What do global *p*-modes tell us about large scale solar flows?

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What this work is about

- (Re)calculation of effects of Zonal (rotation) and poloidal solar flows (meridional circulation , giant cells) on *p*-mode splitting coefficients. *Leads to much lower estimates for frequency shifts!*

Previous related work:

Roth, Howe and Komm, 2002, A&A, 396, 243

Roth and Stix (1999, 2003, 2008)

- Compare theoretical splitting coefficients with GONG as well as MDI data sets and *put upper limits on the magnitudes of these flows.*

The technique

- Quasi-degenerate Perturbation Theory (Lavely & Ritzwoller, 1992)

Couples two modes with slighly different un-perturbed frequencies (for us $|\omega_2 - \omega_1| < 100\ \mu Hz$)

$$\Delta = \omega_2^2 - \omega_1^2$$

$$\begin{bmatrix} H_{11} - \Delta & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \rightarrow \lambda = H_{22} - \frac{|H_{12}|^2}{H_{11} - H_{22} - \Delta}$$

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$$\begin{bmatrix} H_{11}-\Delta & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \rightarrow \lambda=H_{22}-\frac{\cancel{|H_{12}|^2}}{H_{11}-H_{22}-\Delta} \quad 0$$

In contrast *degenerate perturbation theory* (DPT) couples only exactly degenerate un pertubed modes

The Model

- Standard solar model with OPAL equation of state, OPAL opacities, convective flux calculation (Canuto & Mazzitelli, 1991)
- Eigenfunctions and frequencies from a *Solar pulsation code* (with non-adiabatic effects and without Cowling's approx, H. M. Antia, 2002, ver 2.2)

QDPT: Application to rotation

$$\Omega r \sin\theta = -w_1^0 \partial_\theta Y_1^0 - w_3^0 \partial_\theta Y_3^0 \qquad \omega_{nlm} = \omega_{nl} + \sum_q a_q^{(nl)} P_q^l(m)$$

- Zonal flows affects *p*-modes in two ways:

1. Coriolis force, linear in Ω (odd splitting coefficients a_{2q+1} from DPT)

N-S asymmetric Ω gives even splitting coefficients a_{2q}:

with w_2^0= -7.8±0.3 ms^{-1} (Hathaway et al 1996), we calculate a_2~ 0.1 nHz

2. Centrifugal force, Ω^2 (DPT gives even splitting coeff, a_{2q} *Antia et al. 2000*: a_2 < 25 nHz/l)

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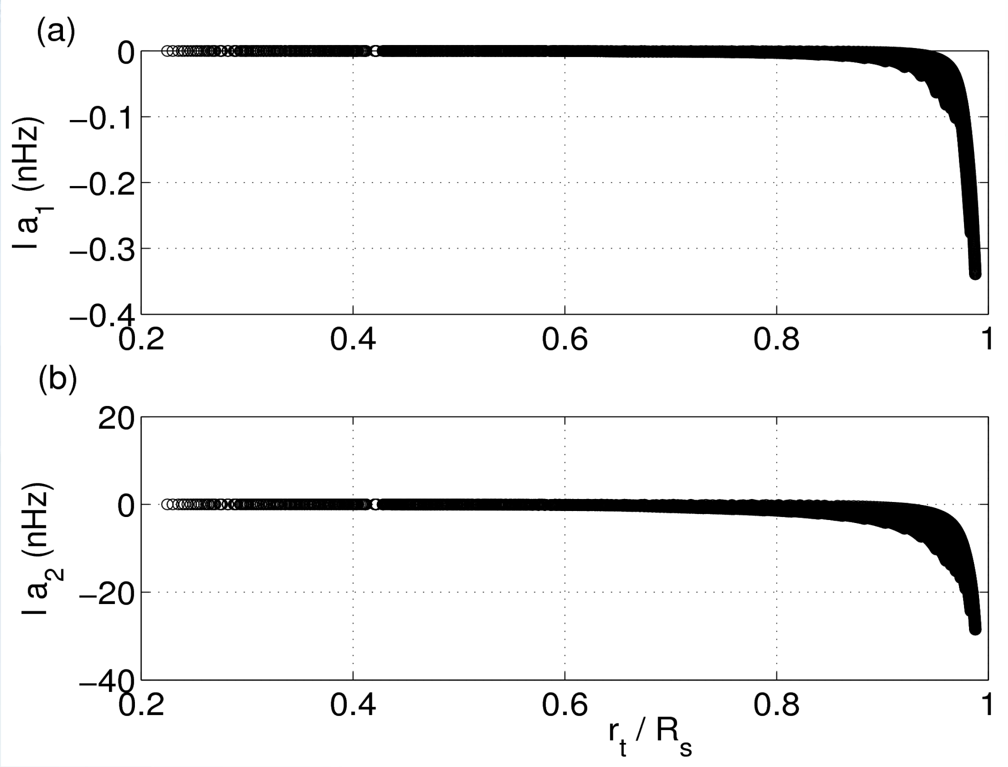
QDPT: Application to rotation

QDPT can calculate effect of Ω (Coriolis force) on a_{2q} ***as a second order correction***

m^4

$$\omega_{nlm} = \left[\omega_{nl}^2 + H_{22} - \frac{|H_{12}|^2}{H_{11} - H_{22} - \Delta} \right]^{1/2} \quad ; H_{22} \approx \mathbf{450}\ m\, nHz \gg H_{21}$$

m^2



For w_1^0 term H_{12}= 0!

