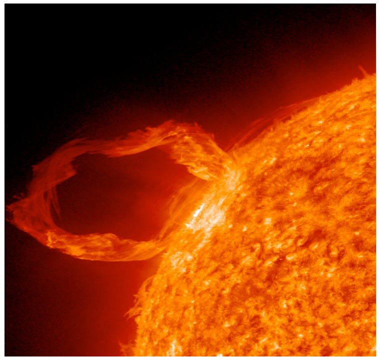
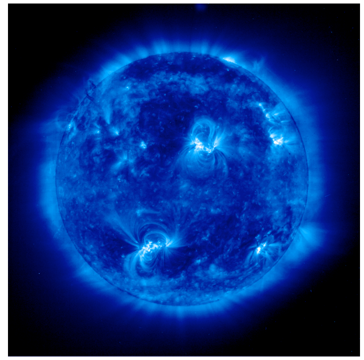
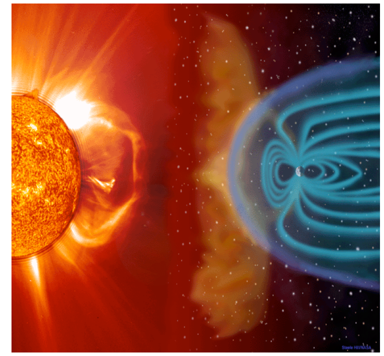





$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$ Induction Equation

$$\frac{D\rho}{Dt} + \rho \nabla . \mathbf{v} = 0$$ Mass Conservation

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g} + \text{Viscous Terms}$$ Motion

$$\frac{\rho^\gamma}{\gamma-1} \frac{D}{Dt}\left(\frac{p}{\rho^\gamma}\right) = \nabla . \left(\kappa_\parallel \nabla T\right) - \rho^2 Q(T) + H(s,t,\mathbf{B},\rho,T)$$ Energy

$$p = \frac{R \rho T}{\mu}$$ Gas Law

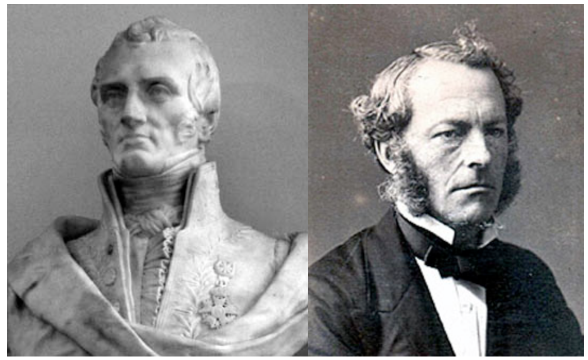
$$\nabla . \mathbf{B} = 0$$ Gauss' Law

Turbulence
(with and without B)

Sébastien GALTIER

IAS - UPS - IUF

Navier-Stokes equations



Claude Navier (1785-1836) ; George Stokes (1819-1903).

$$\partial \mathbf{v}/\partial t + \mathbf{v} \, . \, \nabla \, \mathbf{v} = -\nabla P + \nu \, \Delta \mathbf{v} \, ,$$

