\nu_{n-1,l} + \nu_{n-2,l}$ that has an oscillatory behaviour. It is related with the rapid variation of the adiabatic exponent due to the HeII ionization, therefore this quantity may provide a diagnostic to determine the helium abundance (Audard and Provost, 1994).

2.1 Echelle Diagram

The method used to detect the large separation, $\Delta\nu_0$, is the Echelle diagram. The Echelle diagram consists in cutting the spectrum into pieces of width $\Delta\nu_0$ and pile them up atop of each other. If $\Delta\nu_0$ is the correct one, "vertical" ridges will appear clearly in the diagram.

In the case of HD57006 "clearly" is not the best word to describe it. The distribution of the modes in the Echelle diagram in the Figure 1 does not permit to take any conclusion about any l but l=0, although for high orders it starts to be messy.

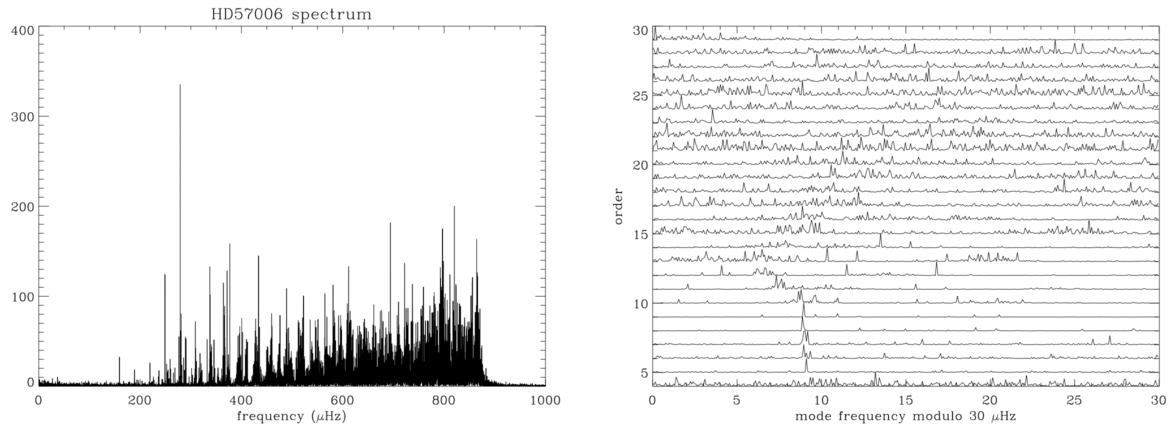


Figure 1: On the left hand side is the spectrum of the star HD57006, and on the right hand side the Echelle diagram with $\Delta\nu_0$=30μHz.

Because of the oddness of the HD57006 Echelle diagram, additional theoretical information about this star was necessary. Two sets of frequencies were provided by Ian Roxburgh and by Mário Monteiro with the collaboration of João Marques.

As one can see on Figure 2, l=1 in both sets of frequencies behaves almost chaotic. After the 15^{th} order, $l = 2$ starts to have the familiar behaviour next to the $l = 0$ on the left hand side, as predicted by the asymptotic theory. However if one looks again at the Echelle diagram in Figure 1, it is very hard, if not impossible to recognize or to guess its location, therefore it wasn't possible to fit it. We only fitted $l = 0$, the results can be seen in Figure 3 and in Table 2.1.

Above 500μHz, since the $l = 0$ and $l = 2$ modes are very close to each other and apparently very wide, it is difficult to affirm that only $l = 0$ is fitted, because it is possible that some $l = 2$ were fitted with or