For w_3^0= 34.9 ms^{-1} r > 0.7,

la_1 ~ 0.4 nHz; la_2 < 35 nHz

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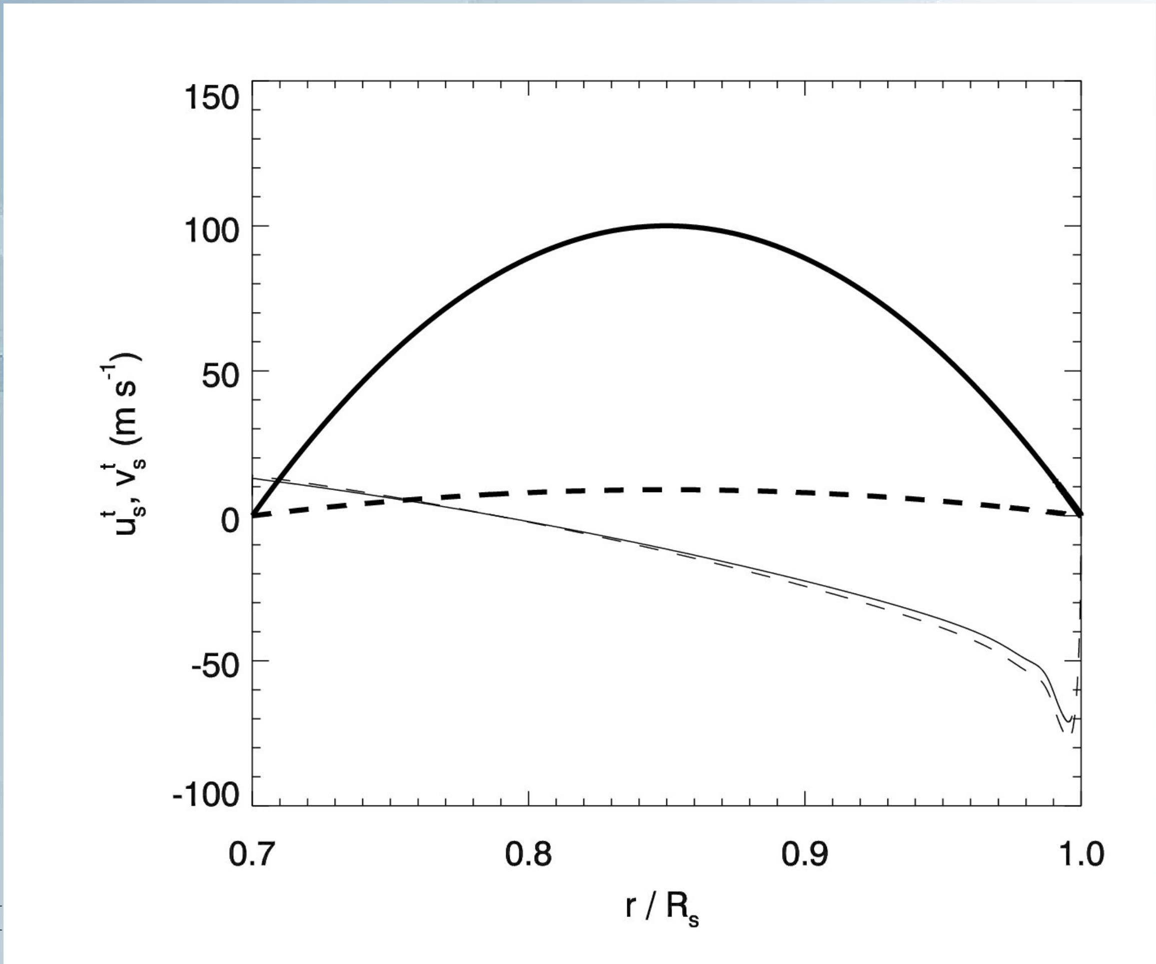
QDPT: Non Zonal flows

($u_0 = 9\ \mathrm{ms}^{-1}$, $s = 2$, $t=0$, Y_2^0)

- Meridional circulation

$$v(r,\theta,\varphi) = u_s^t(r)\, Y_s^t + v_s^t(r)\, \partial_\theta Y_s^t$$



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$$v(r,\theta,\varphi) = u_s^t(r) Y_s^t + v_s^t(r) \partial_\theta Y_s^t$$

$$\omega_{nlm} = \left[\omega_{nl}^2 + H_{22} - \frac{|H_{12}|^2}{H_{11} - H_{22} - \Delta} \right]^{1/2}$$