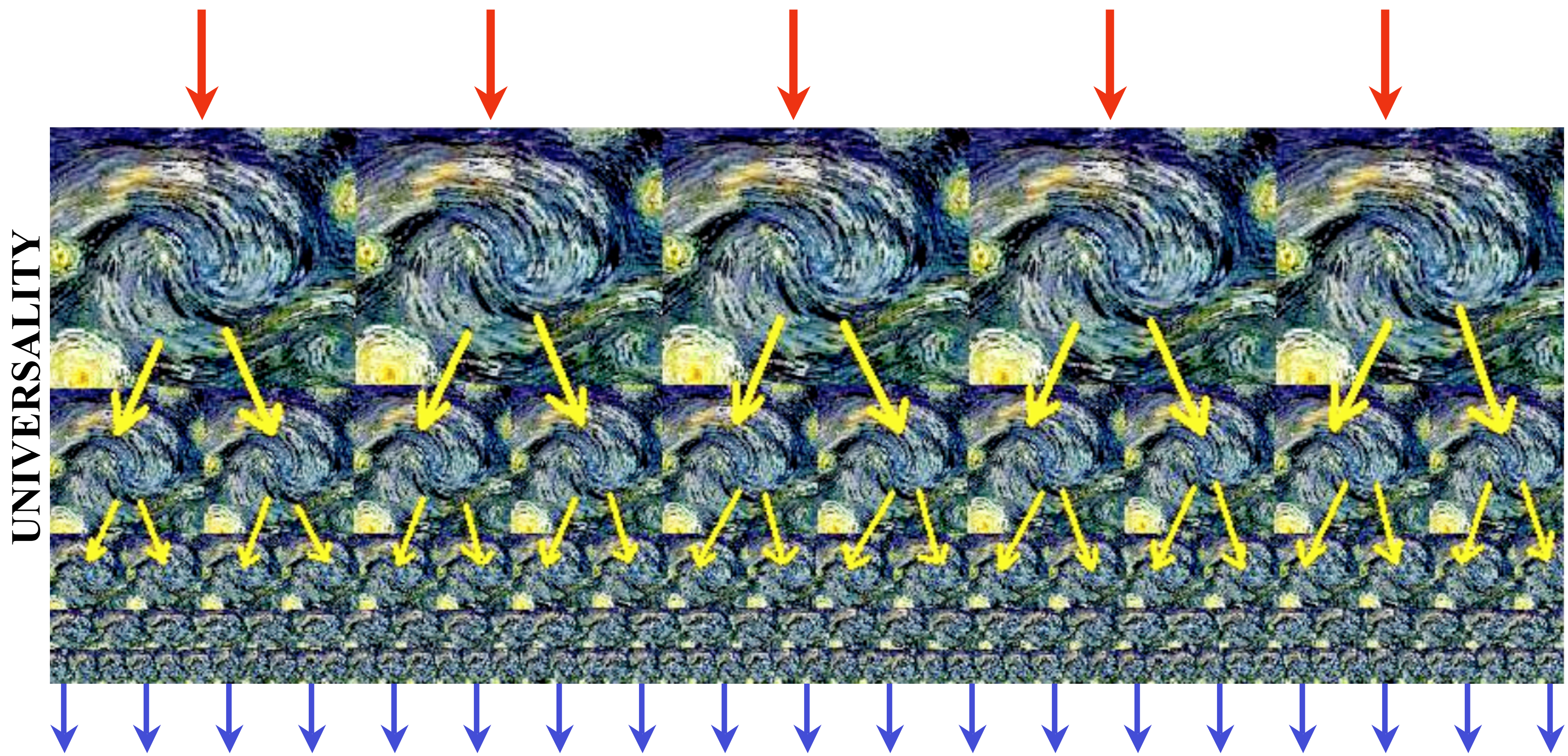
$$\nabla \, . \, \mathbf{v} = 0 \, ,$$

Reynolds number: $R_e = U \, L / \nu$

$$R_e > 1000 >> 1$$

Concept of cascade

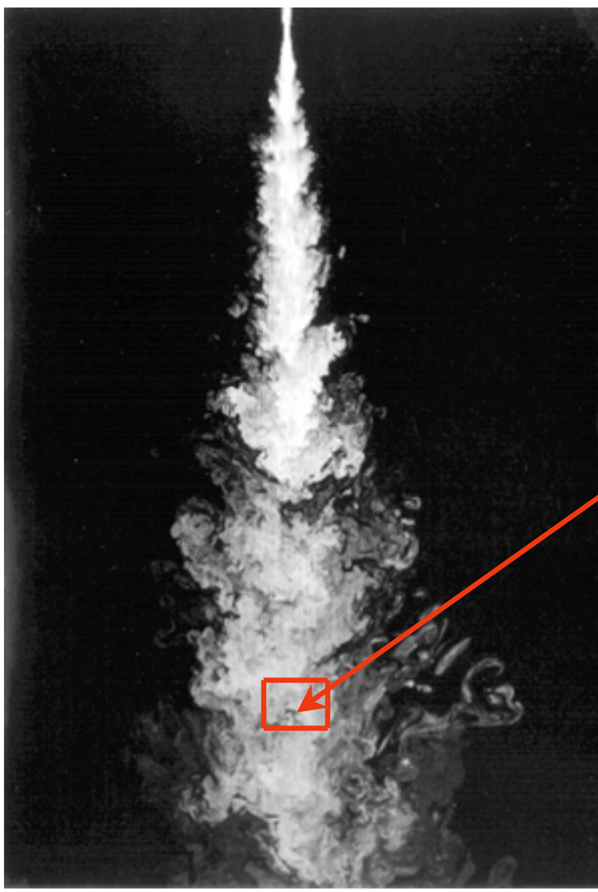
Turbulence source : generation of large eddies



Turbulence sink : dissipation of small eddies

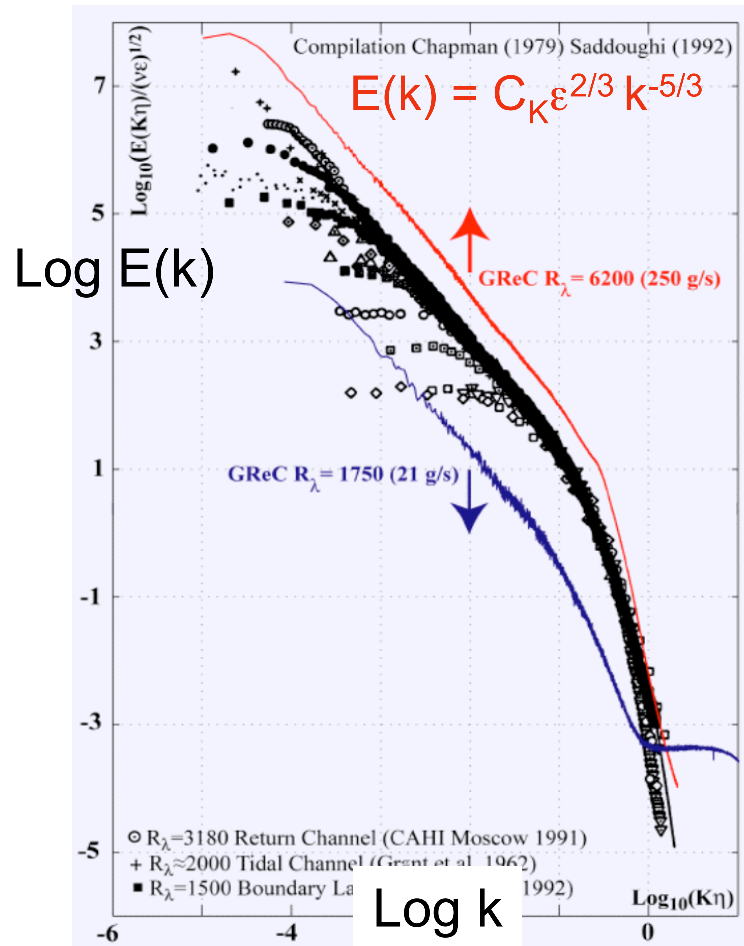
Homogeneous turbulence spectrum

$$\begin{cases} R_{ij} = \langle u_i(\mathbf{x})\, u_j(\mathbf{x}+\mathbf{r}) \rangle = R_{ij}(\mathbf{r}) \\ \phi_{ij}(\mathbf{k}) = (1/2\pi)^3 \iiint_{R^3} R_{ij}(\mathbf{r})\, e^{-i\mathbf{k}.\mathbf{r}}\, d\mathbf{r} \end{cases} \rightarrow \quad E(\mathbf{k}) = (1/2)\, \phi_{ii}(\mathbf{k})$$



Here

Kolmogorov's spectrum



Solar wind turbulence

Taylor hypothesis:

