CNRS — université PARIS-SACLAY — DÉPARTEMENT Sciences de la Planète et de l'Univers — IAS Institut d'Astrophysique Spatiale Orsay

Characterising turbulence in clusters with X-rays

Edouard LECOQ

ByoPiC Workshop #1 – Malta

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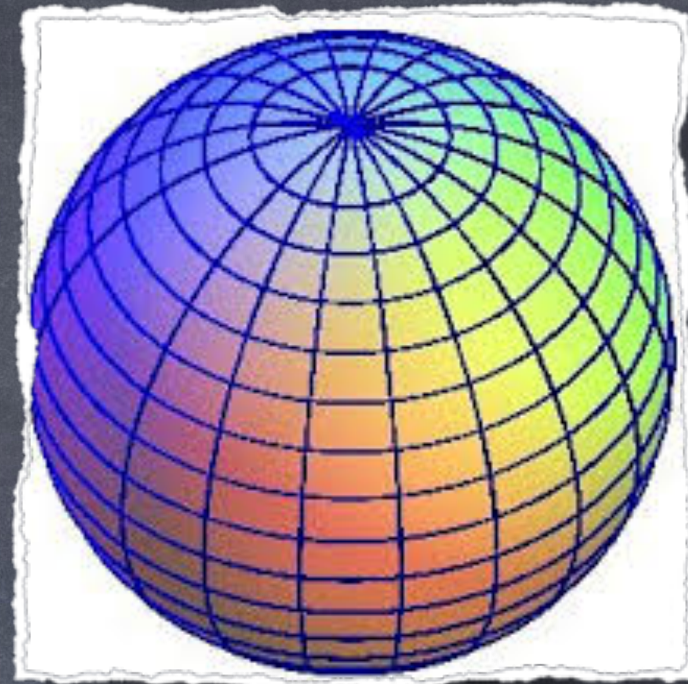
European Research Council
Established by the European Commission

WHY ?

Nowadays →



A cluster is a sphere at Hydrostatic Equilibrium

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

In the scope of this approximation

$$M_X = -\frac{k_B T(r) r}{\mu m_p G}\left(\frac{\mathrm{d}ln(\rho(r))}{\mathrm{d}ln(r)} + \frac{\mathrm{d}ln(T(r))}{\mathrm{d}ln(r)}\right)$$

However – The measure mass due to lensing

$$M_{lens} = M_{true} = (1-b)M_X$$

b The mass bias

WHY ?

Trusting Einstein (GR)
+ non-thermal processes (turbulence, etc)

$$M_X = -\frac{k_B T(r) r}{\mu m_p G}\left(\frac{\mathrm{d}ln(\rho(r))}{\mathrm{d}ln(r)} + \frac{\mathrm{d}ln(T(r))}{\mathrm{d}ln(r)} + \frac{P_{n-th}}{P_{th}}\frac{\mathrm{d}ln(P_{th}(r))}{\mathrm{d}ln(r)}\right)$$

$$M_X + \frac{k_B T(r) r}{\mu m_p G}\frac{P_{n-th}}{P_{th}}\frac{\mathrm{d}ln(P_{th}(r))}{\mathrm{d}ln(r)} = -\frac{k_B T(r) r}{\mu m_p G}\left(\frac{\mathrm{d}ln(\rho(r))}{\mathrm{d}ln(r)} + \frac{\mathrm{d}ln(T(r))}{\mathrm{d}ln(r)}\right)$$

