Constraints on T-τ Laws from 3D Models

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Topics in Part I

- Box-in-a-Star Simulations
- Solar Surface: Code Comparisons
- Mean Thermal Structure
- Kinetic Energy and Kurtosis
- Implications for Modelling

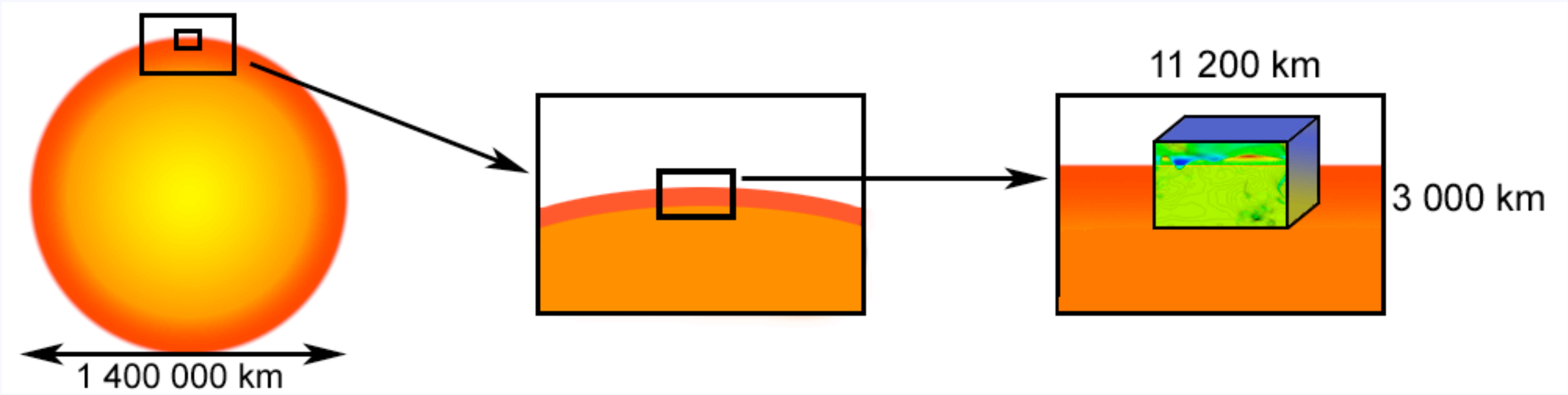
Box-in-a-Star Simulations I

In surface convection zones of main sequence stars

surface pressure scale height $P/(\rho g) = H_p \ll R$, the stellar radius

➔ Standard idea: use a simulation box representing only a small volume of the entire convection zone ("box-in-a-star") ➔ Solar granulation simulations ! Solve equations numerically for that volume on a grid in space and time.

➔ Compute horizontal averages or averages over identical optical depth, followed by time averages (assuming a quasi-ergodic hypothesis to hold)



(illustration courtesy of F. Zaussinger)

Solar Surface: Code Comparisons I

ANTARES solar granulation simulations used in the following

- closed simulation by F. Zaussinger
  - closed vertical boundaries; G93 composition; grey (1 bin) RT; WENO5 scheme
  - size: 6*6*3 Mm^3, grid: 150*150*190, h_h = 40 km, h_v = 16 km
  - time: ~3 hours 30 min
- osc6 simulation by H. Grimm-Strele
  - open vertical boundaries; G93 composition; grey (1 bin) RT; WENO5 scheme
  - size: 6*6*4 Mm^3, grid: 159*159*316, h_h = 40 km, h_v = 13 km
  - time: 2 hours 33 min
- cosc13 simulation by H. Grimm-Strele & F. Kupka
  - open vertical boundaries; G93 composition non-grey (4 bin) RT; WENO5 scheme
  - size: 6*6*3.88 Mm^3, grid: 179*179*359, h_h = 35.3 km, h_v = 11.1 km
  - time: 11 hours 6 min (126 sound crossing times)
- wide4 simulation by H. Grimm-Strele
  - open vertical boundaries; G93 composition; non-grey (4 bin) RT; WENO5 scheme
  - size: 18*18*4.45 Mm^3, grid: 519*519*414, h_h = 35.3 km, h_v = 11.1 km
  - time: 3 hours 27 min (39 sound crossing times)

Solar Surface: Code Comparisons II

Solar granulation simulations based on other codes

(data by courtesy of cited authors; cf. Kupka, F. 2009, Mem.S.A.It. 80, 701)