v²

0

QDPT in necessary to provide the first correction due to poloidal flows

$$\delta\omega_{nlm} \approx \frac{H_{12}^2}{2\omega_{nl}\Delta}$$

Where, Δ = difference in squared frequency of the coupling modes

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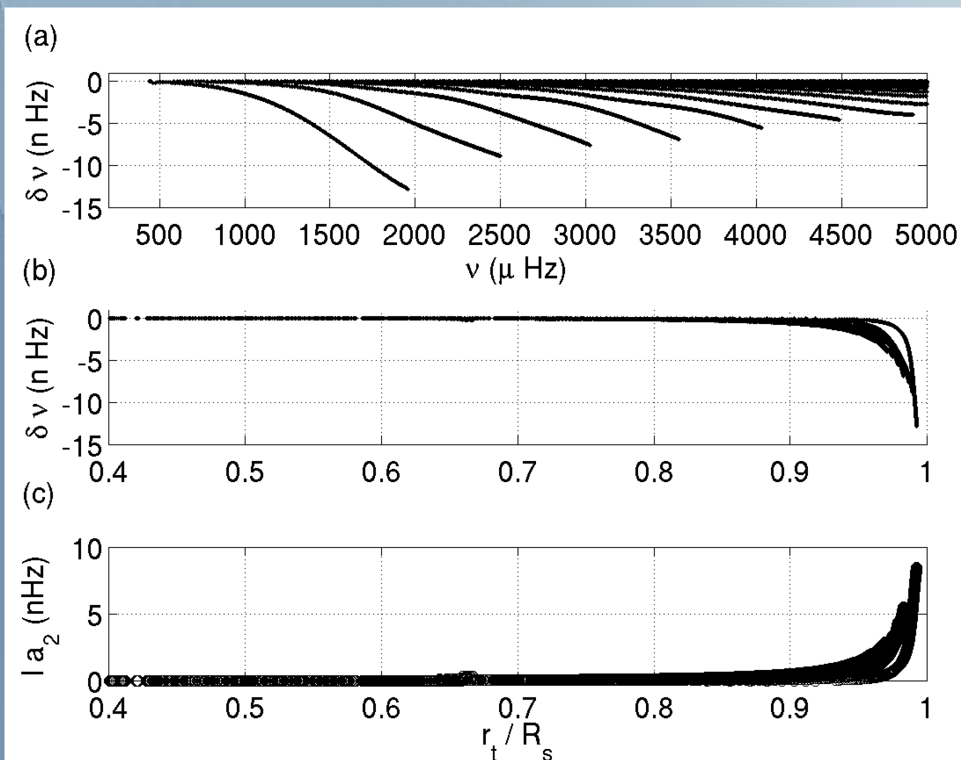
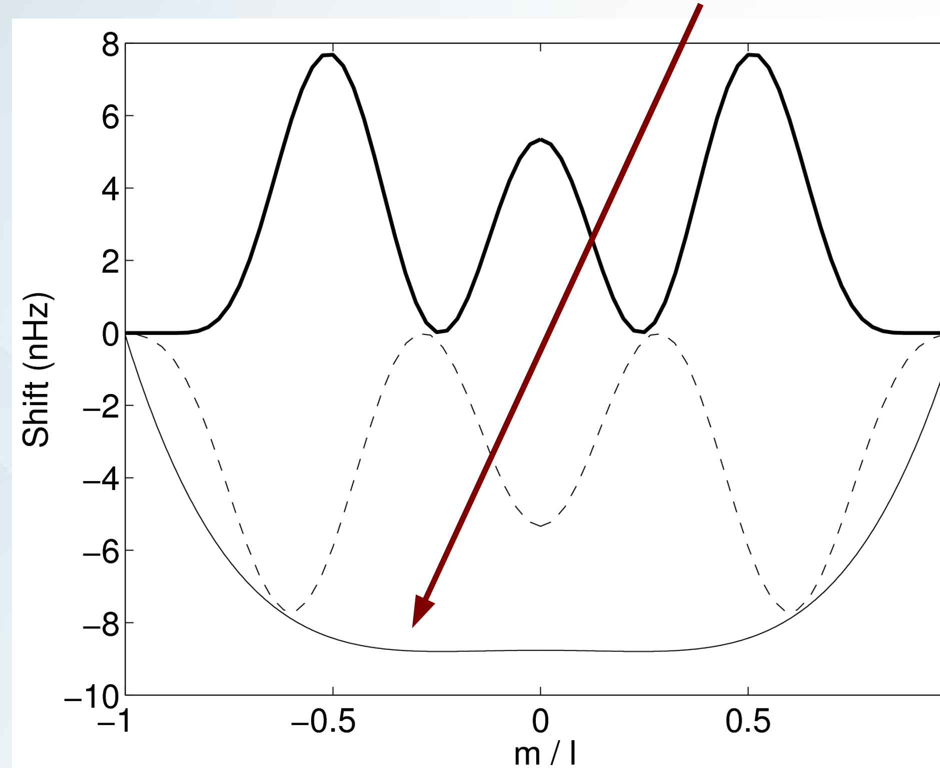
QDPT: Meridional flow

($u_0 = 9\ ms^{-1}$, $s = 2$, t=0, Y_2^0)



$$\delta\omega\left(m\right)\propto m^{2q}\ ,u_0^2$$

For *(n, l)*=(1, 292)



la_1 and la_2 as a function of turning point radius r_t

Max δω ~ −12.5 nHz!

Max la2 ~ 8.5 nHz !

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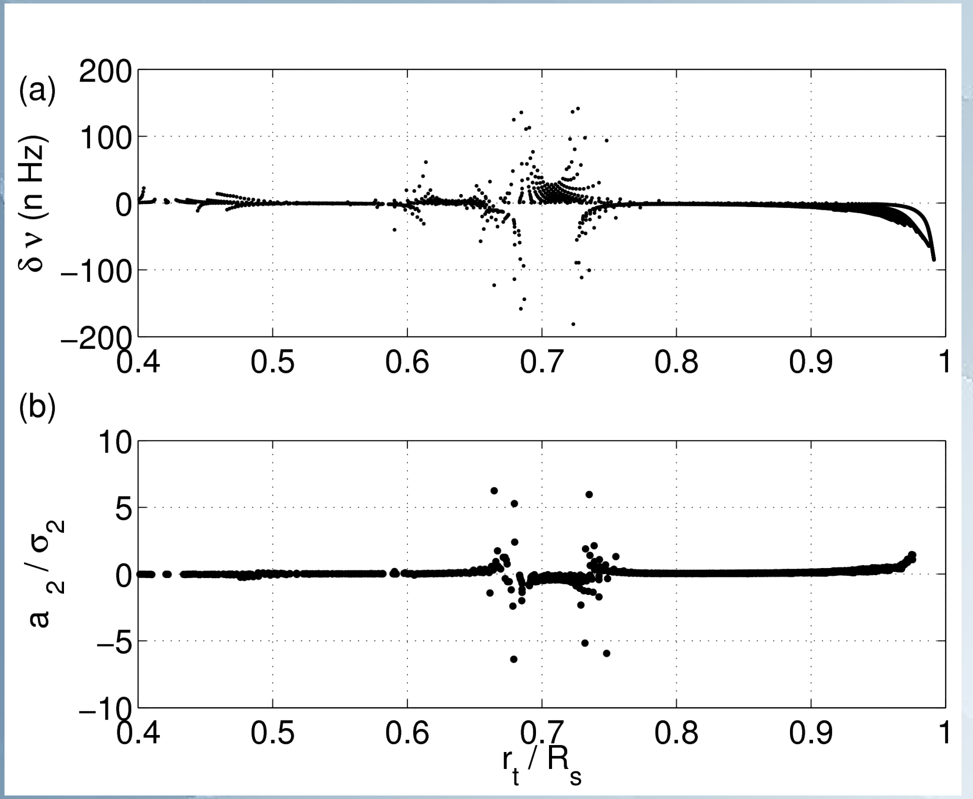
QDPT: Meridional flow