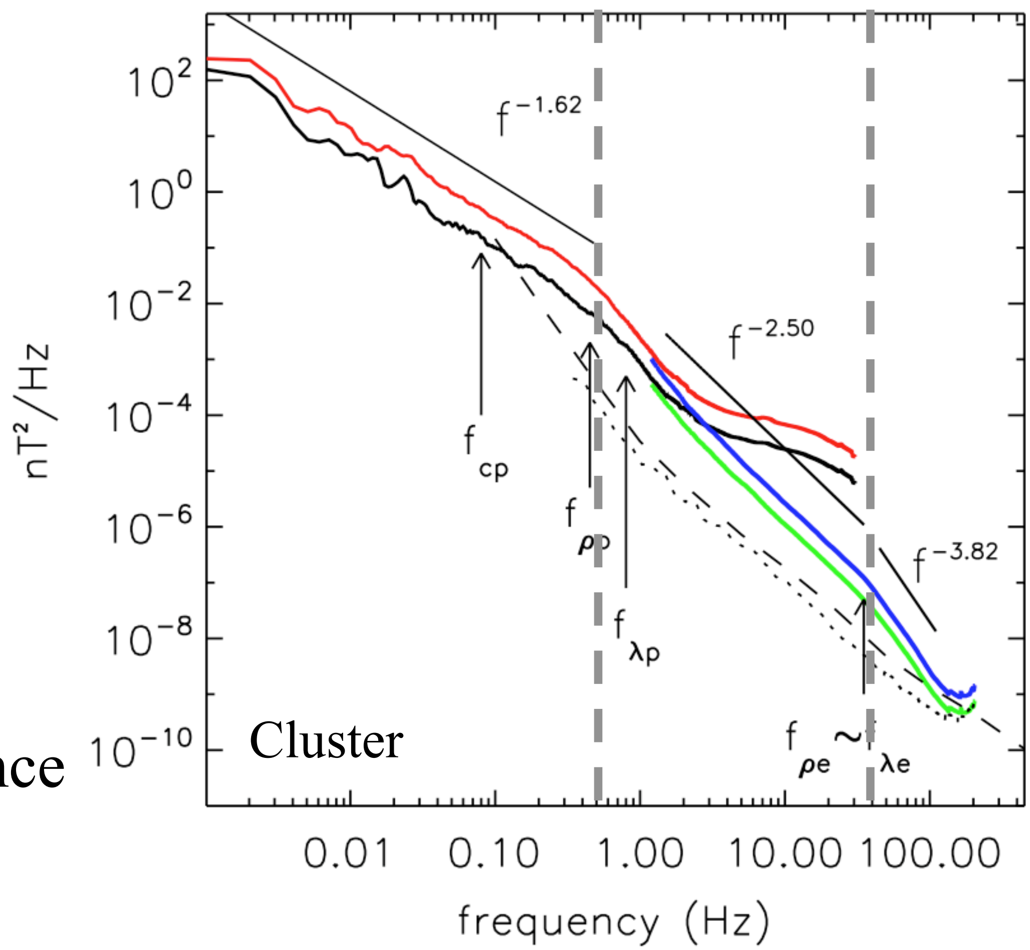
$k \approx 2\pi f / V_{SW}$

Possible models:

f < 1/2Hz : **MHD** turbulence

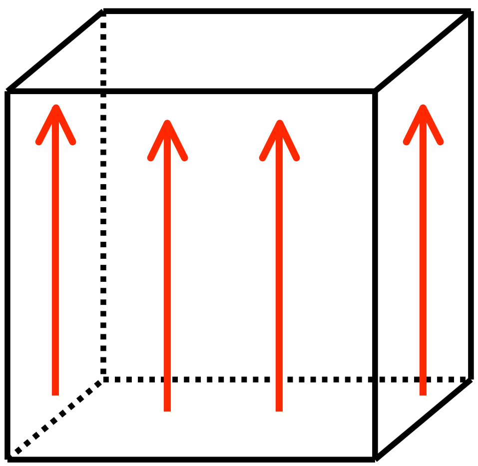
f > 1/2Hz : **Hall MHD** turbulence

Magnetic fluctuations spectrum



[Sahraoui et al., PRL, 2009]

Regimes of turbulence in MHD

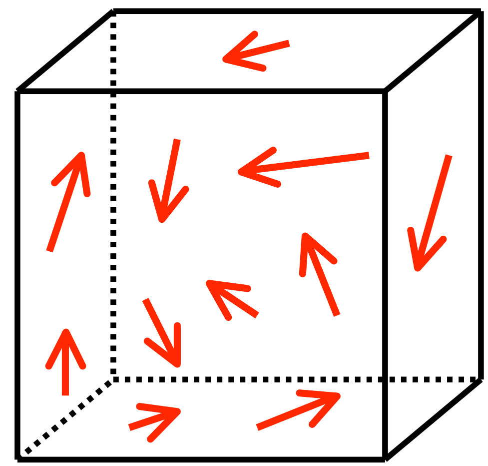


$\mathbf{B} = \mathbf{B_o} + \boldsymbol{\varepsilon}\mathbf{b}$

Wave turbulence

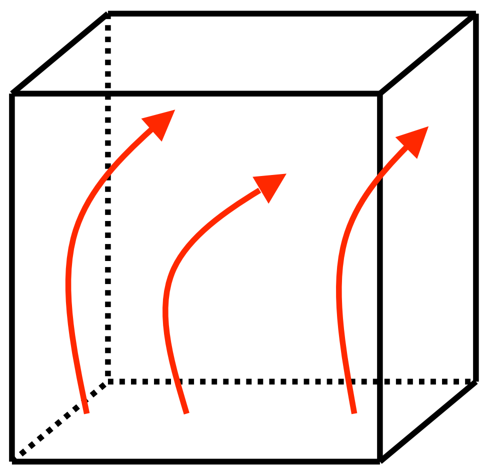
(ε<<1)

MHD is non invariant under Galilean transformation



$\mathbf{B} = \mathbf{b}$

Isotropic strong turbulence



$\mathbf{B} = \mathbf{B_o} + \mathbf{b}$

Anisotropic strong turbulence

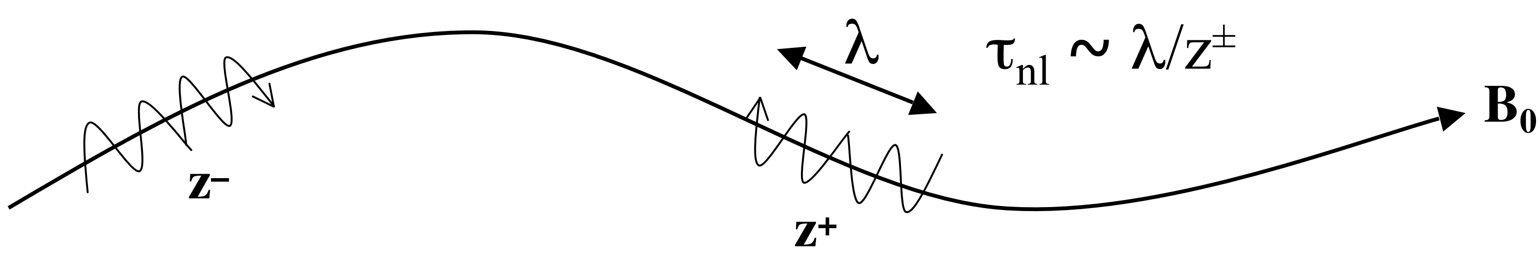
Concept in MHD turbulence

The main difference between neutral fluids and MHD is the presence of **Alfvén waves** [Alfvén, Nature, 1942]

