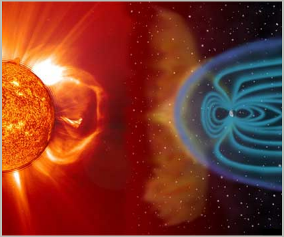
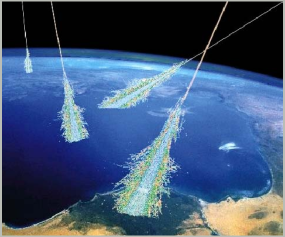
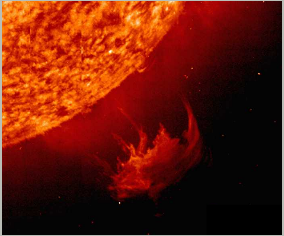
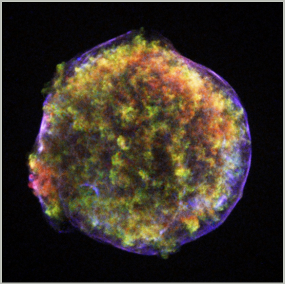
A Review: Theories for Diffusion of Charged Particles

by Andreas Shalchi, University of Manitoba, Canada









Stochastic Particle Motion

Mean square deviation (MSD)

$$\left\langle (\Delta x)^2 \right\rangle = \left\langle (x(t) - x(0))^2 \right\rangle \sim t^{\sigma}$$

Different cases:

$$\begin{aligned} 0 < \sigma < 1 \quad &: \quad \text{Subdiffusion} \\ \sigma = 1 \quad &: \quad \text{(Markovian) Diffusion} \\ \sigma > 1 \quad &: \quad \text{Superdiffusion} \end{aligned}$$

Diffusion coefficient (Kubo-formula)

$$\kappa_{xx} = \lim_{t \to \infty} \frac{\left\langle (\Delta x)^2 \right\rangle}{2t} = \int_0^{\infty} dt \ \langle v_x(t) v_x(0) \rangle$$

Equations of Motion

Parallel diffusion:

$$\begin{aligned}
\dot{\mu} &= \frac{\Omega}{v}\left(v_x \frac{\delta B_y}{B_0} - v_y \frac{\delta B_x}{B_0}\right) \\
\rightarrow D_{\mu\mu}(\mu) &= \int_0^\infty dt \ \langle \dot{\mu}(t)\dot{\mu}(0)\rangle \\
\rightarrow \kappa_\parallel &= \frac{v^2}{8}\int_{-1}^{+1} d\mu \ \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)}
\end{aligned}$$

Perpendicular diffusion:

$$\begin{aligned}
v_x &= v_z \frac{\delta B_x}{B_0} \\
\rightarrow \kappa_\perp &= \int_0^\infty dt \ \langle v_x(t) v_x(0)\rangle
\end{aligned}$$

Turbulence and Transport Theory

In Cosmic Ray diffusion theory terms of the form

$$\langle \delta B_i(\vec{x}(t)) \delta B_j(\vec{x}(0)) \rangle$$

occur!

Employ a Fourier Transformation

$$\delta B_i(\vec{x}(t)) = \int d^3k \, \delta B_i(\vec{k}, t) e^{i\vec{k}\cdot\vec{x}(t)}$$

to find

$$P_{ij}(\vec{k}, t) = \left\langle \delta B_i(\vec{k}, t) \delta B_j^*(\vec{k}, 0) \right\rangle .$$

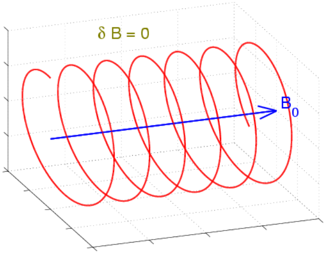
Problem I: What are the magnetic fields/correlation functions described by $P_{ij}(\vec{k}, t)$? $\Rightarrow$ see first talk!

Problem II: How can we replace the particle trajectory $\vec{x}(t)$?

Quasilinear Theory

Principle of the quasilinear theory (QLT, Jokipii 1966):

replace $\vec{x}$ by the unperturbed trajectory



$\Rightarrow$ QLT is a first-order perturbation theory!