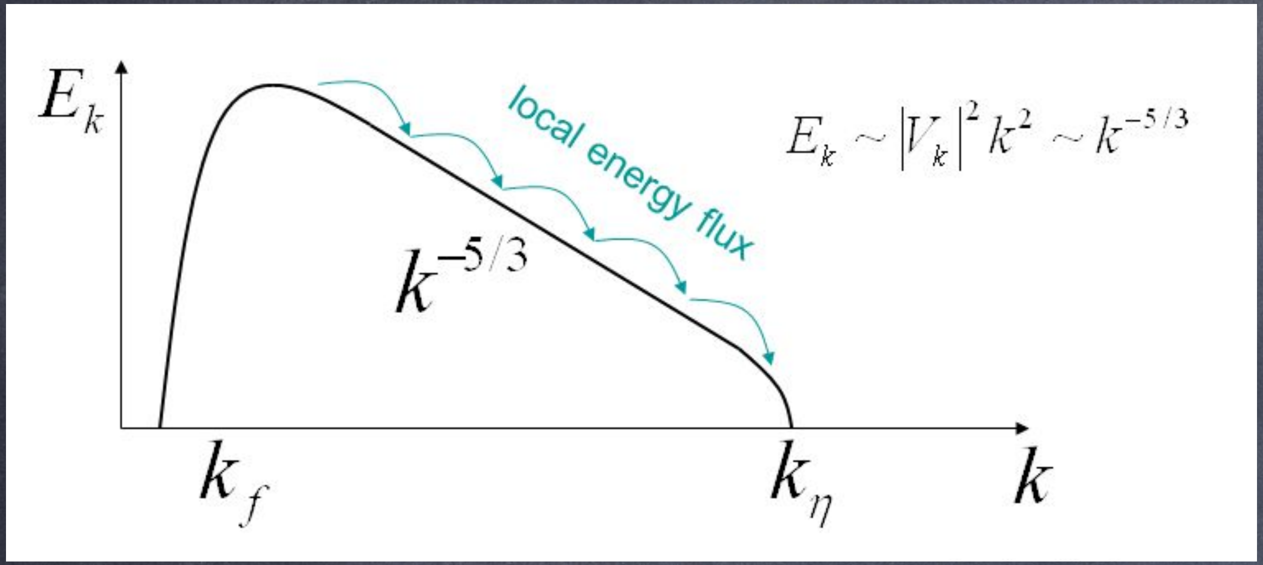
$\propto -b$

Candidate for (part of) the mass bias

Overestimation of the "true mass"

HOW ?

Quantify + characterise Turbulence



Kolmogorov Spectrum

k_f Injection wavenumber

k_η Dissipation wavenumber

Detect

XARM (X-Ray Astronomy Recovery Mission)

Athena Observatory

X-Ray Facilities

STUDY THE DETECTABILITY OF TURBULENCE TROUGH THOSE NEW GENERATION X-RAY SATELLITES

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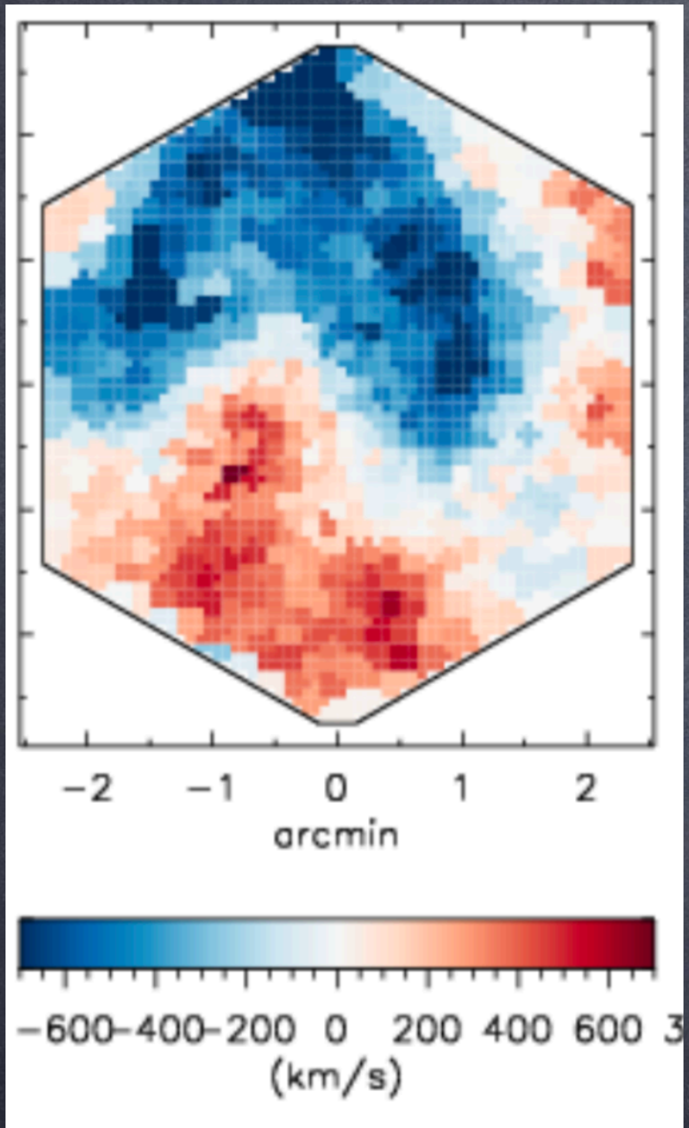
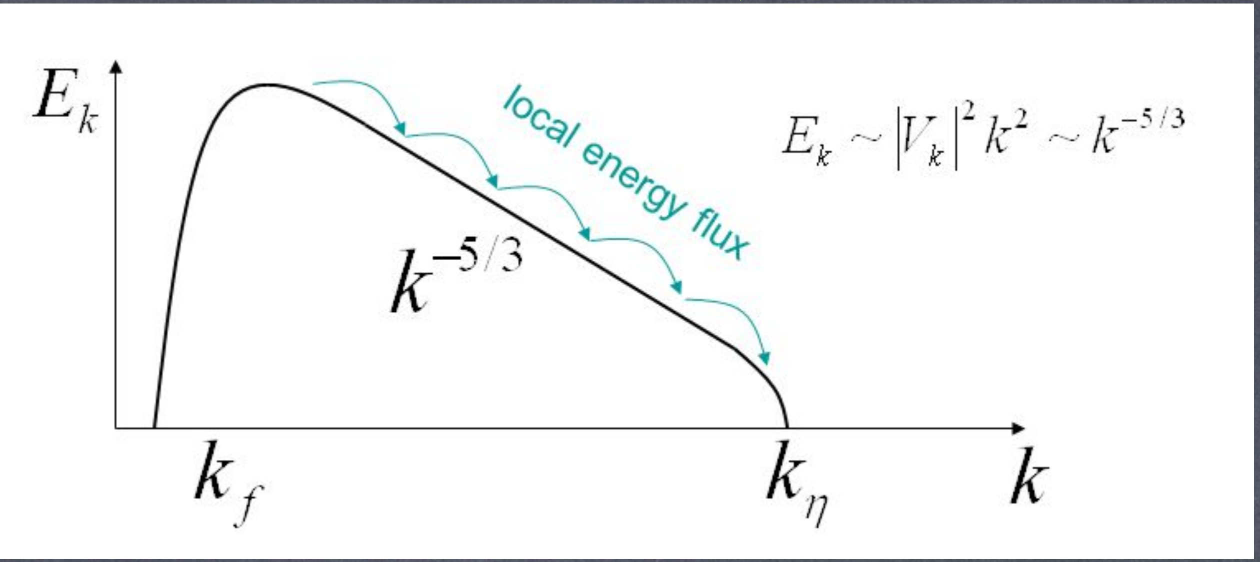
ME

