A new method for 3D reconstruction of polar plumes

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Tomography with time evolution

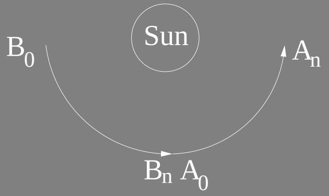


Fig.: STEREO spacecrafts trajectory scheme

- 2 simultaneous points of view
- 2n points of view : rotational tomography
- But polar plumes show time evolution

Can we get a 3D + time reconstruction of poles using all this information ?

An inverse linear problem

$$y_t = H_t x_t + b_t \qquad (1)$$

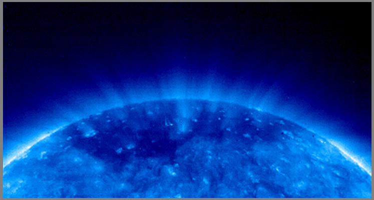


Fig.: The image y_t depends linearly on the emission x_t

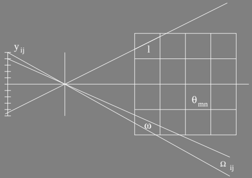


Fig.: H_t describes the projection of one image

This problem is heavily underdetermined since $x = (x_t)_{t \in [0,T]}$ is of dimension 4.

A model of time evolution

But we can add some kind of *a priori* directly into the model :

$$\boldsymbol{y}_t = \boldsymbol{H}_t(\boldsymbol{x}. * \boldsymbol{A}\alpha_t) + \boldsymbol{b}_t \tag{2}$$

- $\boldsymbol{y_t}$ and $\boldsymbol{H_t}$ are still an image and the projection matrix
- $\boldsymbol{x}$ is a time independent emission cube
- $\boldsymbol{A}$ describes areas of homogeneous temporal evolution
- α is the time evolving gain on each area

Note that the problem is bilinear on $(\boldsymbol{x}, \alpha)$

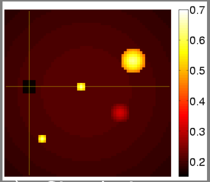
Regularized least-square

We define our solution as the minimum of :

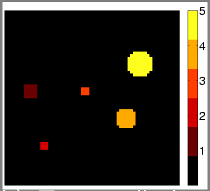
$$J = \|y - H(x.*A\alpha)\|^2 + \lambda\|D_r x\|^2 + \mu\|D_t\alpha\|^2 \quad (3)$$

- $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ are user-chosen parameters
- $\boldsymbol{D_r}$, $\boldsymbol{D_t}$ are finite differentiation matrices on space and time
- We minimize this criterion with a conjugate-gradient algorithm on $\boldsymbol{x}$ and explicitly on $\boldsymbol{\alpha}$

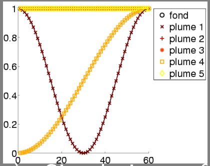
Simulated data Results



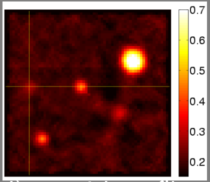
(a) Simulation at $t = \frac{T}{2}$



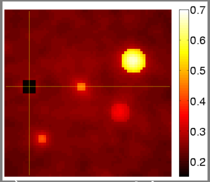
(b) Temporally homogeneous areas



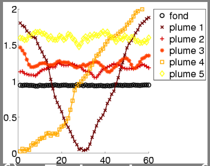
(c) Simulation of α_t for each plume



(d) x with a filtered back-projection algorithm

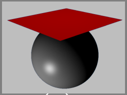


(e) $x. * g_t$ with our algorithm at $t = \frac{T}{2}$

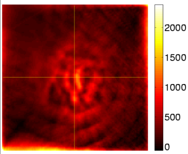


(f) Estimation of α_t with our algorithm

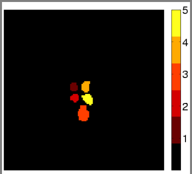
Preliminary results on SOHO data



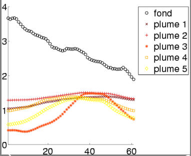
(g)



