General Introduction to Charged Particle Transport in Turbulence

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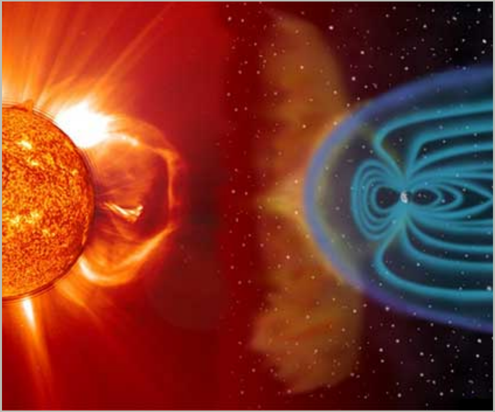
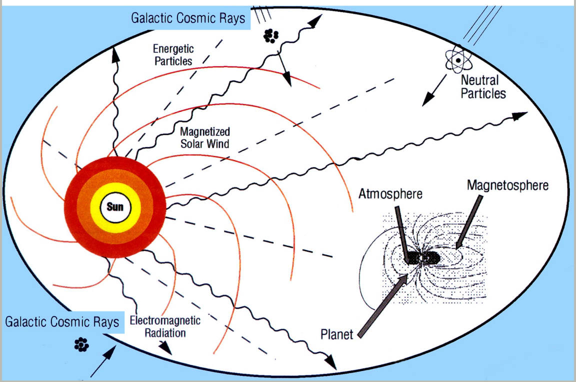


Example: Transport of Cosmic Rays in the Solar System



Interaction of charged particles with:

- the magnetic field of the Sun $\vec{B}_0 \approx B_0 \vec{e}_z \approx const.$
- the turbulent magnetic fields of the solar wind $\delta \vec{B}$

Magnetic Correlation Functions

Dominant mean magnetic field $\vec{B}_0 = B_0 \vec{e}_z$ (magnetic field of the Sun).

Total magnetic field is a composition of the mean field and a turbulent component: $\vec{B} = \vec{B}_0 + \delta\vec{B}$

Two-Point-Two-Time correlation function in homogeneous turbulence:

$$\left\langle \delta B_i(\vec{x}, t) \delta B_j(\vec{0}, 0) \right\rangle = \int d^3k \; P_{ij}(\vec{k}, t) e^{i\vec{x}\cdot\vec{k}}$$

where $P_{ij}(\vec{k}, t) =< \delta B_i(\vec{k}, t) \delta B_j(\vec{k}, 0) >$ is the correlation tensor in the wavenumber space.

Models for Solar Wind Turbulence

Standards assumption (same temporal behavior of all tensor components):

$$P_{ij}(\vec{k}, t) = \Gamma(\vec{k}, t) P_{ij}(\vec{k}).$$

$P_{ij}(\vec{k})$ is the static correlation tensor and $\Gamma(\vec{k}, t)$ is the dynamical correlation function describing wave propagation effects and dynamical turbulence effects.

General form for axisymmetric turbulence (Matthaeus & Smith 1981):

$$P_{ij}(\vec{k}) = A(k_\parallel, k_\perp) \left[\delta_{ij} - \frac{k_i k_j}{k^2} \right].$$

Properties of the spectrum $A(k_\parallel, k_\perp)$ can be measured (in the solar system) or must be approximated by models.

Slab Turbulence

A simple model for approximating the turbulence is the slab model defined as

$$A^{slab}(k_{\parallel}, k_{\perp}) = g^{slab}(k_{\parallel}) \frac{\delta(k_{\perp})}{k_{\perp}}$$

corresponding to $\delta B_i(\vec{x}) = \delta B_i(z)$. $g^{slab}(k_{\parallel})$ is the (slab) wave spectrum.

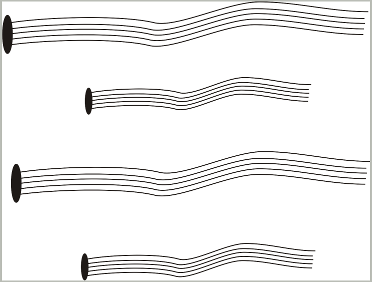


Fig.: The magnetic field lines for slab turbulence.