Observable



Theory



Kolmogorov Spectrum $P_{3D} = |\tilde{v}_{3d}|^2(\vec{k})$

Centroid velocities

$$\bar{v}_z(r_\perp) = \int v_{3d}(r_\perp, r_z)\omega(r_\perp, r_z)\mathrm{d}r_z$$

Where $\omega(r_\perp, r_z) = \frac{\epsilon(r_\perp, r_z)}{\int \epsilon(r_\perp, r_z)}$

$\epsilon(r) = n_e(r)n_H(r)\Lambda(T, Z) \propto n_e^2(r)$ is the emissivity

How do I link theory and observation?

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The Structure Function

$$SF(r_\perp) = \left\langle |\bar{v}_z(\chi_\perp + r_\perp) - \bar{v}_z(\chi_\perp)|^2 \right\rangle_{r_\perp}$$

$$\langle|\bar{v}_z(\chi_\perp + r_\perp)\bar{v}_z(\chi_\perp)|\rangle = \int_{-\infty}^{+\infty} e^{2\pi i r_\perp . k_\perp} P_{2D}(k_\perp) dk_\perp .$$

Wiener-Khinchin theorem

$$SF(r) = 4\pi \int_0^\infty (1 - J_0(2\pi k_\perp r)) P_{2\mathrm{D}}(k_\perp) k_\perp dk_\perp$$

$$P_{2\mathrm{D}}(k_\perp) = |\tilde{\bar{v}}_z(k_\perp)|^2$$

What is the relation from $P_{3D}(k_{3d})$ to $P_{2D}(k_\perp)$?

$$\langle P_{2D}(\vec{k}_\perp)\rangle_S = \int d\vec{k}_{\perp_1} dk_{z_1} P_{3D}\left(\sqrt{k_{\perp_1}^2 + k_{z_1}^2}\right) P_\omega(\vec{k}_\perp - \vec{k}_{\perp_1}, k_{z_1})$$

- $\langle P_{2D}(\vec{k}_\perp)\rangle_S$: Emissivity weighted line of sight velocity power spectrum = Centroid velocity power spectrum
- P_{3D}: Velocity power spectrum (described by a Kolmogorov spectrum)
- P_ω: Emissivity power spectrum

Computation of the emissivity power spectrum

Starting from the emissivity field $\epsilon(r) = n_e(r) n_H(r) \Lambda(T, Z) \propto n_e^2(r)$

Taking $n_e = \left(1 + \left(\frac{r}{r_c}\right)^2\right)^{-3\beta/2}$ r_c the cluster core radius

We got $\omega(r_\perp, z) = \frac{2}{\pi} \frac{(r_c^2 + r_\perp^2)^{3/2}}{(r_c^2 + r_\perp^2 + z^2)^2}$ In the coordinate system $(r_\perp = \sqrt{x^2 + y^2}, \theta, z)$

Fourier tranform (+ "unshowable" computations)

$$\tilde{\omega}(k_\perp, k_z) = \frac{1}{\sqrt{2\pi}} \int_{r_c}^{+\infty} (|c^2 k_z| + c)\ J_0\left(k_\perp (c^2 - r_c^2)^{1/2}\right) e^{-|c k_z|} dc$$

So we get

$$P_\omega(k_\perp, k_z) = |\tilde{\omega}(k_\perp, k_z)|^2 = \frac{1}{2\pi} \left(\int_{r_c}^{+\infty} (|c^2 k_z| + c)\ J_0\left(k_\perp (c^2 - r_c^2)^{1/2}\right) e^{-|c k_z|} dc\right)^2$$