- CO^{5}BOLD simulations by M. Steffen
  - open vertical boundaries; chem. composition: G98
  - non-grey (5 bin) RT (grey for deep case); Roe-scheme & SGS-viscosity
  - size: 11.2*11.2*3.1 Mm^3, grid: 400*400*165, h_h = 28 km, h_v = 12-28 km
  - deep case: 11.2*11.2*5.2 Mm^3, grid: 200*200*250, h_h = 56 km, h_v = 21 km
- CKSR (Chan-Kim-Sofia-Robinson) simulation: “model 2008” by F.J. Robinson
  - closed vertical boundaries; chem. composition: G98
  - grey RT: 3D Eddington approximation; shock-smoothing & SGS viscosity
  - size: 5.4*5.4*3.6 Mm^3, grid: 117*117*190, h_h = 46 km, h_v = 19 km
- STAGGER (Nordlund & Stein) simulation by R. Samadi & K. Belkacem
  - open vertical boundaries; chem. composition: G98
  - non-grey (4 bin) RT; shock-smoothing & hyperviscosity
  - size: 6*6*3 Mm^3, grid: 150*150*150, h_h = 40 km, h_v ~ 20 km (variable)

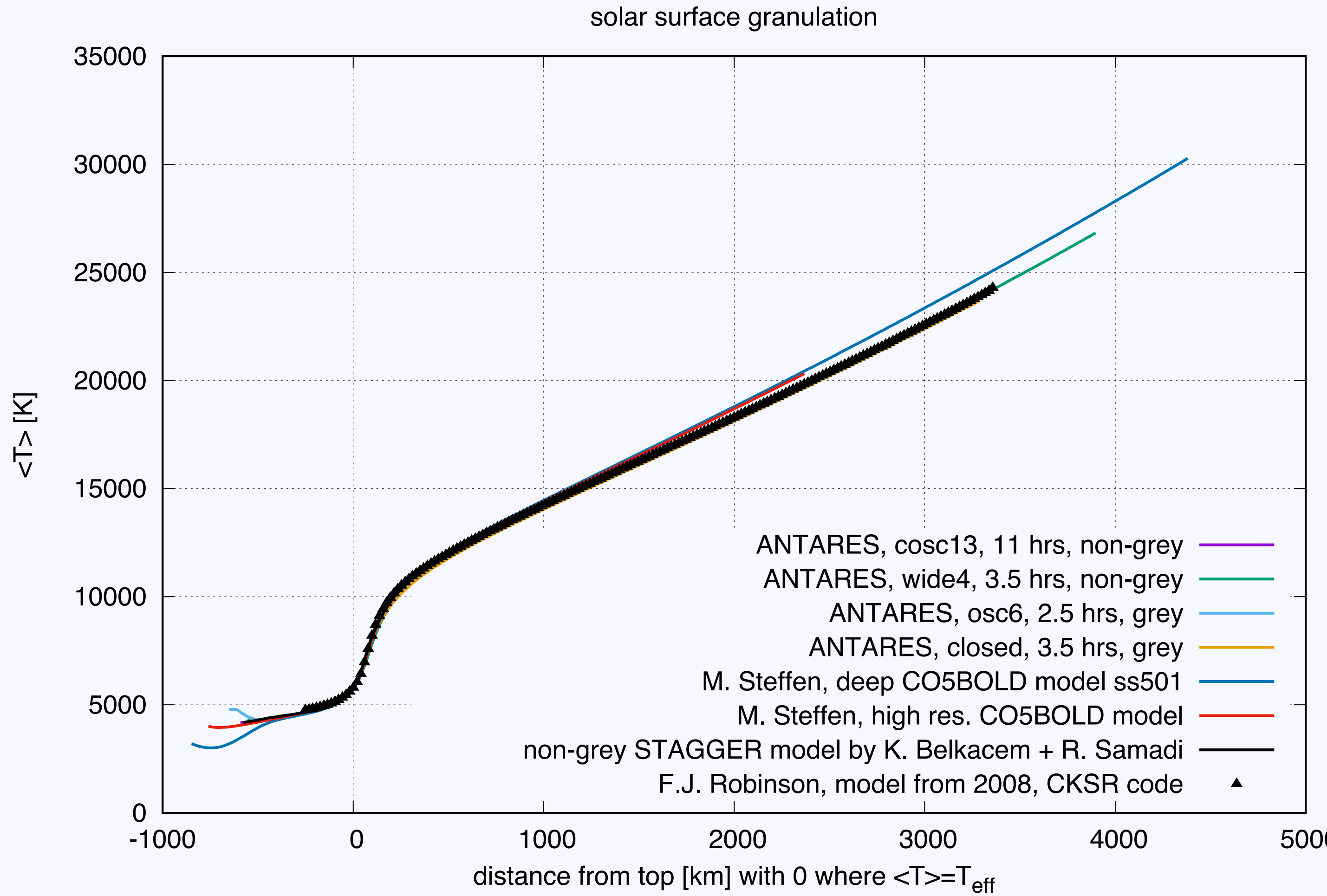
Mean Thermal Structure I

Temperature profile from granulation simulations

mean temperature as a function of height

$T_{eff} \sim 5777$ K,
$g = 274$ m sec^{-2}
($\log g = 4.4377$),
$M = 1\ M_\odot$

3D simulations, averaged horizontally and in time



good agreement except for the photosphere (boundary conditions, ...) and a slightly different gradient in the interior in CO5BOLD simulations (equation of state ?)