Wavepackets interact nonlinearly on a crossing time: $\tau_A \sim \lambda / B_0$

$$\begin{cases} \partial_t \mathbf{z}^{\pm} \mp \mathbf{B_0} \cdot \nabla \mathbf{z}^{\pm} = -\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} - \nabla P_* \\ \nabla \cdot \mathbf{z}^{\pm} = 0 \end{cases}$$

$\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}$ are the Elsässer fields



Need N **collisions** between wavepackets : $\tau_{tr} \sim N\tau_A \sim (\tau_{nl}/\tau_A)^2 \, \tau_A$

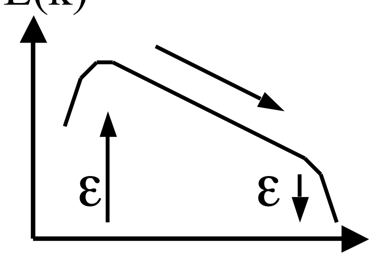
[Iroshnikov, Sov. Astron., 1964; Kraichnan, Phys. Fluids,1965]

Theories for isotropic MHD

[Iroshnikov, Sov. Astron., 1964; Kraichnan, Phys. Fluids,1965]

$$\varepsilon \sim z_\lambda^4/(\lambda B_0) \sim (E_k k)^2 k/B_0 \sim E_k^2 k^3/B_0$$

$$E(k) \sim (\varepsilon B_0)^{1/2}\, k^{-3/2}$$

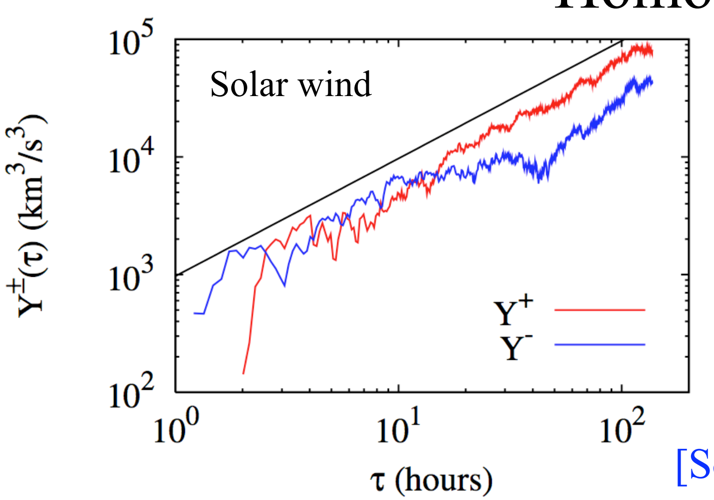


1) Problem: **isotropy** is assumed !!
2) Very **close** to the Kolmogorov prediction ($k^{-5/3}$)
3) Heuristic result **not compatible** with the 4/3's exact law

Homogeneous isotropic medium, at Re, Rm → +∞

[Politano & Pouquet, PRE, 1998]

$$-\frac{4}{3}\varepsilon^{\pm} r = \underbrace{\langle \delta z_L^{\mp} (\delta z^{\pm})^2 \rangle}_{Y^{\pm}} \Rightarrow E(k) \sim k^{-5/3}$$



[Sorriso-Valvo et al., PRL, 2007]

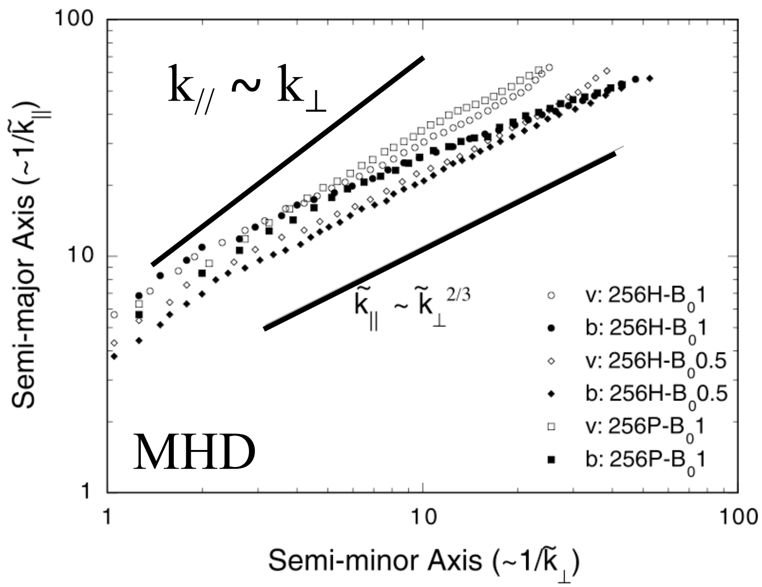
Theories for anisotropic MHD

- Critical balance : $\tau_{nl} \sim \tau_A$ at **all** scales [Goldreich & Sridhar, ApJ, 1995]