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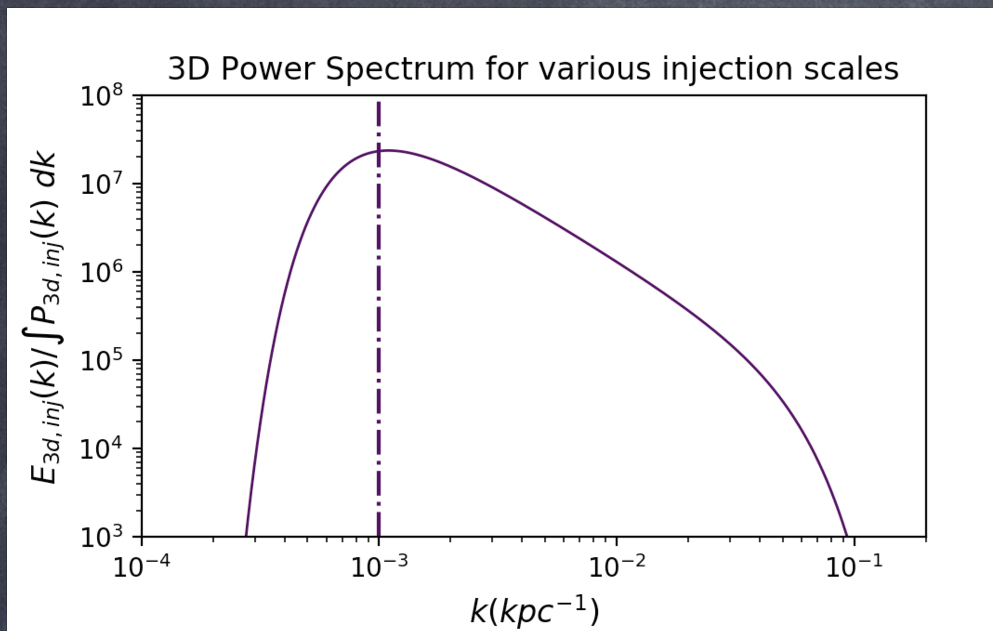
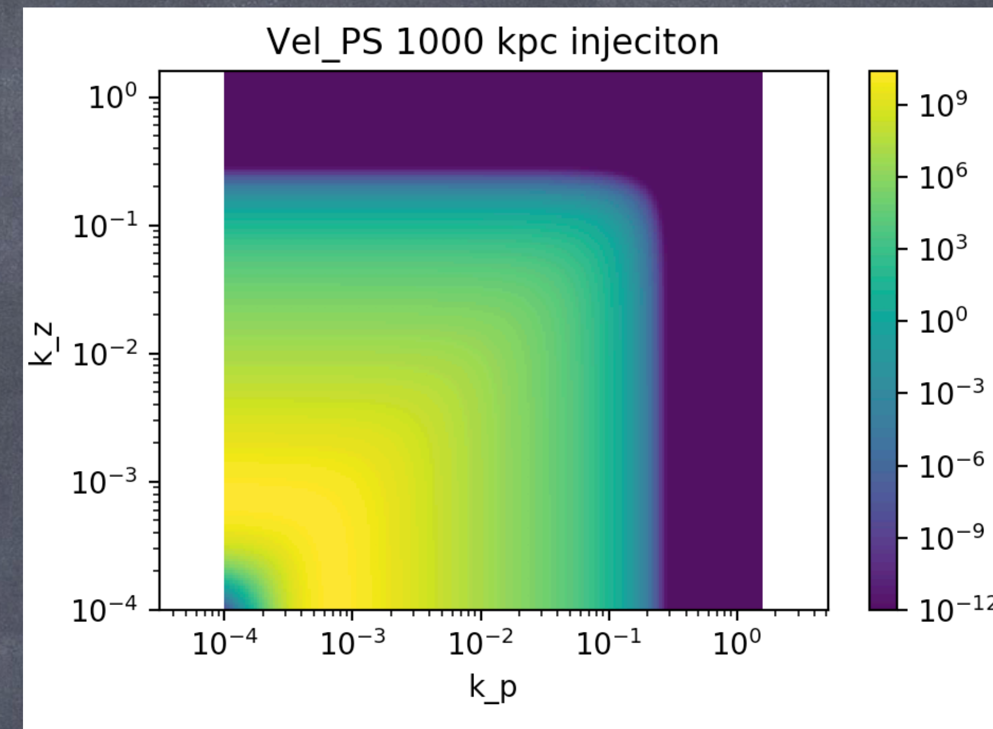
Needs to go numerical !!!