Quasilinear Theory

The resonance functions $R_\pm$ describe the interaction between plasma and particle. Example for a quasilinear result:

$$D_{\mu\mu} = \frac{2\pi v^2(1-\mu^2)}{B_0^2 R_L^2} \int_0^\infty dk_\parallel \, g^{slab}(k_\parallel) \left[R_-(k_\parallel) + R_+(k_\parallel)\right]$$

(Particles, Fields, Interaction)

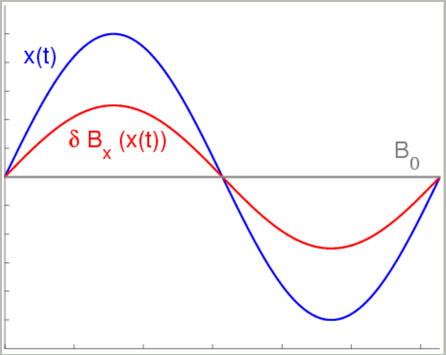
Resonance function of QLT

$$R_\pm^{(QLT)} = \pi\delta\left(k_\parallel v_\parallel \pm \Omega\right)$$

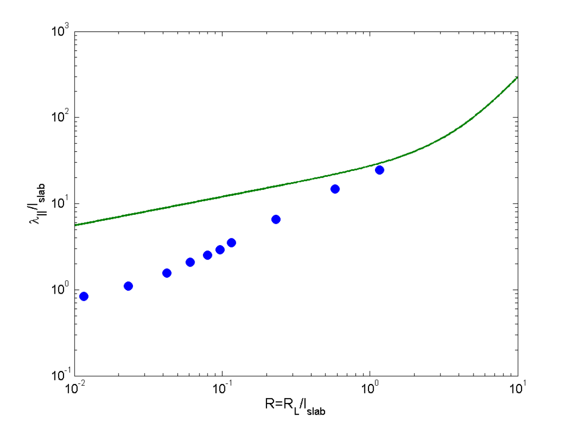
$\Rightarrow$ only interaction for

$$\frac{v_\parallel}{\Omega} \equiv \mu R_L = \frac{1}{k_\parallel}$$

(Gyro-resonance)

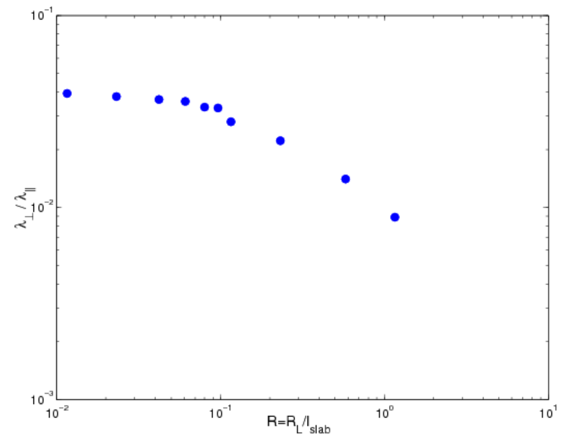


Comparison with Simulations



The parallel mean free path: simulations and quasilinear theory.

Comparison with Simulations



The perpendicular mean free path: simulations.
In this case we find by employing QLT $\lambda_\perp/\lambda_\parallel = \infty$.

Nonlinear Particle Diffusion Theory

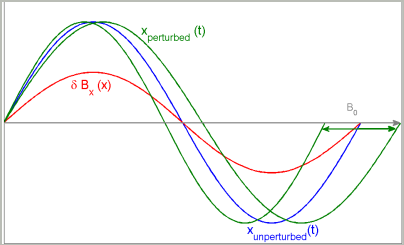
Weakly Nonlinear Theory (WNLT) developed by Shalchi et al. (2004):

Idea: nonlinearity due to resonance broadening

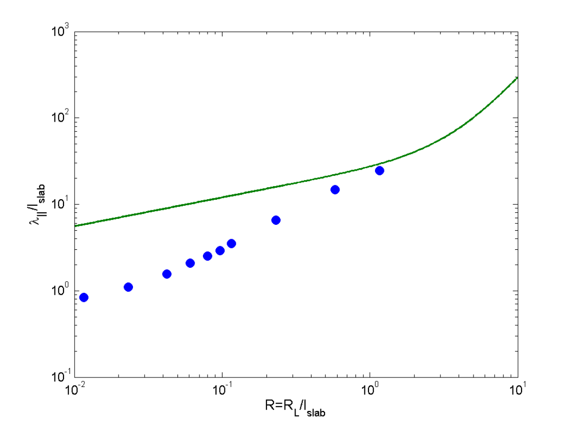
$$R_{\pm}^{(QLT)} = \pi\delta\left(k_{\parallel} v_{\parallel} \pm \Omega\right)$$

$$\rightarrow R_{\pm}^{(WNLT)} = \frac{\alpha}{\alpha^2 + (k_{\parallel} v_{\parallel} \pm \Omega)^2}$$

with $\alpha = \alpha\left(\kappa_i\right)$.