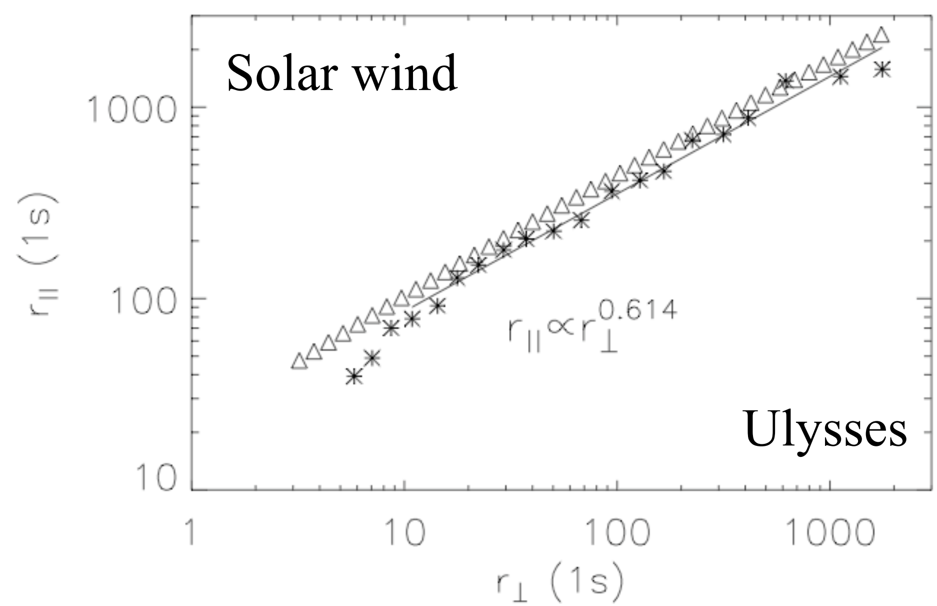
  → phenomenology

  → $E(k_\perp) \sim k_\perp^{-5/3}$ and $E(k_{//}) \sim k_{//}^{-2}$

  → increase of anisotropy at small scales : $k_{//} \sim k_\perp^{2/3}$



[Cho & Vishniac, ApJ, 2000]

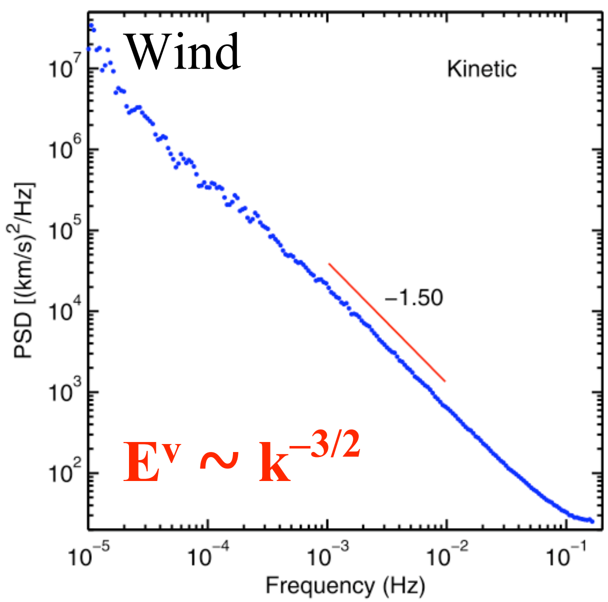


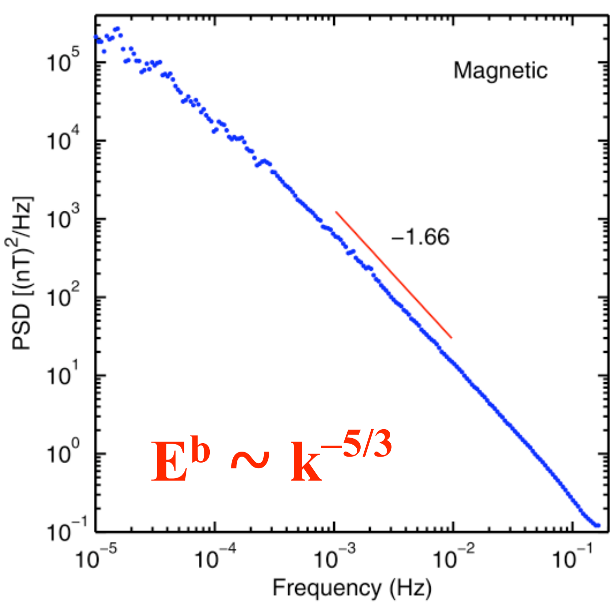
[Luo & Wu, ApJ, 2010]

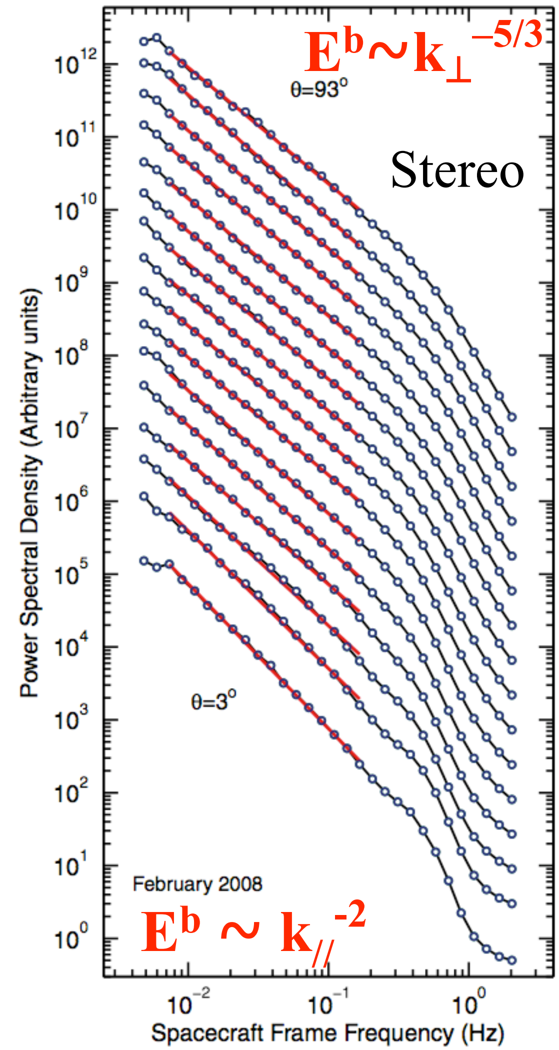
Solar wind energy spectra

Taylor hypothesis: $k \approx 2\pi f / V_{SW}$

No unique solution !