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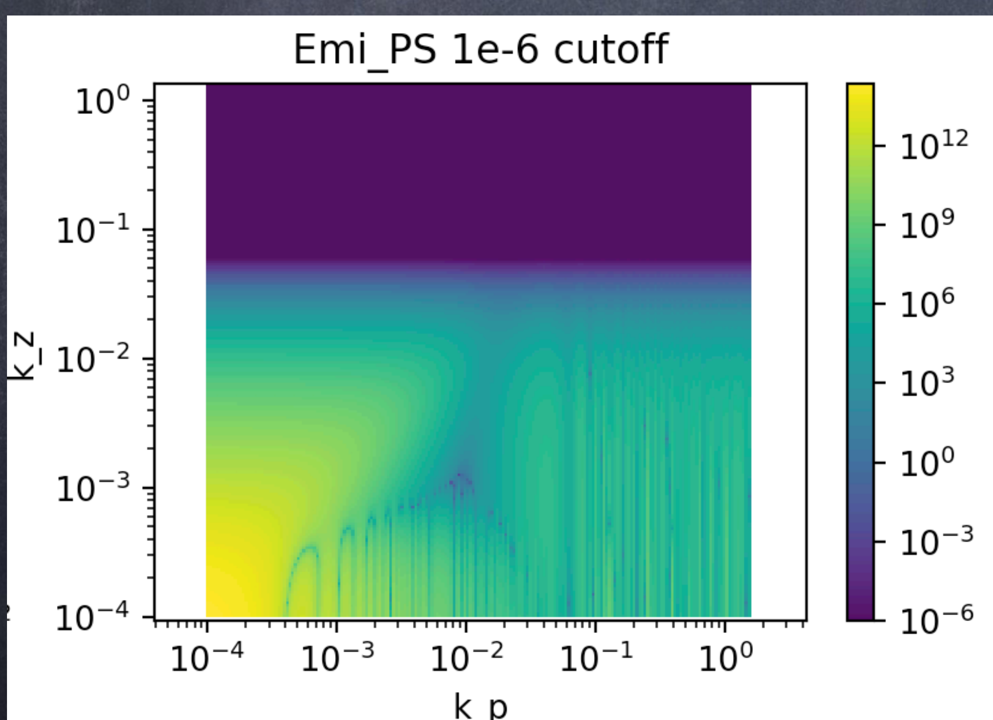
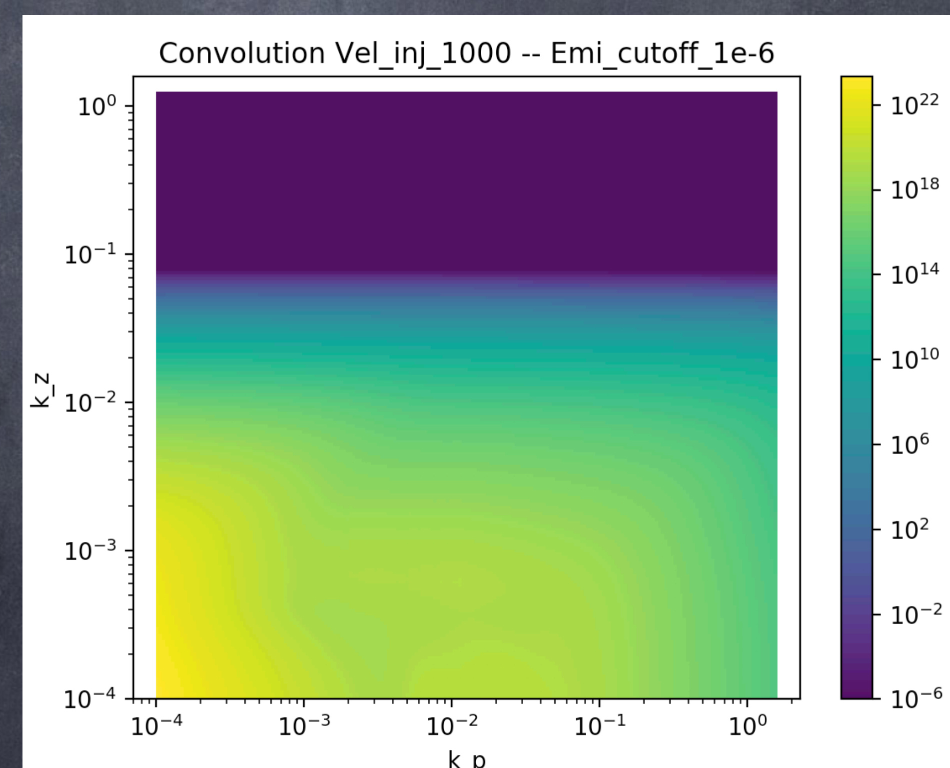