(u_0 = 100 ms^{-1}, s = 8, t=0, Y_8^0)



Increasing s allows more multiplets with $|\omega_1 - \omega_2| < 100\ \mu Hz$ to couple.

More instances of "near degeneracy" such that $\delta\nu \propto \Delta^{-1}$ can become very large

where, Δ = difference in squared frequency of the coupling modes

Example: (n, l) = (17, 56) and (16, 64) have $\Delta / 2\nu$ = -0.06μHz
la_2 = 480 nHz

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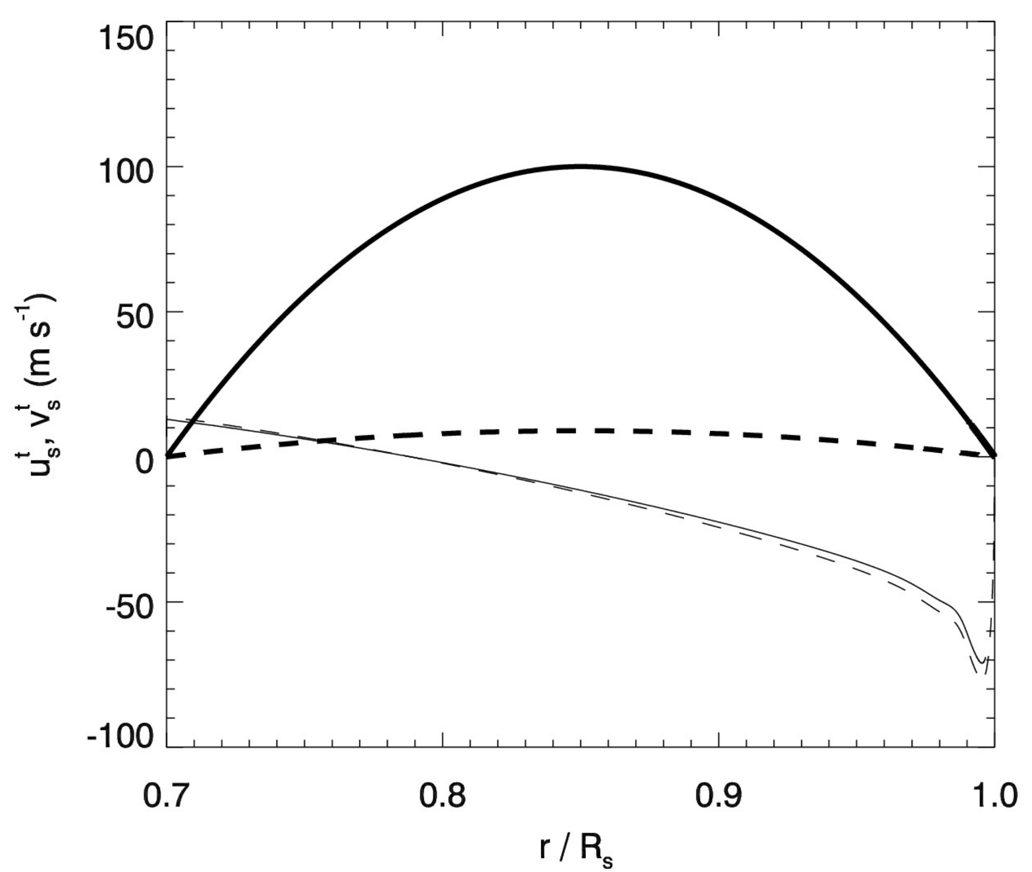
QDPT: Giant Cells

(u_0 = 100 ms^{-1}, s = 8, t=4, 8,$Y_8^{\,8}$ $Y_8^{\,4}$)



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QDPT: Giant Cells

($u_0 = 100$ ms^{-1}, $s = 8$, t=4, 8, Y_8^8 Y_8^4)

- Expression for $u(r, \theta, \varphi)$ involves $\boldsymbol{Y}_8^4(\theta, \varphi)$ or $\boldsymbol{Y}_8^8(\theta, \varphi)$ (banana cells) $\Rightarrow$ coupling between different m, m' of p-modes.
- *Important to take effect of rotational splitting on* Δ, before calculating the effect of flows with $t \neq 0$.
- Basically $\Delta \equiv \Delta(m)$ unlike for $t = 0$ cases.
- Rotational splittings obtained from temporally averaged GONG data.
- Giant cells cause splittings asymmetric about $m = 0$; odd a_{2q+1} affected. To what extent is rotational inversion affected?