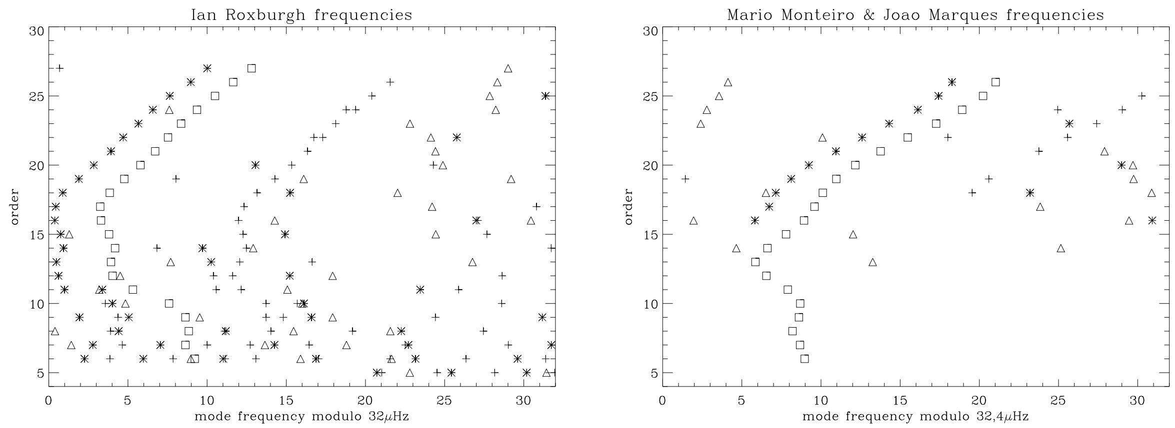


Figure 2: The Echelle diagram of the set of frequency given by the two stellar models of the star HD57006. l=0 is represented by a **square**, l=1 by a **triangle**, l=2 by an **asterisk** and l=3 by a **plus** sign.

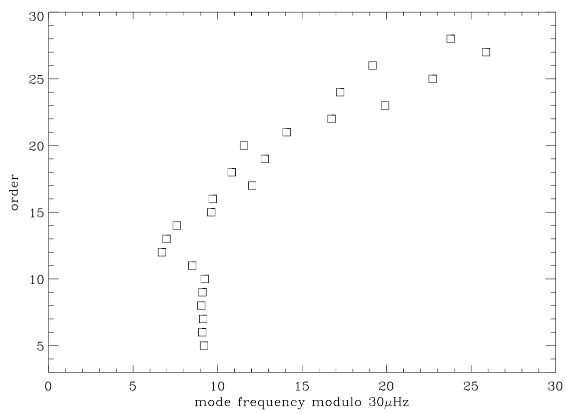


Figure 3: The result of the l=0 mode fitting. For high orders (n>15) the fit is rather erratic. Since the star spectrum is very messy, at least after n>15, the l=2 modes are sufficiently close to that of the l=0 modes (as it can be seen in the Echelle diagrams of the stellar models) for being mixed with the l=0 . It is possible that some l = 2 are among the fitted l=0.

instead of $l = 0$. Therefore we only present in Table 2.1 the frequencies fitted for $l = 0$ below 580 μHz.

$l = 0$
159.19391±0.038500000
189.08200±0.044807799
219.14301±0.036794599
249.03085±0.013279705
279.10117±0.038500000
309.23514±0.099702701
338.49667±0.14239374
366.45700±0.17383800
396.97388±0.40610036
427.58200±0.28380781
459.62387±0.15837938
490.79501±0.81077600
522.04315±0.44192797
550.83026±6.6242681
582.78986±6.8992927e-06

Table 1: $l = 0$ mode frequencies and their formal errors in μHz as determined by maximum likelihood estimators. The errors determined for the frequency 582μHz seems to be way too low to be reliable, this identified mode belongs to the high-order mess that can be observed in the echelle diagram. This is the reason why we concentrated our attention on frequencies below 500μHz.

2.2 Mode identification challenge

The main goal in asteroseismology is to derive from the oscillation frequencies of the modes, the internal structure and rotation of the stars. With l=0 only we can measure $\Delta\nu_0$, the information that can be derived is the acoustic radius of the star. In order to know the internal structure and the rotational of HD57006 it is necessary to extract the frequencies of the other modes.

For the high orders, $n > 15$, the spectrum seems too messy to be able to understand it and extract useful information from it. The existence of large power narrow peaks spread along the lower part of the Echelle diagram $n < 15$ can be noticed. These peaks may lead us to additional information about the internal structure of the star. We believe that we may learn something from their frequency extraction. We concentrated our attention to the frequency range below the 500 μHz.

Analyzing the power spectrum below 500 μHz, one can notice two distinct features:

- the isolated narrow peaks and
- a bunch of peaks close to each other.

Two bunches of peaks can be well noticed on the left hand side of the Echelle diagram (Figure 1) within the 13^{th} and 15^{th} orders. These two kind of features can not correspond to the same physics, since that the isolated narrow peaks are probably modes which their energy fall only into one bin, therefore it can correspond to long-lived modes, while the bunches of peaks may correspond just to one broad mode, and it can correspond to short-lived modes.

To extract the frequencies of the isolated narrow peaks we need to assess whether they may be due to noise. In some cases they can be embedded in the bunches of peaks that are supposed to be short-lived modes. Therefore we also need to assess whether we can detect and distinguish them among the broad modes.