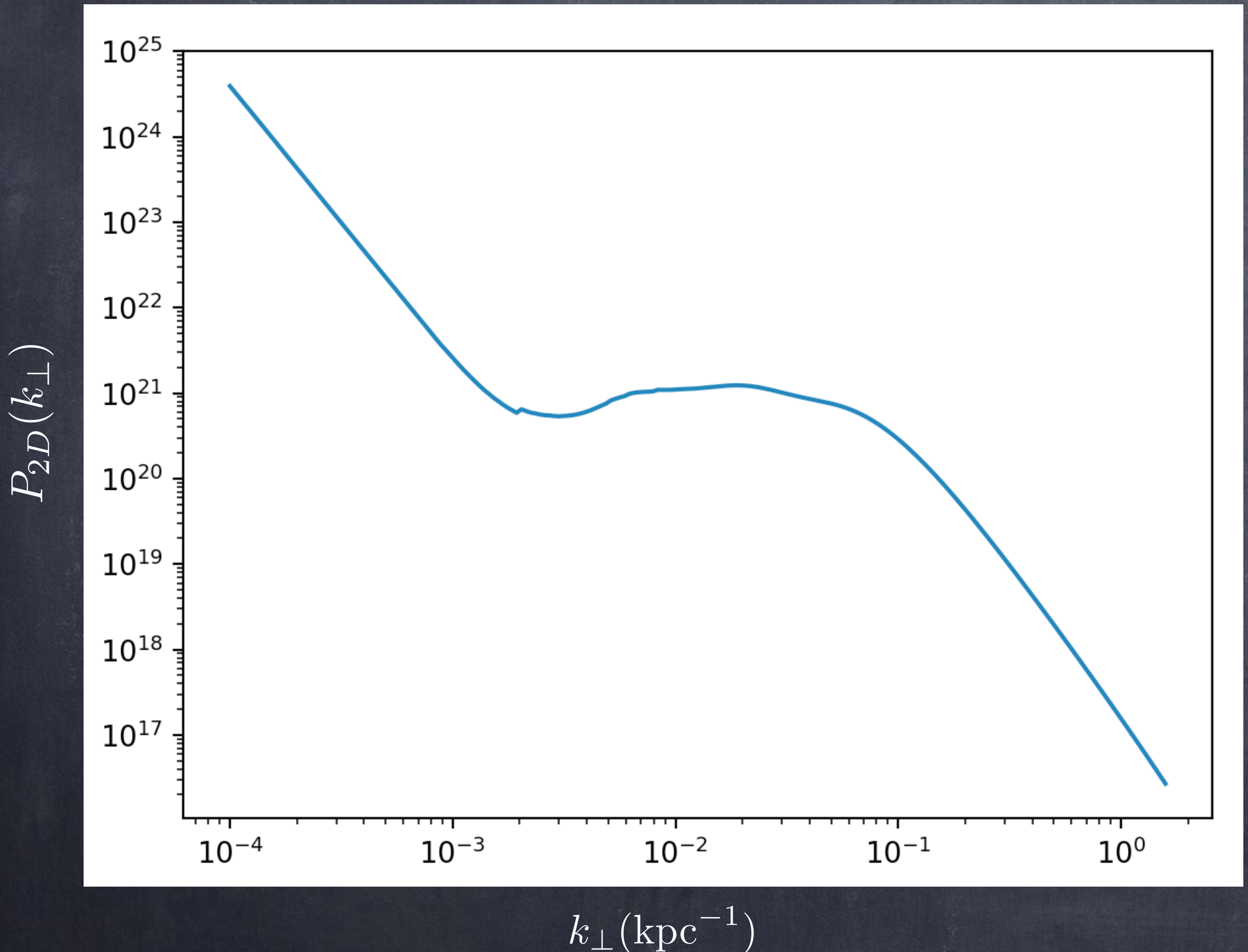
CONVOLUTION
=
ALONG X-AXIS



PROBLEMS :(

Integrating over k_z we got :

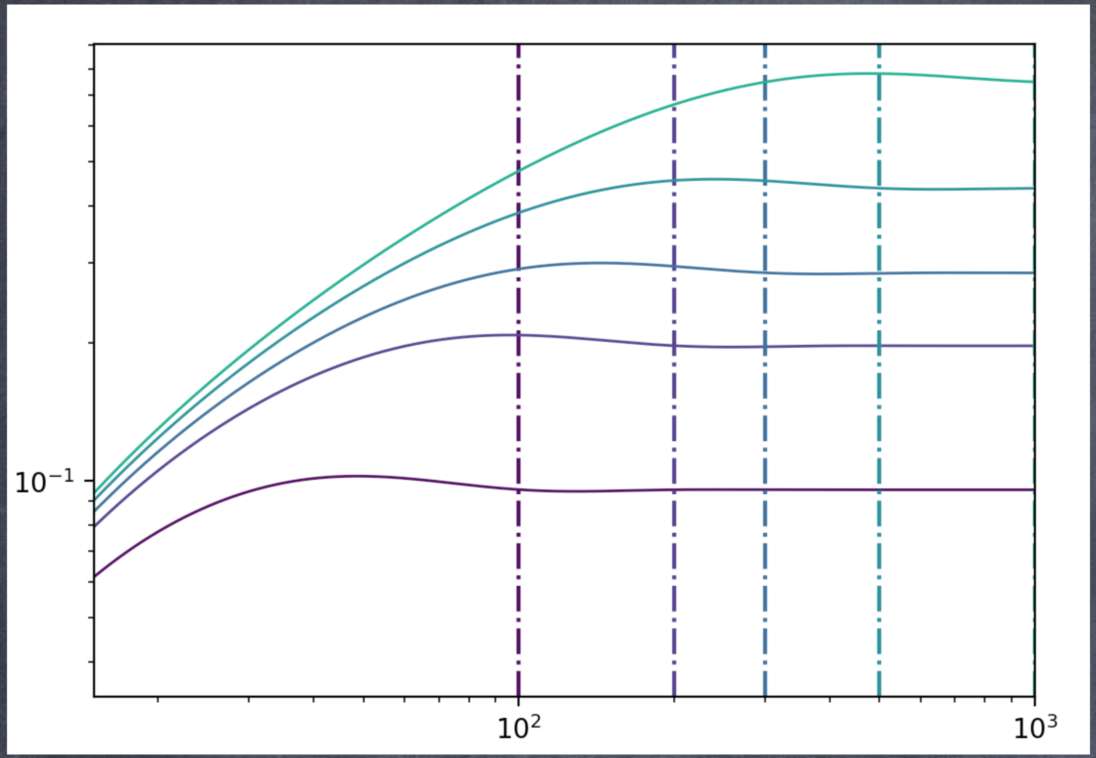
Velocity centroid power spectrum $P_{2D}(k_\perp)$