[Podesta, J Geophys. Res., 2006; Podesta, ApJ, 2009]

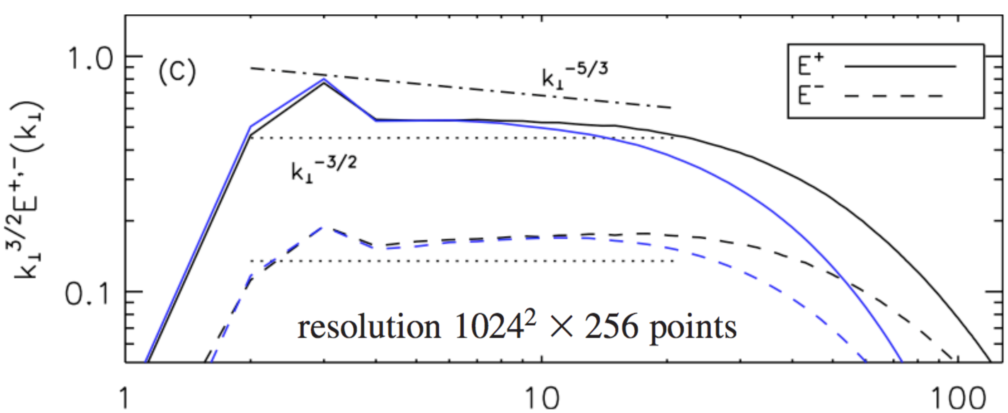
Theories for anisotropic MHD

- Dynamic alignment : $\mathbf{v}.\mathbf{b} = \pm vb \cos\theta$ [Boldyrev, PRL, 2006]

→ phenomenology

→ $\theta \sim \lambda_\perp^{1/4}$ and $E(k_\perp) \sim k_\perp^{-3/2}$

→ anisotropic relation : $\mathbf{k_{//} \sim k_x^{1/2}}$



[Perez & Boldyrev, PRL, 2009]

But DNS reveals that :

$E(k_\perp, k_{//}=0) \sim k_\perp^{-3/2}$

[Bigot et al., PRE, 2008]

Theories for anisotropic MHD

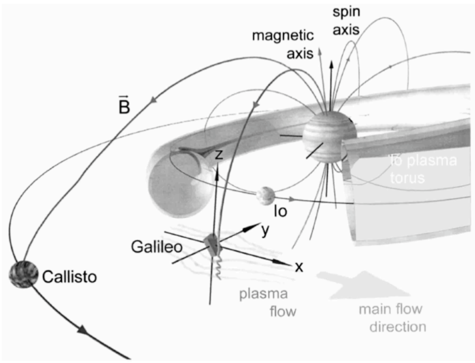
- Alfvén wave turbulence ($B_0 \rightarrow +\infty$; $\tau_A \ll \tau_{nl}$) [SG et al., J. Plasma Phys., 2000]

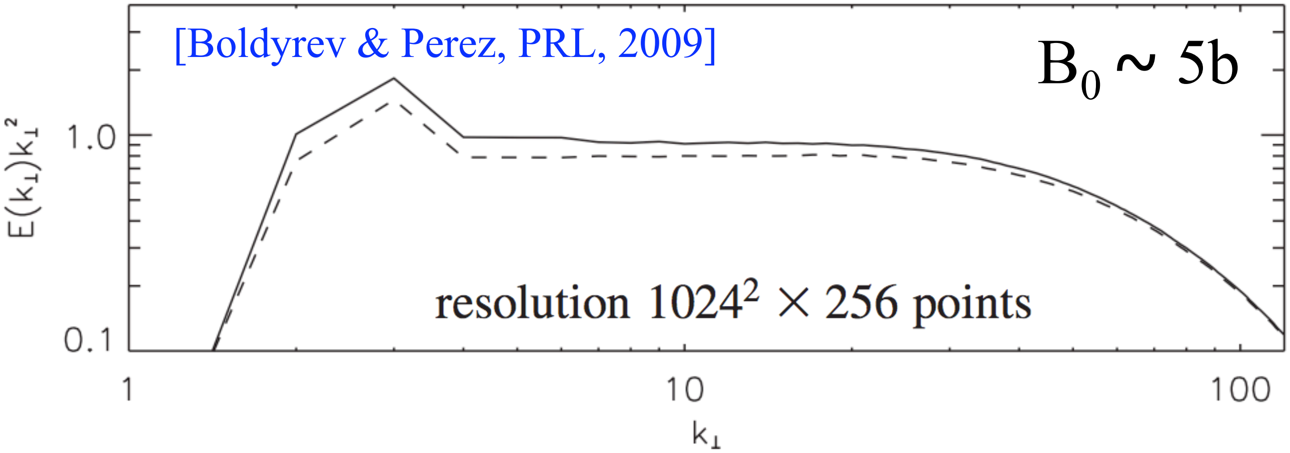
→ **exact** asymptotic closure

→ $E(k_\perp) \sim k_\perp^{-2}$: exact solution!

→ **observed** in Jupiter's magnetosphere

[Saur et al., A&A, 2002]