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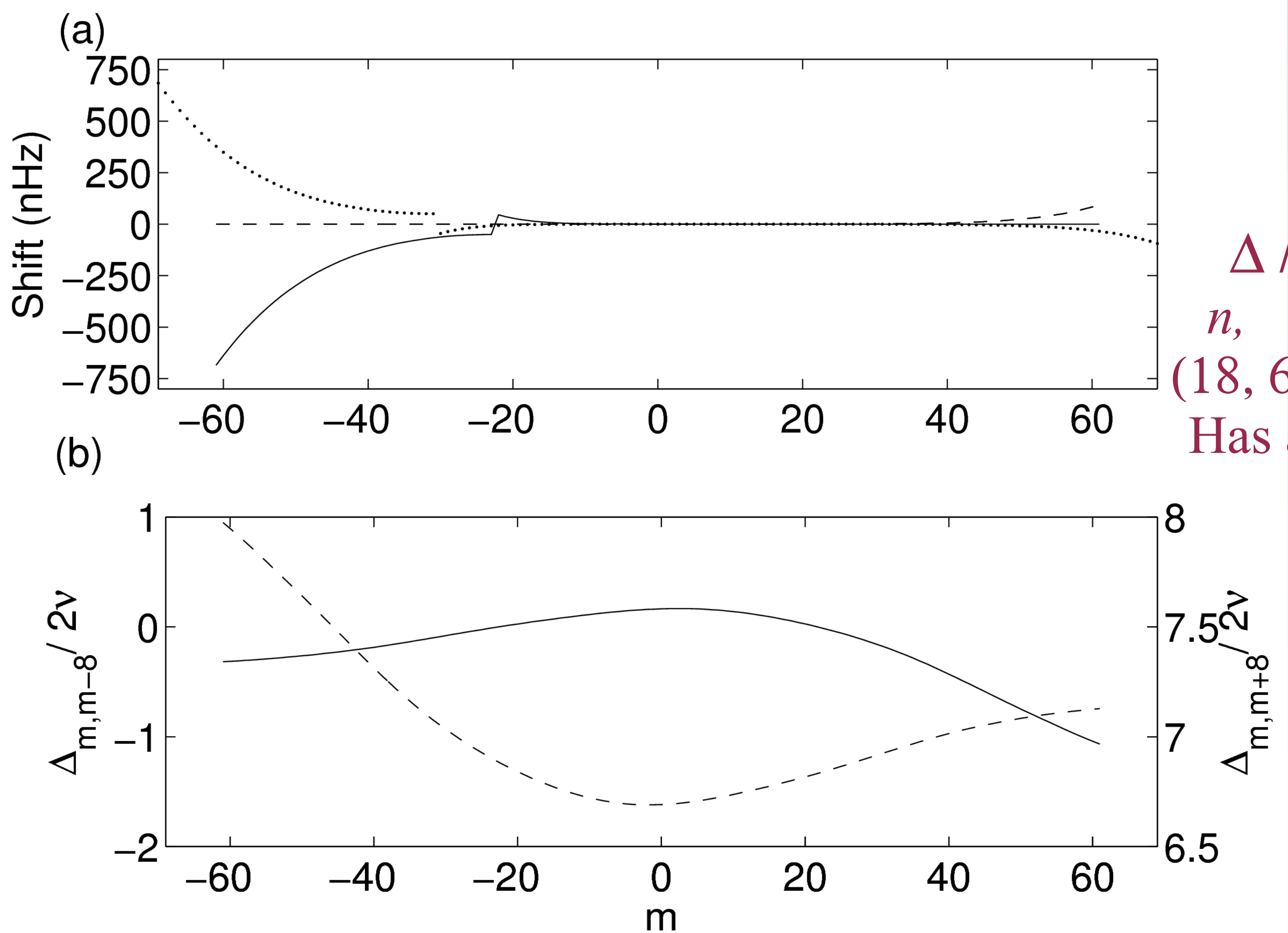
QDPT: Giant Cells

($u_0 = 100\ ms^{-1}$, $s = 8$, $t=8$, Y_8^8)



Δ / 2ν (μ*Hz*) due to

n, *l* *n*, *l*

(18, 61, *m*)⇔(17, 69, *m*±8)

Has a "near degeneracy"

at m=-22

$\Delta(m)=0$

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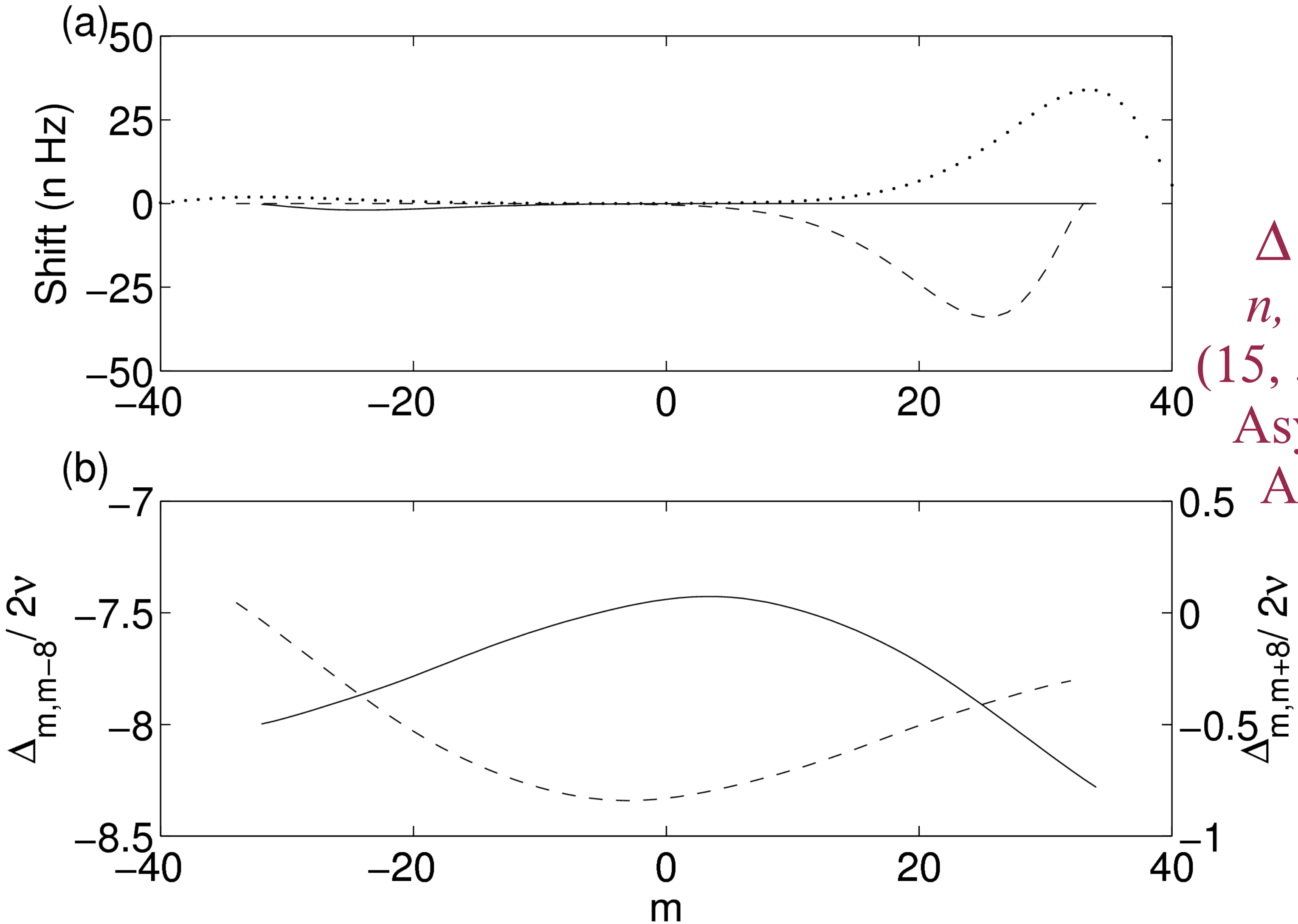
QDPT: Giant Cells