The Turbulence Spectrum

An important property of turbulence is the scale dependence:

- Large scales: the turbulence gains energy ⇒ energy range
- Intermediate scales: energy is transfered from large to small scales ⇒ inertial range
- Small scales: turbulence loses energy due to dissipation ⇒ dissipation range

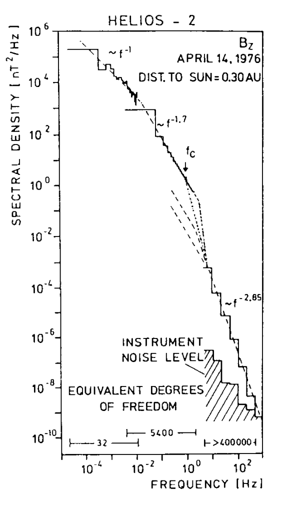


Fig.: The turbulence spectrum as measured in the solar wind (from Denskat & Neubauer 1982).

Spectral Anisotropy

The slab model is not a good approximation for solar wind turbulence. According to Matthaeus et al. (1990) there is a strong perpendicular component (*Maltese cross*):

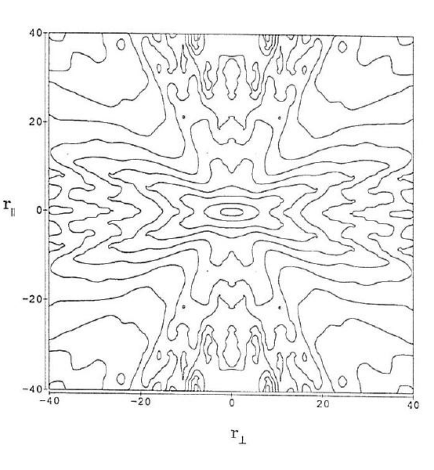


Fig.: Contour plot of correlations of interplanetary magnetic field fluctuations as a function of parallel and perpendicular distance with respect to the mean magnetic field (from Matthaeus et al. 1990).

Two-component Turbulence

Approximation: Two-component or Slab / 2D composite model:

$$\delta B_i(\vec{x}) = \delta B_i(z) + \delta B_i(x, y).$$

The correlation functions of the 2D modes is

$$A^{2D}(k_\parallel, k_\perp) = g^{2D}(k_\perp)\frac{\delta(k_\parallel)}{k_\perp}$$

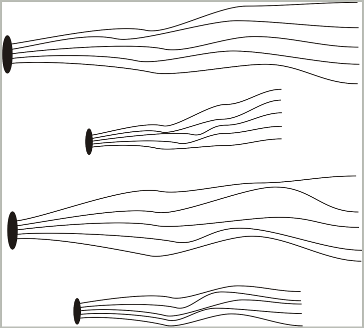


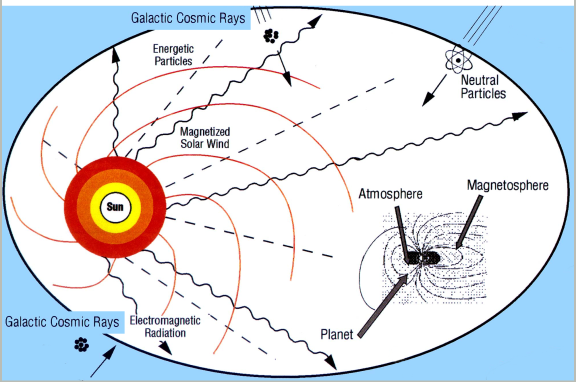
Fig.: The magnetic field lines for combined slab/2D turbulence.

Alternative Models for Turbulence

The slab/2D model is used by the solar wind community. However, there are other models to approximate turbulence. Some examples are:

- Isotropic Turbulence: No preferred direction.
- Anisotropic Turbulence Models (see, e.g., Lerche & Schlickeiser 2001).
- Models based on the Goldreich-Sridhar model (see, e.g., Chandran 2000, Cho et al. 2002).