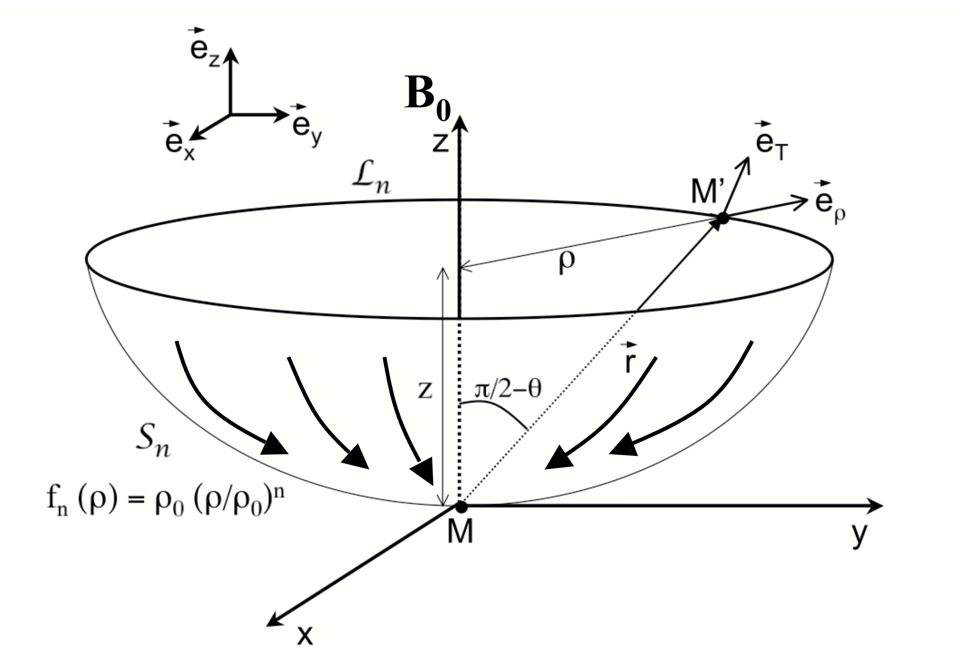


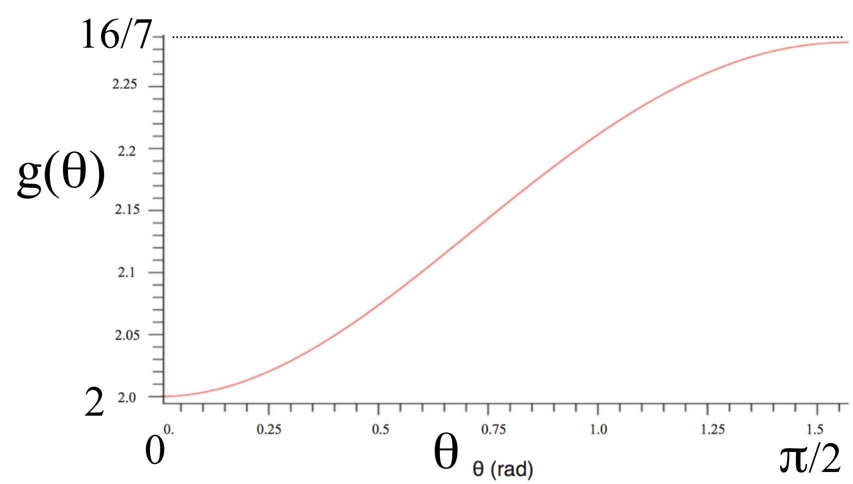
Theories for anisotropic MHD

Critical balance: $k_{//} \sim k_{\perp}^{2/3}$ ⇔ space correlation is **foliated**

[SG, 2010]

$$-g(\theta)\varepsilon^{\pm} r\, \mathbf{e_T} = \mathbf{F_T^{\pm}}(\mathbf{r}) \qquad \mathbf{F}^{\pm}(\mathbf{r}) = \langle \delta\mathbf{z}^{\mp}(\delta\mathbf{z}^{\pm})^2 \rangle$$



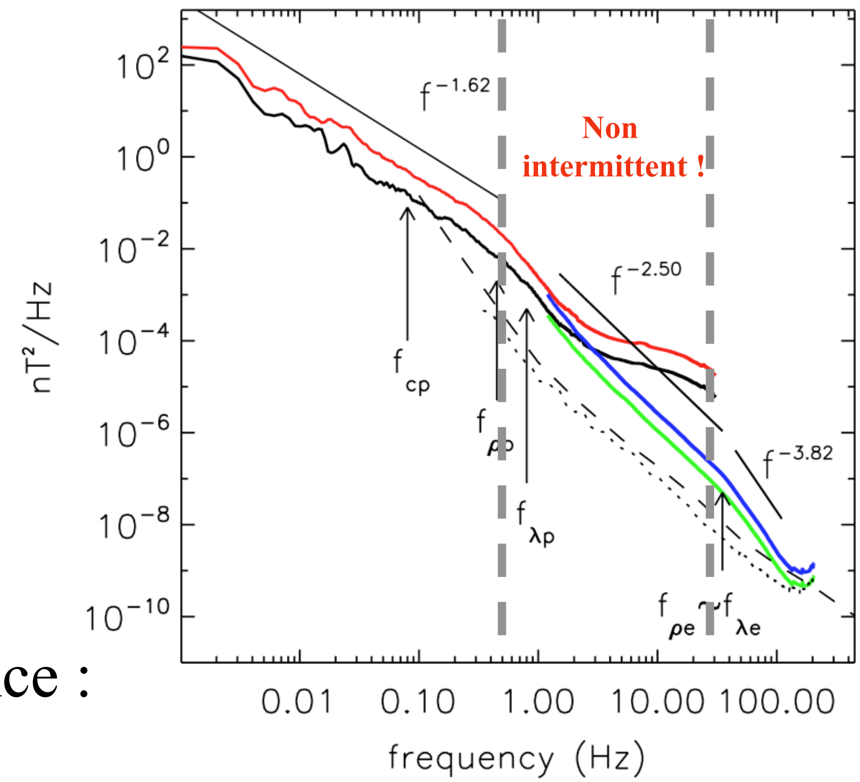


Beyond MHD : Hall MHD

- **Exact** law for (isotropic) Hall MHD :
  [SG, PRE, 2008]

$$-\frac{4}{3}\varepsilon^T r = \overbrace{B^{vvv}_{\|ii} + B^{vbb}_{\|ii} - 2B^{bvb}_{\|ii}}^{E \sim k^{-5/3}} + \underbrace{4d_I\langle[(\mathbf{J}\times\mathbf{b})\times\mathbf{b}']_\|\rangle}_{E^b \sim k^{-7/3}}$$

[Kiyani et al., PRL, 2009]



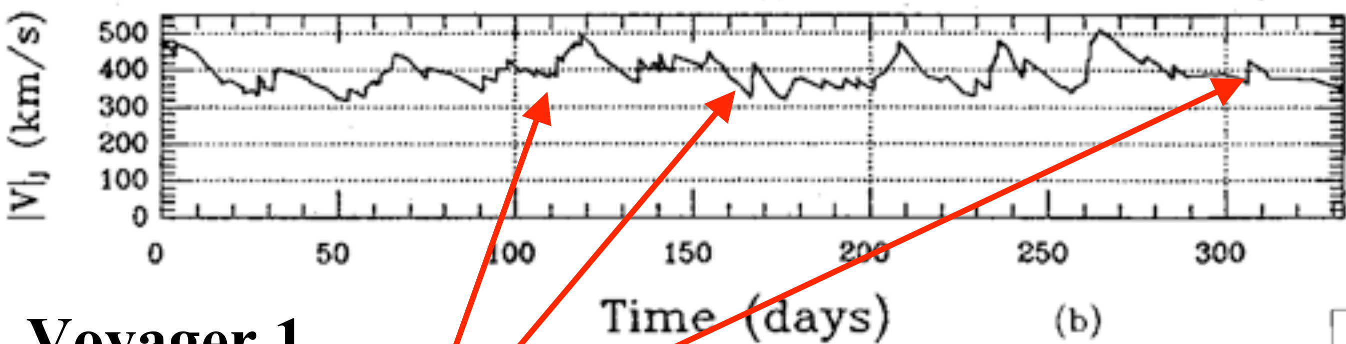
- **Whistler** (anisotropic) wave turbulence :