(h) $\boldsymbol{x}$ without time evolution

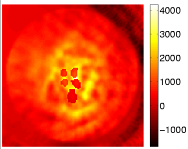


(i) Temporally homogeneous areas

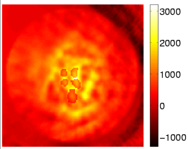


(j) Estimation of α_t with our algorithm

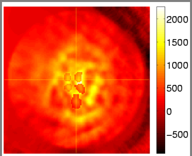
Movie



(k) $\boldsymbol{x}.*\boldsymbol{g}_t$ with our algorithm at $t = 0$

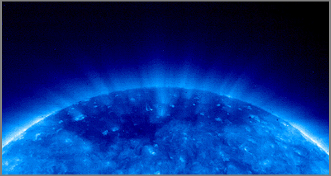


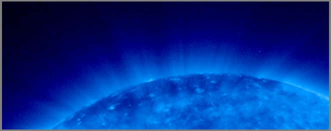
(l) $\boldsymbol{x}.*\boldsymbol{g}_t$ with our algorithm at $t = \frac{T}{2}$



(m) $\boldsymbol{x}.*\boldsymbol{g}_t$ with our algorithm at $t = T$

Upcoming results on EUVI





But we need accurate parameters to define the projector $\boldsymbol{H_t}$:

- The roll angle needs to be taken into account.
- The sun center has to be localized precisely

Ideal data set

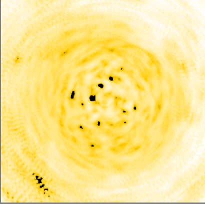
- Lossless compression - underdetermination increases noise
- Simultaneous images - it constrains more the problem
- STEREO spacecrafts at 90° of each other in longitude
- or STEREO + SOHO spacecrafts at 60° of each other
  in order to minimize the time of data acquisition

What's next ?

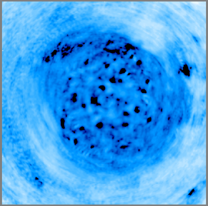
- Continue analysis of the algorithm with simulated data
- Apply our algorithm with 2 or 3 points of view
- Enforce positivity of the unknowns
- Find a method to estimate the plumes positions
- Estimate electron density and temperature from the emission at 17,1 nm 19,5 and 28,4 nm

Electron density and temperature

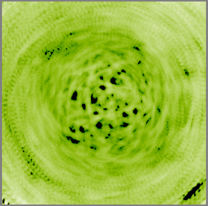
It is similar to DEM but we do not have LOS ambiguity



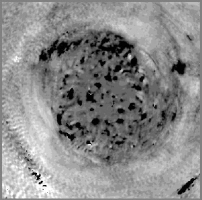
17.1 nm



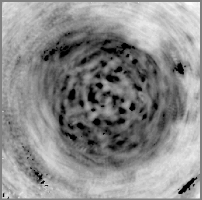
19.5 nm



28.4 nm



T_e



n_e

Bayesian methods

Tomography can be seen as an inverse problem of the form :

$$y_t = H_t x + b_t \tag{4}$$

	general formulation	tomography
likelihood	$f(y\|x)$	$e^{-\frac{\\|y-Hx\\|^2}{\sigma_n^2}}$
a priori	$f(x)$	$e^{-\frac{\\|Dx\\|^2}{\sigma_r^2}}$
a posteriori	$f(x\|y) = \frac{f(x)f(y\|x)}{f(y)}$	$e^{-\\|y-Hx\\|^2-\lambda\\|Dx\\|^2}$

Minimization Algorithm

We could estimate jointly the parameters but in practice it is more efficient to alternate estimation on $\boldsymbol{x}$ and $\boldsymbol{\alpha}$.
We have the explicit minimum on $\boldsymbol{\alpha}$ since $\boldsymbol{M}$ is a matrix of small size :

$$\alpha_{min} = M(M^t M)^{-1} M^t y \text{ with } M = Hdiag(x)A \tag{5}$$

We make use of a gradient algorithm on $\boldsymbol{x}$:

$$x^{n+1} = x^n - a^n_{opt} \nabla_{x^n} J \tag{6}$$

Degenerate case with no plume

$$y_t = H_t x \alpha_t, \text{ with } \alpha_t \in R \quad (7)$$

We can estimate the global intensity variation on each image.
It can be rewritten as a non-linear problem on x only :

$$y_t = \left[I - H_t x \left((H_t x)^T H_t x \right)^{-1} (H_t x)^T \right]^{-1} b \quad (8)$$

which can be solved with a gradient algorithm.