($u_0 = 100\ \mathrm{ms}^{-1}$, $s = 8$, $t=8$, Banana cells)



$\Delta / 2\nu$ (μHz) due to

n, l n, l

(15, 34, m)$\Leftrightarrow$(14, 40, $m\pm8$)

Asymmetric about m=0

Affects odd splitting coeff.

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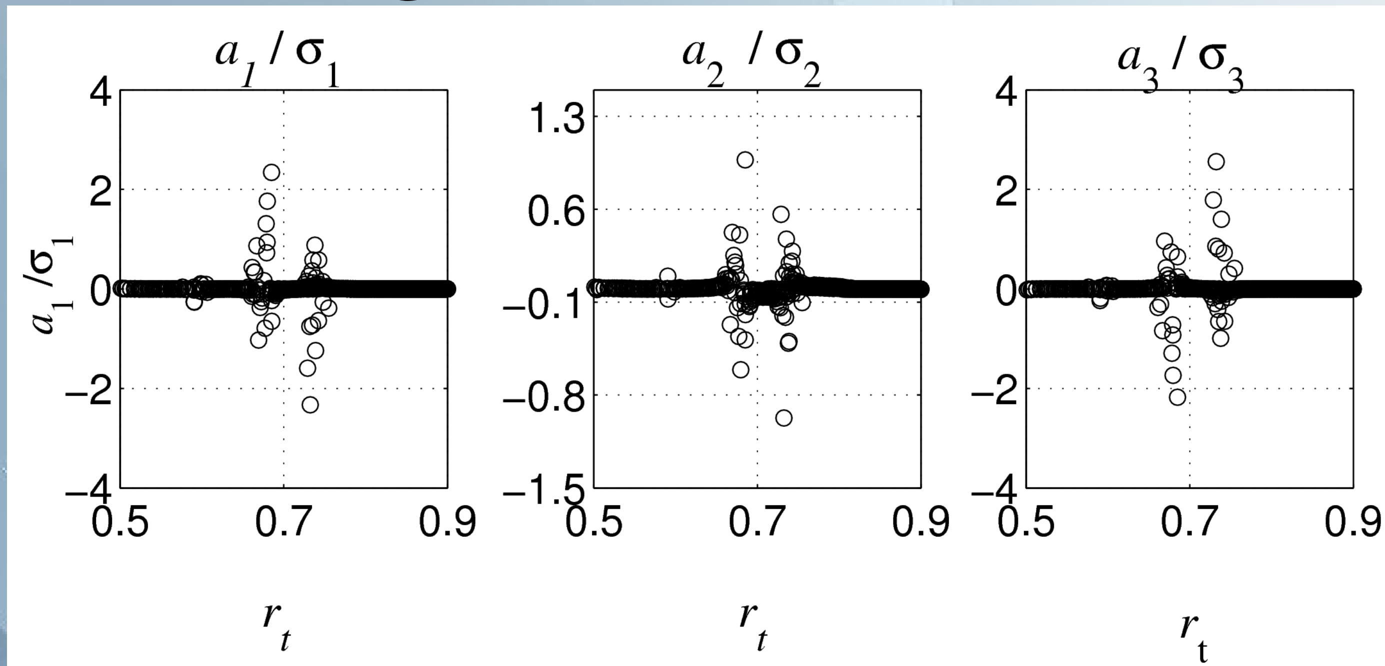
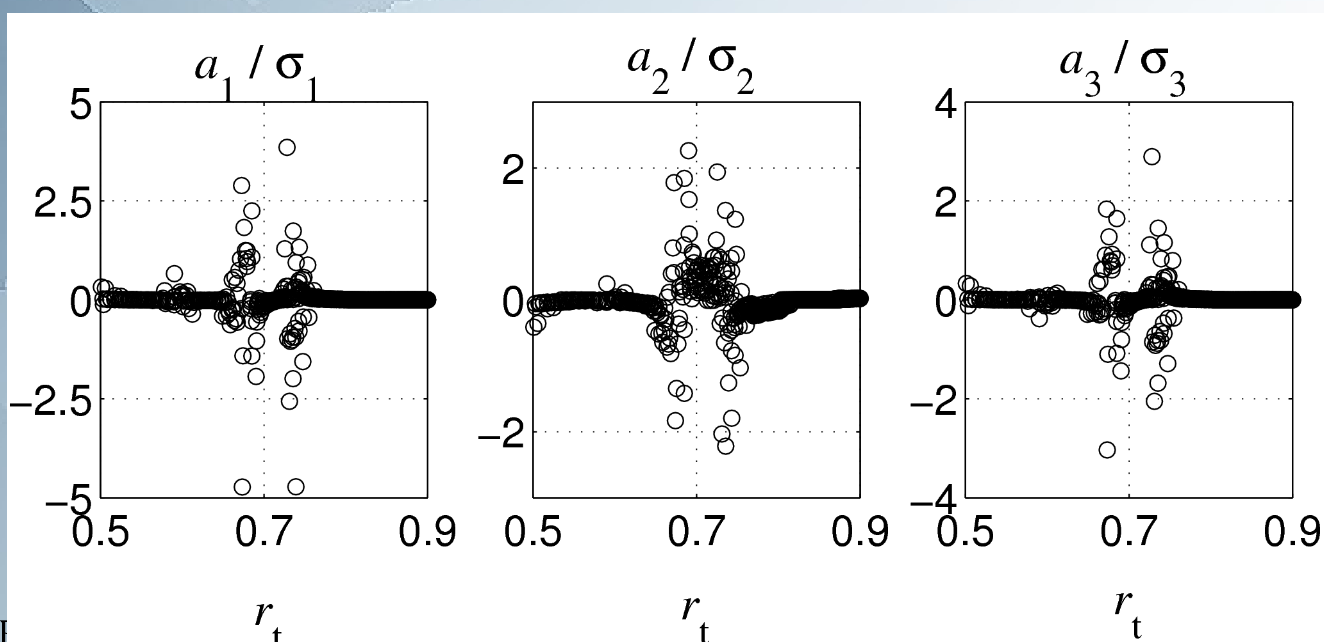
QDPT: Giant Cells