$P_{2D}(k_\perp)$ depends of $\vec{k}_\perp$ and not only its norm

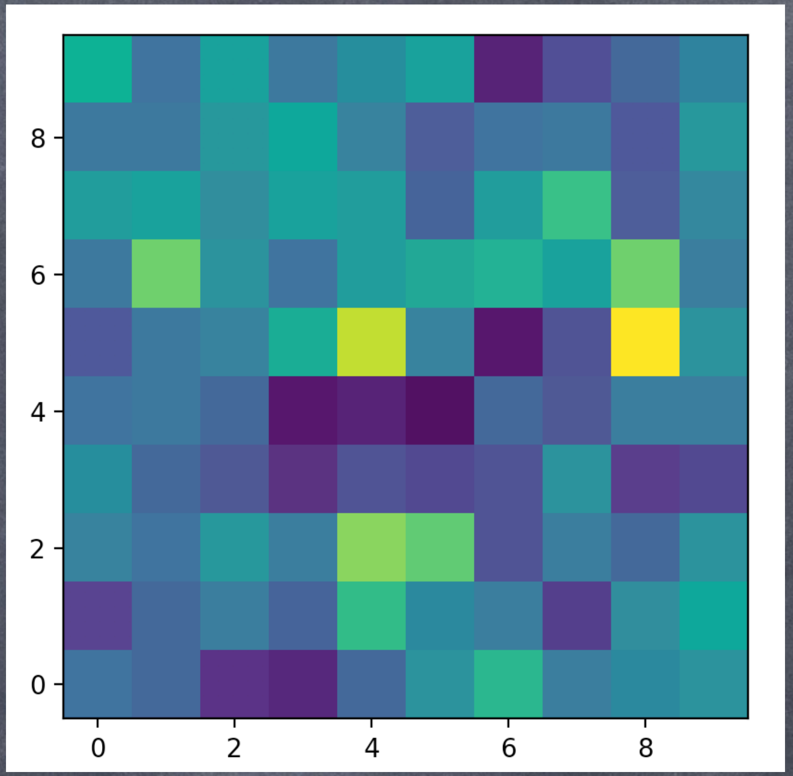
From observable to the Structure Function

Structure Function

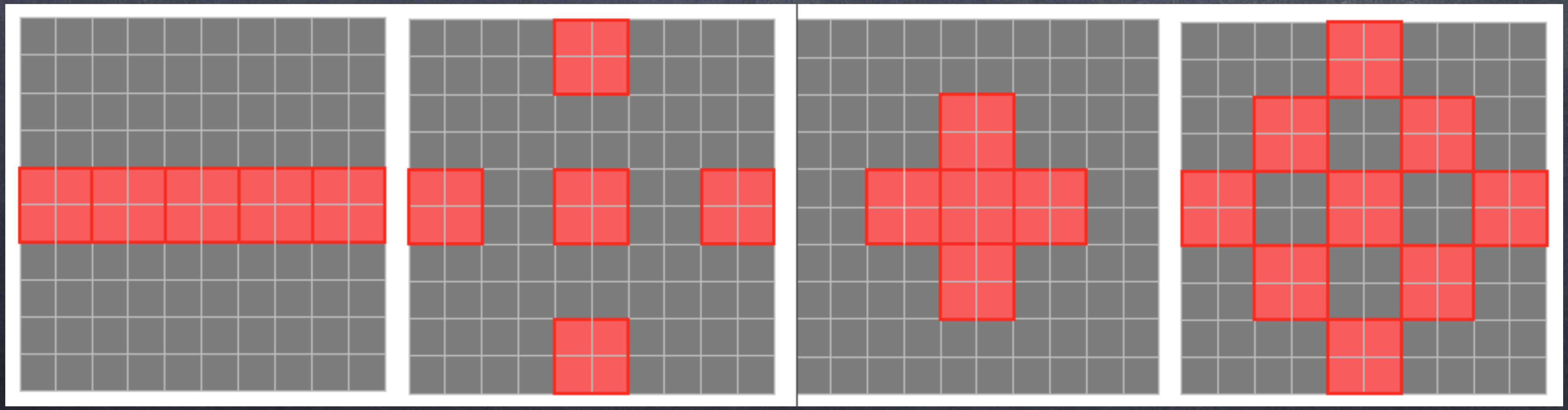


$$SF(r_\perp) = \left\langle |\bar{v}_z(\chi_\perp + r_\perp) - \bar{v}_z(\chi_\perp)|^2 \right\rangle_{r_\perp}$$