Parallel Diffusion: Nonlinear Theory and Simulations



The parallel mean free path: simulations, quasilinear theory, and nonlinear theory of Shalchi et al. 2004.

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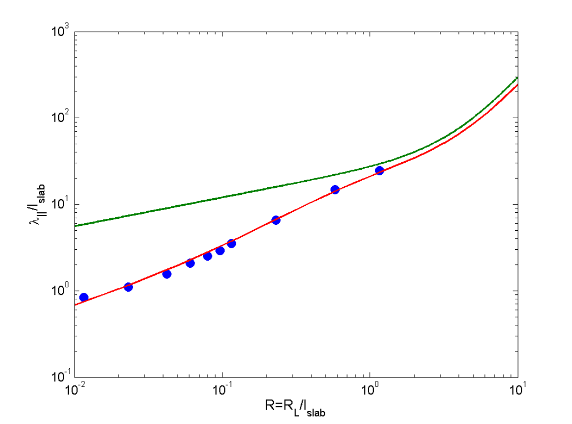

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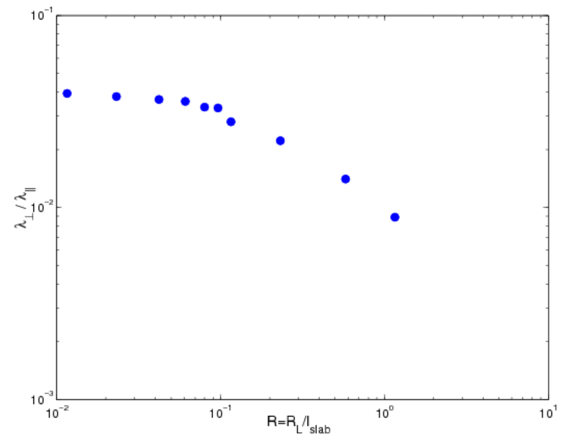

The perpendicular mean free path: simulations and nonlinear theory of Shalchi et al. 2004. The QLT-result is $\lambda_\perp/\lambda_\parallel = \infty$.

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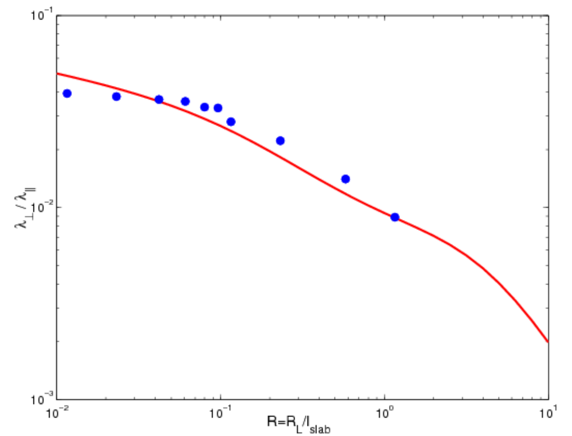

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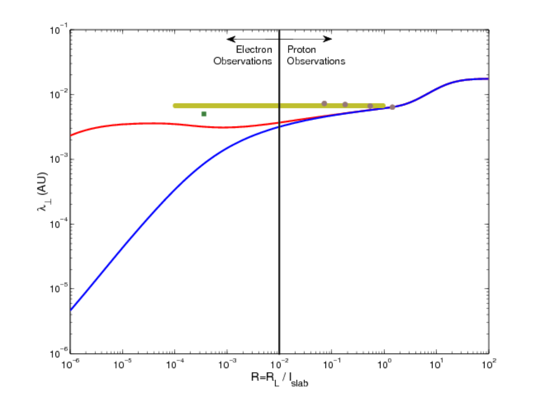
Application I - Particles in the Solar System



The perpendicular mean free path: theoretical results from Shalchi et al. (2006) for electrons and protons in comparison with different observations: Jovian electrons, Ulysses measurements of Galactic protons, and the value suggested by Palmer (1982).