  → $E^b(k_\perp) \sim k_\perp^{-2.5}$: exact solution

  [SG, J. Plasma Phys., 2006]

Compressible turbulence

[Roberts et al., JGR, 1987]

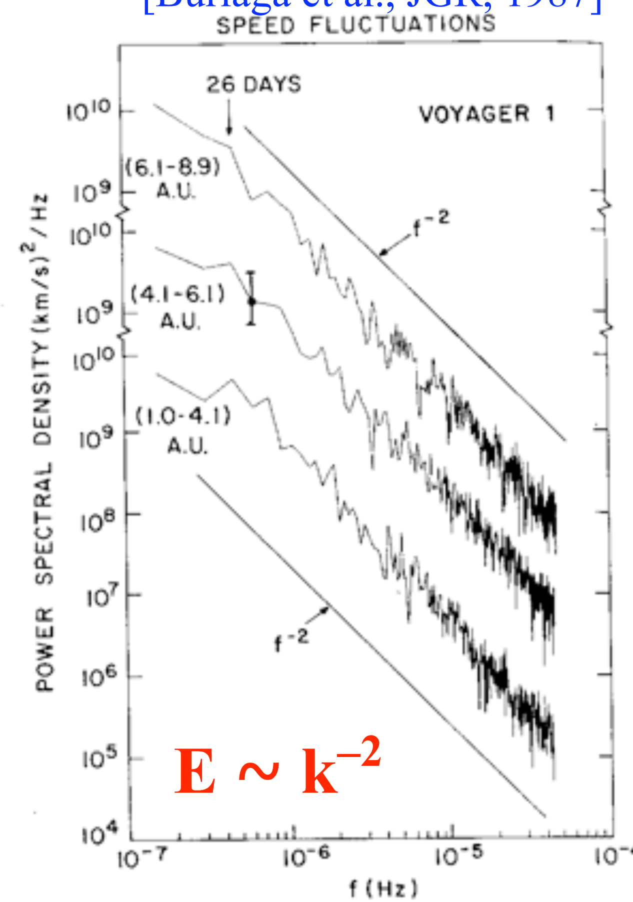


Solar wind
(>1AU)

Voyager 1

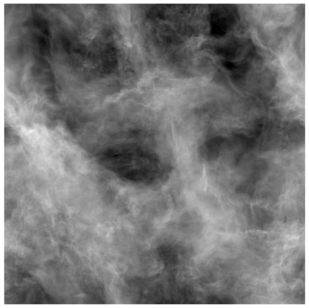
Shocks

[Burlaga et al., JGR, 1987]

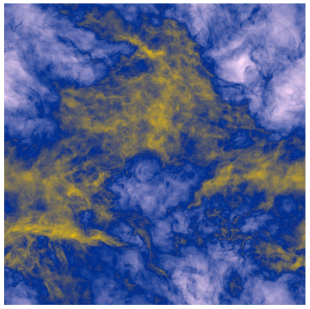


$E \sim k^{-2}$

There is **no exact** law for
3D compressible turbulence !



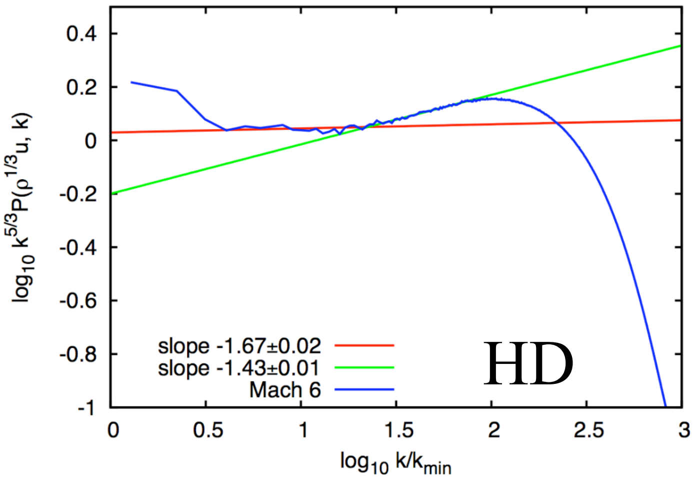
Compressible turbulence



- 3D M-HD simulations (adiabatic case):

[Kritsuk et al., ApJ, 2007; ASP, 2009]

$v = \rho^{1/3} u \quad \rightarrow \quad E^{v}(k) \sim k^{-5/3}$



2048^3

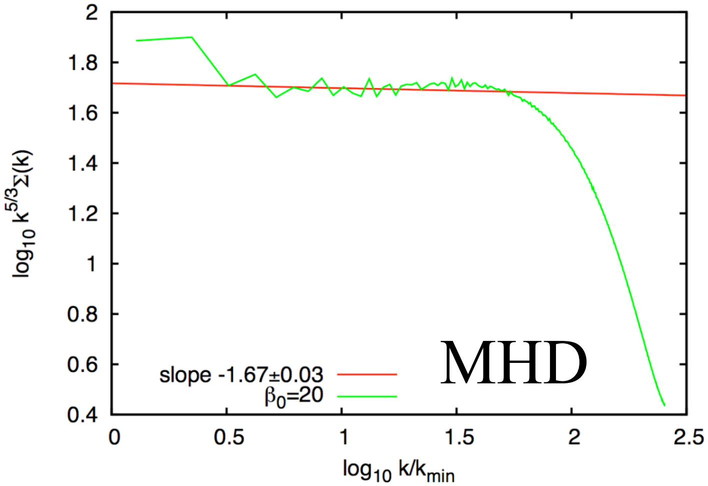
- First analysis (HD + adiabatic):

$$-\varepsilon = \nabla . \mathbf{F}(\rho,u) + S(\nabla.u,\rho,u)$$

[SG, 2010]

F: energy flux vector

S: compressible source



Conclusion

Thank you !