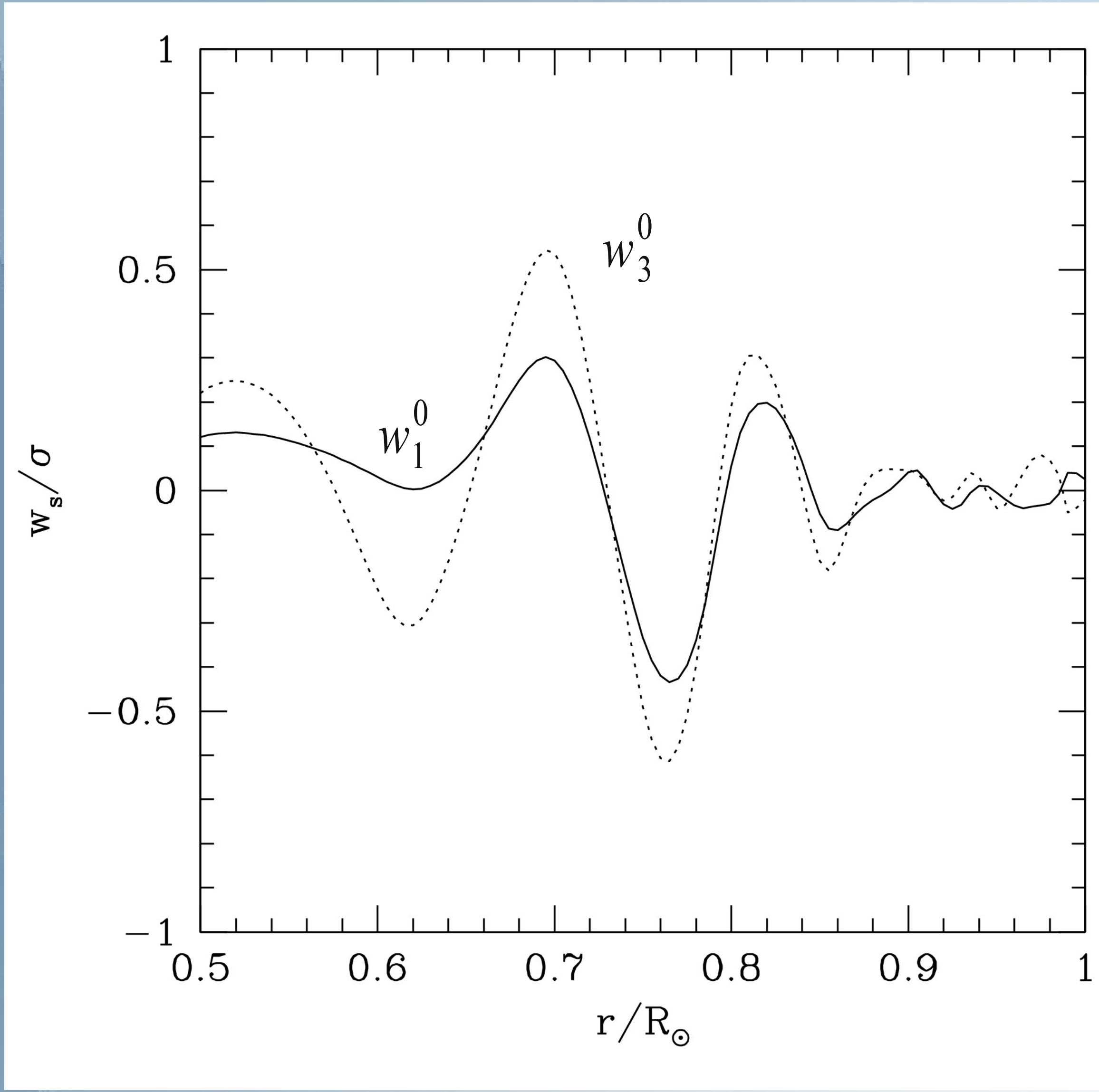


$Y_8^4(\theta,\varphi)$



$Y_8^8(\theta,\varphi)$
(Banana cells)