Transport of Cosmic Rays in the Solar System



Interaction of charged particles with:

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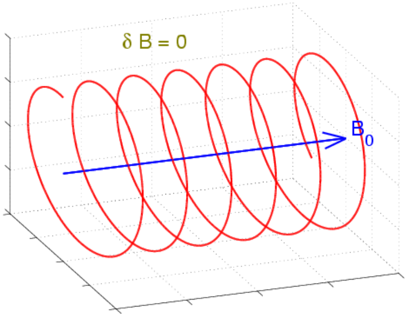
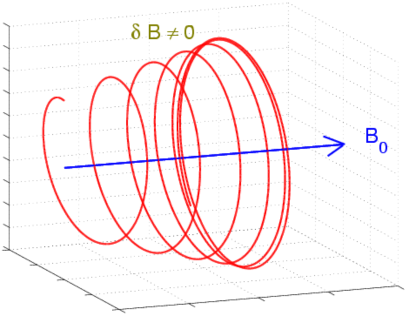
The Unperturbed Orbit



$$
\begin{aligned}
x(t) &\approx \frac{v}{\Omega}\sqrt{1-\mu^2}\sin(\Omega t) \\
y(t) &\approx \frac{v}{\Omega}\sqrt{1-\mu^2}\cos(\Omega t) \\
z(t) &\approx v\mu t, \quad \mu = const.
\end{aligned}
$$

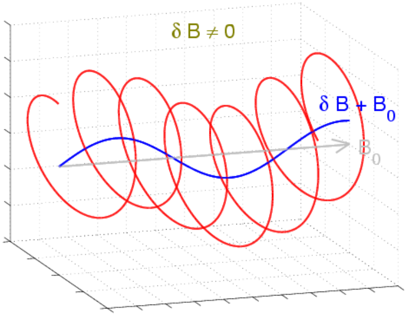
Parallel Scattering of Cosmic Rays



Physical process: pitch-angle diffusion

$$\dot{v}_{\parallel}/v \sim \dot{\mu} = \frac{\Omega}{v}\left(v_x \frac{\delta B_y}{B_0} - v_y \frac{\delta B_x}{B_0}\right) \neq 0$$

Perpendicular Scattering of Cosmic Rays



Physical process: field line random walk

$$dx = \frac{\delta B_x}{B_0} dz \quad \Rightarrow \quad v_x = v_z \frac{\delta B_x}{B_0}$$

Stochastic Particle Motion

Mean square deviation (MSD)

$$\left\langle (\Delta x)^2 \right\rangle = \left\langle \left(x(t) - x(0) \right)^2 \right\rangle \sim t^{\sigma}$$

Different cases:

$$\begin{aligned} 0 < \sigma < 1 \quad &: \quad \text{Subdiffusion} \\ \sigma = 1 \quad &: \quad \text{(Markovian) Diffusion} \\ \sigma > 1 \quad &: \quad \text{Superdiffusion} \end{aligned}$$

Diffusion coefficient (Kubo-formula)

$$\kappa_{xx} = \lim_{t \to \infty} \frac{\left\langle (\Delta x)^2 \right\rangle}{2t} = \int_0^{\infty} dt \ \langle v_x(t) v_x(0) \rangle$$

Turbulence and Transport Theory

$\Rightarrow$ Terms of the form

$$\langle \delta B_i(\vec{x}(t)) \delta B_j(\vec{x}(0)) \rangle$$

occur!

Problem I: What are the magnetic fields/correlation functions?

$$\begin{aligned} \delta B_i(\vec{x}(t)) &= \int d^3k \, \delta B_i(\vec{k}, t) e^{i\vec{k}\cdot\vec{x}(t)} \\ \Rightarrow P_{ij}(\vec{k}, t) &= \left\langle \delta B_i(\vec{k}, t) \delta B_j^*(\vec{k}, 0) \right\rangle \end{aligned}$$

(see first part of this talk).

Problem II: What is the particle trajectory $\vec{x}(t)$?

Summery

The most important questions in Cosmic Ray diffusion theory:

1. What is the nature of turbulence described by the tensor $P_{ij}(\vec{k}, t)$?
   $\Rightarrow$ Cosmic Ray diffusion theory is directly related to plasma physics and turbulence theory!
2. How can we describe the stochastic motion of charged particles in turbulence?
   $\Rightarrow$ Different approaches to solve this problem will be reviewed in the second talk.