2.2.1 Long-lived modes detection

Narrow peaks alone

If one considered a pure noise signal with a χ^2 statistical distribution, the probability that the power within one bin is greater than m times the mean of the noise power, σ, is:

$$\mathcal{P}(m) = e^{-m} \tag{1}$$

For a frequency band containing N independent bins, the probability that there aren't any bins with power greater than m becomes:

$$\mathcal{P}_N(m) = (1 - e^{-m})^N \tag{2}$$

Therefore the probability that at least one bin has a power greater than m becomes:

$$\mathcal{P}_N(m) = 1 - (1 - e^{-m})^N \tag{3}$$

then if $e^{-m} \ll 1$ the equation (3) can be approximated to:

$$\mathcal{P}_N(m) = Ne^{-m} \tag{4}$$

Setting a given value for $\mathcal{P}_N(m)$, for instance 10% (which means 10% probability that one bin above m is due to the noise), choosing a window range in our spectrum which contains N bins, and estimating σ, one can derive using the equation (4) the correct value for m. This way, we can create a statistical test that can detect the bins that can be considered as not being due to the noise (see also Appourchaux et al., 2000; Gabriel et al.,2002).

Below 300 μHz there aren't any broad modes and we applied the test directly to $P(\nu)$ we filtered out the bins corresponding to the identified $l = 0$.

Narrow peaks embedded in short-lived modes

Above 300 μHz the peaks that we wanted to analyze are among broad modes. Therefore, to apply the aforementionned test directly to the power spectrum is not very useful because it detects bins that are part of a single broad mode, in this case we can not assume that the detected bins are all individual modes. Furthermore in some cases the power peaks are very close to the broad modes, and is not possible to assume reliably if they are sharp modes or if they are just part of the broad mode. We need to find a test that can distinguish the broad modes from the sharp modes.

In order to solve this problem, we devised a technique that:

1. Fit the short-lived modes using MLE
2. Correct the spectrum for the fitted model
3. Apply the aforementionned test as if we had only narrow peaks

Step 1: Assuming that p modes are stochastically excited oscillator, one can derive that the power spectrum of p modes oscillator is distributed around a mean Lorentzian profiles with a χ^2 probability distribution (Toutain and Fröhlich, 1992; Appourchaux et al., 1998), therefore it is possible to apply a statistical test. In our case we want to extract the frequency corresponding to the sharp peaks in the power spectrum, within the frequency range below 500 μHZ, that have high probability not to be due to noise. The power spectrum of the p modes can be described as:

$$P(\nu) = M(\nu)F(\nu) \tag{5}$$

Where $F(\nu)$ is a random function with a χ^2 statistical distribution, and $M(\nu)$ is the model of the fitted mode made of a single Lorentzian profile plus noise.

Step 2: One can fit this model to the observed power spectra using the Maximum Likelihood Estimators technique (Toutain and Appourchaux, 1994). If one divide the power spectrum by the fitted profile, let's call it $M'(\nu)$, one obtain:

$$P'(\nu) = \frac{P(\nu)}{M'(\nu)} \sim F(\nu) \tag{6}$$

In a first approximation $P'(\nu)$ has a χ^2 statistical distribution. This is an approximation because $M'(\nu)$ is derived from data and has also a statistical distribution that should be taken into account[1]. In this way we solved the problem of the mixing between the the sharp modes and the broad modes.

Step 3: Applying the χ^2 test to HD57006 spectrum, we set $\mathcal{P}_N(m) = 10\%$, $\sigma = 1$, and a window size of 30 μHz (corresponding 389 bins). For each window we fitted the broad modes within using the MLE technique and after divided the power spectrum by the resulted fitting we applied the statistical test. The result can be seen in Figure 4.