Application II - Transport of Cosmic Rays in the Galaxy

Transport of cosmic rays in the Galaxy:

Interaction between cosmic rays and the interstellar medium

$\Rightarrow$ Primary cosmic particles decay in lighter nuclei

$\Rightarrow$ Secundary cosmic rays are generated

$\Rightarrow$ The measured decrease of the abundance ratio of secondary to primary cosmic ray nuclei as B/C and N/O at kinetic energies above $1GeV$ per nucleon allows the determination of the life time of the particles in the Galaxy

Application II - Transport of Cosmic Rays in the Galaxy

The measurements indicate that (see, e.g., Swordy et al. 1990):

$$v\tau \sim R^{-a}, \quad a = 0.6 \pm 0.1$$

$\tau =$life time of the particles in the Galaxy

For diffusive transport:

$$\kappa_{\parallel} = \frac{< (\Delta z)^2 >}{2t} \sim \frac{L^2}{2\tau} \sim v\lambda_{\parallel}$$

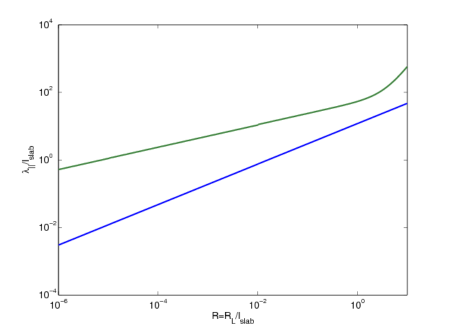
$L =$thickness of the Galactic disk

and, therefore,

$$\lambda_{\parallel} \sim \frac{1}{v\tau} \sim R^{a}$$

Application II - Transport of Cosmic Rays in the Galaxy

The measurements indicate that: $\lambda_{\|} \sim R^{0.6} \sim E^{0.6}$ (Swordy et al. 1990)



The parallel mean free path in the interstellar medium: quasilinear result in comparison with a $R^{0.6}-$fit and the nonlinear result from Shalchi & Schlickeiser (2005).

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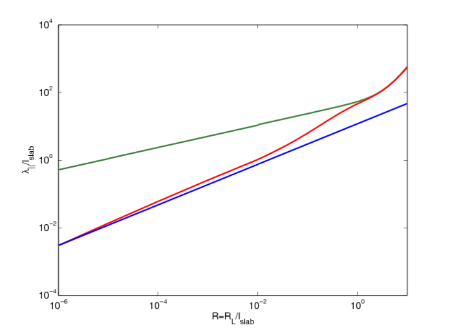

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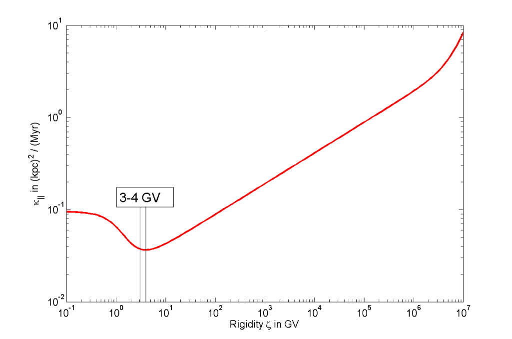
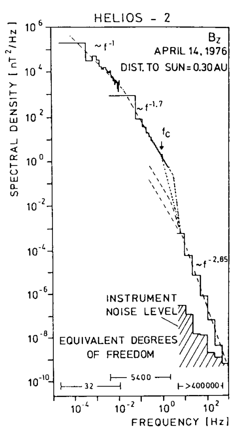
Application II - Transport of Cosmic Rays in the Galaxy

Nonlinear scattering of low energy cosmic rays in the Galaxy
(see Shalchi & Büsching, 2010, submitted)





$\Rightarrow$ Break in the rigidity dependence at $R \approx 3GV$!

Summery & Outlook

Summery:

- To understand the motion of charged cosmic rays we need an improved understanding of MHD-turbulence
- Nonlinear effects are essential for computing charged particles orbits
- Nonlinear diffusion theories are relevant for describing cosmic ray propagation and acceleration in the solar system, the ISM, ...

Ongoing and Future work:

- Improved (phenomenological) models for turbulence (Maltese cross, general wave spectrum, dynamical turbulence)
- Further improvement of the (nonlinear) scattering theory and further applications
- Computer-simulations for realistic turbulence models

For a review see Book:

Shalchi, A., *Nonlinear Cosmic Ray Diffusion Theories,*
2009, Astrophysics and Space Science Library, Vol. 362. Berlin: Springer