Mock observation maps

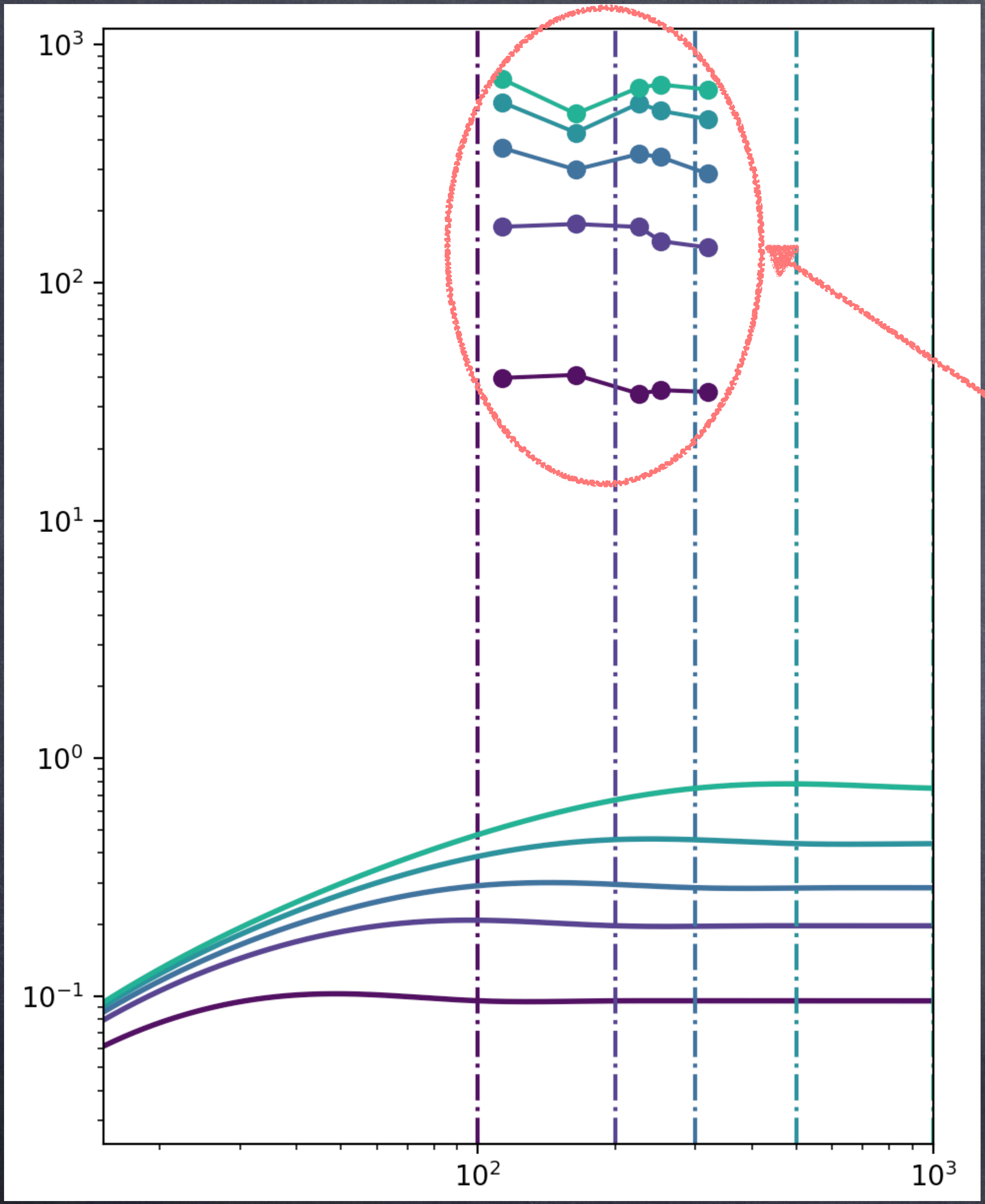


$$P(\nu, \phi) d\nu d\phi = \frac{\nu}{\Sigma^2} \exp\left(-\frac{\nu^2}{2\Sigma^2}\right) d\nu \frac{d\phi}{2\pi}$$



Pointing configuration for reconstruction algorithm

PROBLEMS :(



Normalisation ?

- Of the theoretical SF ?
- Of the reconstructed values?
- Of the mock observations maps?

THANKS !