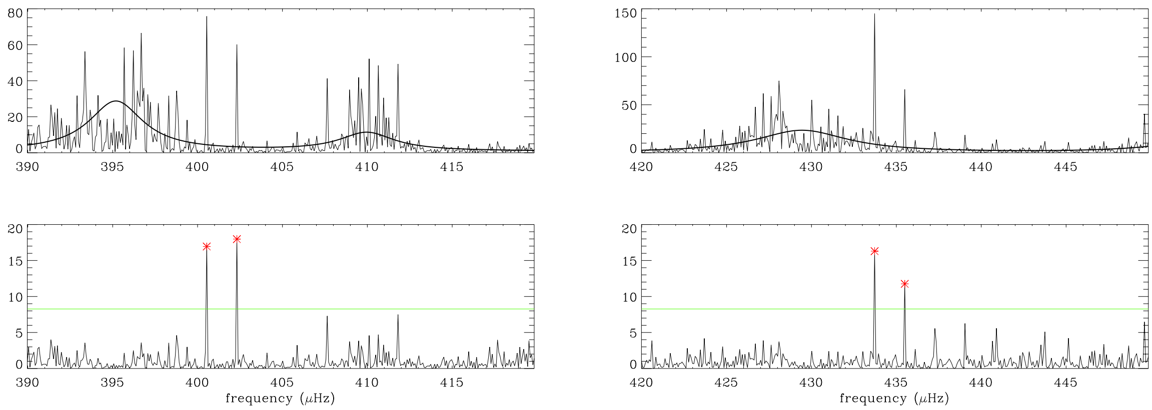


Figure 4: Result of the test for two different ranges of frequencies.The fitting is presented on the top of the figure, and the division of the power spectrum by the fitting, as well as the bins above the 10% probability level, are presented on the bottom of the figure.

The assumption that the g modes have long life-time implies that almost all of the energy from a single g mode will fall in one bin or, at most, in two neighbouring bins (Gabriel et al., 2002). Therefore we assumed that the sharp modes could be the g modes (and of course the broad modes are the p modes). The results are in Tables 2 and 3, and the distribution of the modes frequencies in the Echelle diagram can be seen in Figure 5.

non$-\ell = 0$
320.195±0.256729
353.860±0.246180
373.999±0.324451
409.970±0.225900
451.725±1.45902
474.797±0.512622

Table 2: Frequencies and their formal errors in μHz of the non-$l = 0$ modes as determined by the MLE technique for the broad modes corresponding to the unidentified p modes below 500μHz.

[1]This shall be the subject of a Monte-Carlo analysis

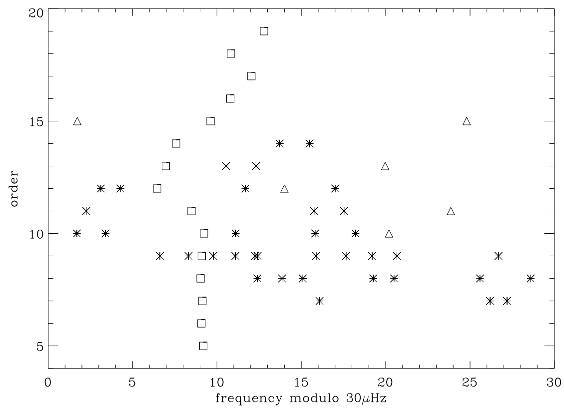


Figure 5: The distribution of the modes in the Echelle diagram. The $l = 0$ modes are presented as squares, the g modes as asterisks, and the non-$l = 0$ modes as triangles.

g modes						
226.08025	236.18827	237.19136	252.39198	253.85802	255.09259	259.25926
260.49383	265.58642	268.59568	276.62037	278.31790	279.78395	281.09568
282.25309	282.40741	285.87963	287.65432	289.19753	290.66358	296.68210
301.69751	303.39504	311.11109	315.81788	318.20986	332.25308	345.75616
347.53085	363.11728	364.27469	371.68210	377.00617	400.54011	402.31481
433.71914	435.49383					

Table 3: Frequencies of the g modes extracted using the statistical test described in the subsection 2.2.1

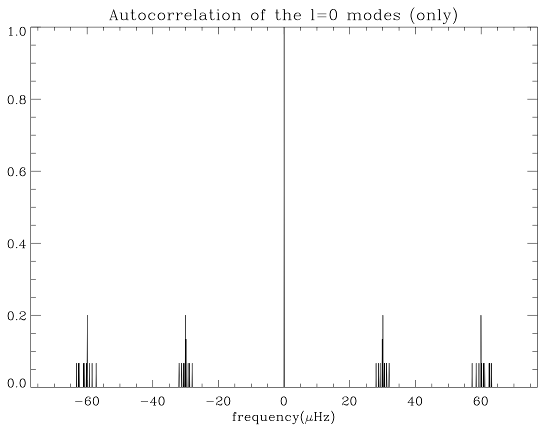
2.2.2 Autocorrelation and correlation

The correlation determines the degree of similarity between two signals. If the signals are identical, then the correlation coefficient is 1 (or -1); if they are totally different, the correlation coefficient is small and close to 0.