QDPT: 1.5d inversion



To what extent is rotational inversion distorted?
Ans: No appreciable distortion

For Y_8^8 flow,

1.5d helioseismic inversion using errors in GONG data and theoretical calculated a_1, a_3

$$\Omega r \sin\theta = -w_1^0 \partial_\theta Y_1^0 - w_3^0 \partial_\theta Y_3^0$$

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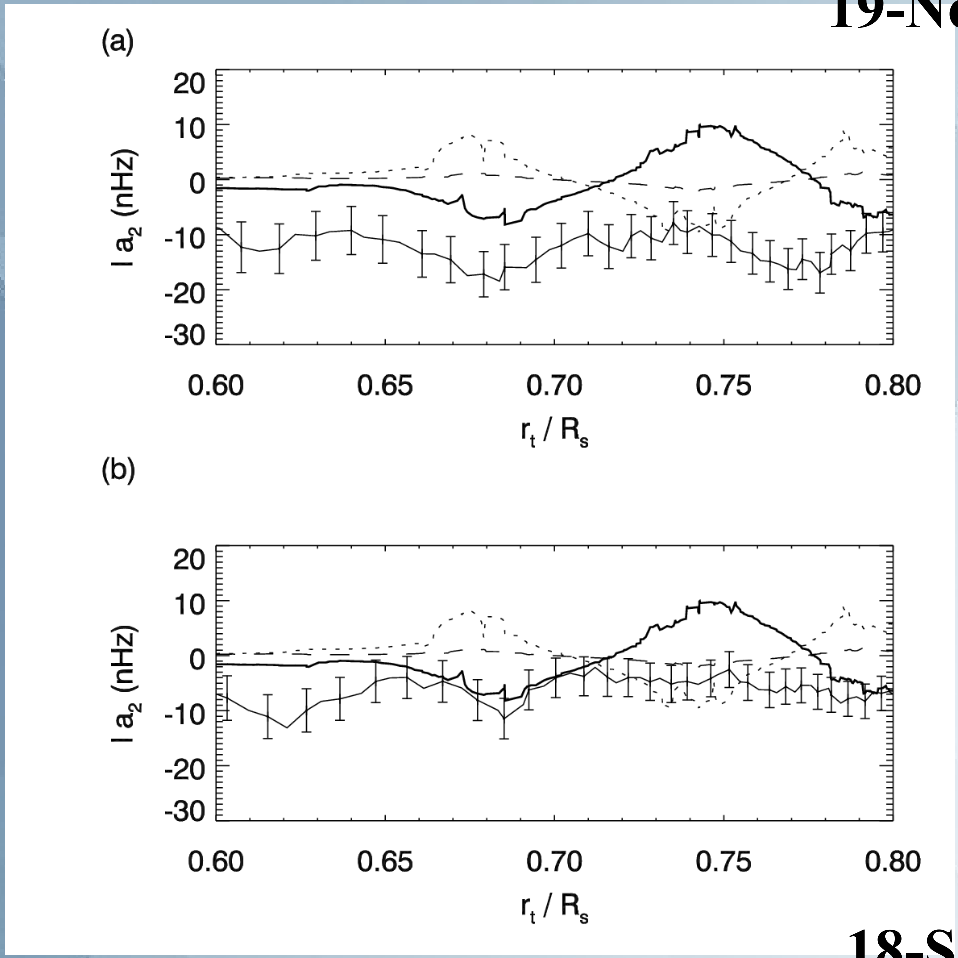
Observational splitting coefficients

- It is only the "nearly degenerate" modes we may hope to detect in observations.
- Use 7 different GONG data sets covering late descending part of cycle 22 – end of cycle 23
- Most contribution to a_2 comes from surface effects. But we are interested in modes with $r_t \sim 0.7$
- Multiplets sorted in order of increasing r_t and smoothed using a 100 point error weighted average.

Observational splitting coefficients

19-Nov-2002



18-Sep-2007

- Only way to put an upper limit on flow is to look for nearly degenerate modes in the data .
- Problem: running mean combines results with positive and negative a_q
- Separate into two groups on basis of theoretical results: $la_2 < -10$ *nHz* & $la_2 > 10$ *nHz*.

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Observational splitting coefficients

- Consider modes with la_2<-10 nHz and search for them in each GONG data set.
- Average la_2 from theory over available multiplets and error σ_2 in a_2 from observations.
- For Y_8^4 flow with u_0= 100 ms^{-1}, $|la_2|$~22 *nHz (theory)*, σ_2 ~ 14 *nHz (obs)*. Ruled out with a CL = la_2 / σ_2 = 1.5
- For Y_8^8 flow with u_0= 50 ms^{-1}, $|la_2|$~28 *nHz (theory)*, σ_2 ~ 13 *nHz (obs)*. Ruled out with a CL = la_2 / σ_2 = 2.0

Summary

- Meridional flow (Y_2^0) : theoretical coefficients la_2 are 7 times smaller than corresponding errors in observations. Not sensitive to no. of cells in radial direction. Impossible to detect return flow by this method.
- Meridional flow (Y_8^0) and giant cells (Y_8^8, Y_8^4): "Nearly degenerate" modes with r_t~0.7 have large values of la_2. Can hope to detect in observations.
- However we do not find any clear signal around r_t~0.7, so we put upper limits on strength of flows by comparing la_2 (theory) with corresponding errors (observations).