The Autocorrelation is a method which is frequently used for the extraction of the fundamental frequency, in this context this is the $l = 0$ ridge. If a copy of the same signal is shifted, the distance between the central peak (corresponding to the non shifted) and the next correlation maximum is taken to be the fundamental frequency, in this case that this the large separation (Figure 6, left). Having this in mind, this can be applied to the extracted frequencies in order to check if there is some hidden pattern, or some kind of periodicity which is not very clearly to human eye.

We determined the autocorrelation of the extracted frequencies corresponding to the g modes plus those corresponding to the non$-l = 0$ p modes (Figure 6, right). We didn't find out a clear evidence for the existence of a correlation as we can find for the autocorrelation of the extracted frequencies of $l = 0$ modes alone.

Assuming that the shapes of $l = 0$ and $l = 1$ in Echelle diagram might be very similar, we correlated the $l = 0$ frequencies with the frequencies of g modes plus non-$l = 0$ p modes to check if one can find a repetition of the $l = 0$ ridge (Figure 7). However, once more we didn't clearly find any evidence for that.



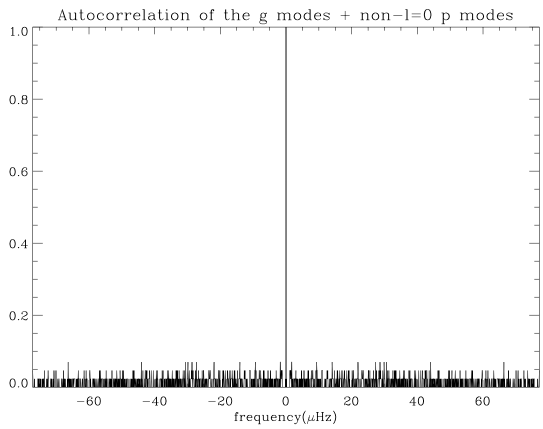


Figure 6: On the left hand side, we have the autocorrelation of the extracted frequencies corresponding the $l = 0$ modes; we can see a fundamental frequency corresponding to the large separation. On the right hand side, the correlation between the frequencies corresponding to the g modes plus the frequencies corresponding to the unidentified p modes; we can not see any fundamental frequency.

3 Conclusion

High-order low-degree p modes of the HD57006 do not behave in the same way as one could predict from the asymptotic theory for solar-like stars in the main sequence. Looking at the Echelle diagram, we cannot see the "vertical" ridges corresponding to $l = 0, 2$ and $l = 1, 3$, therefore it is not possible to determine the $d_{n,l}$ and $\delta_2\nu$, hence is not possible to deduce the internal structure of the star.

If one focus our attention to the low orders, the HD57006 power spectrum seems to be able to provide more information about the star; we can identify clearly the $l = 0$ modes that can be fitted without major difficulties. One can also notice the presence of sharp modes that can possibly correspond to g modes or p modes with mixed character, since in this phase of evolution both type of modes have a larger propagation zone close to the convective core frontier due to the Brunt-väisäla frequency peak.

For the extraction of the frequencies of the sharp modes we devised a technique that can distinguish them from the broad modes. This technique can also be applied to stars where the mixed modes character exists but not so strong as in HD57006.

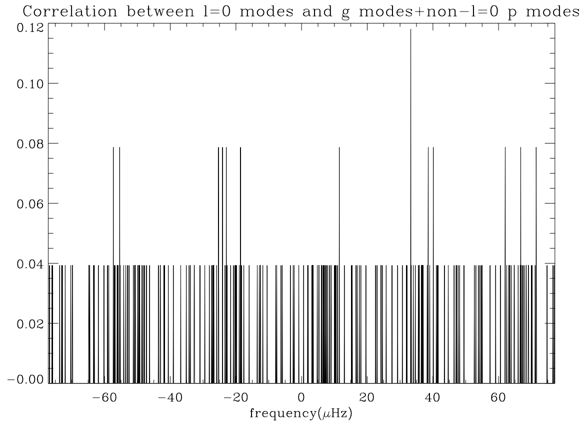


Figure 7: Correlation between the identified $l = 0$ p modes and the unidentified p modes plus the g modes. If one confine our attention to the interval [-30μHz,+30μHz] we can see four maximums but they are not sufficient prominents to affirm that there is a clearly repetition of the $l = 0$ shape.

Last but not least, we were not able to identify any rotational splittings from the non-$l = 0$ modes.

References

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