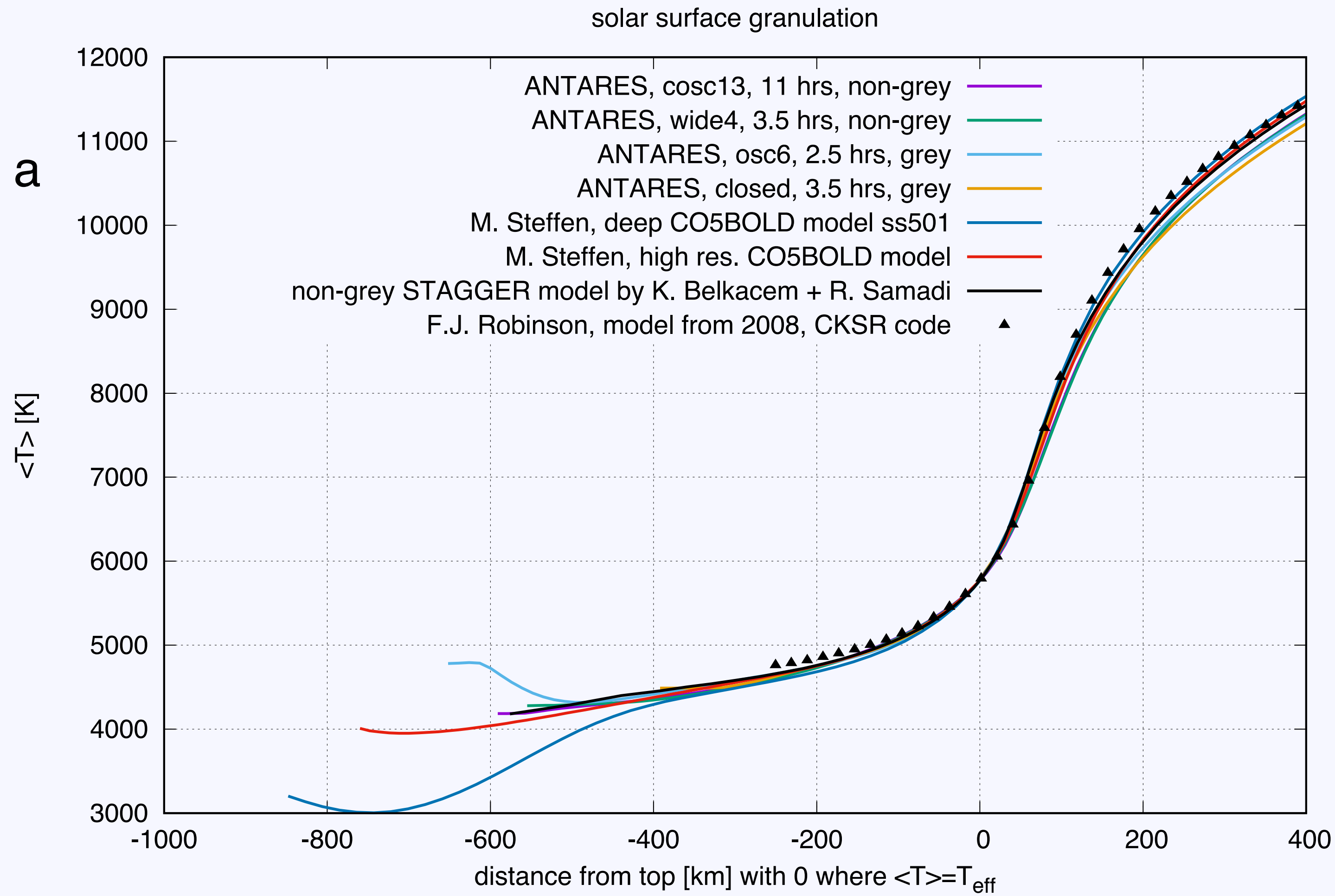
Mean Thermal Structure II

Solar photospheres

mean temperature as a function of height

$T_{eff} \sim 5777$ K,
g = 274 m sec^{-2}
(log g = 4.4377),
M = 1 $M_\odot$

3D simulations, averaged horizontally and in time



Uppers layers very sensitive to details in the boundary conditions ! In addition: CKSR: too shallow ? Eddington approximation ? Closed ANTARES simulations: too shallow ?

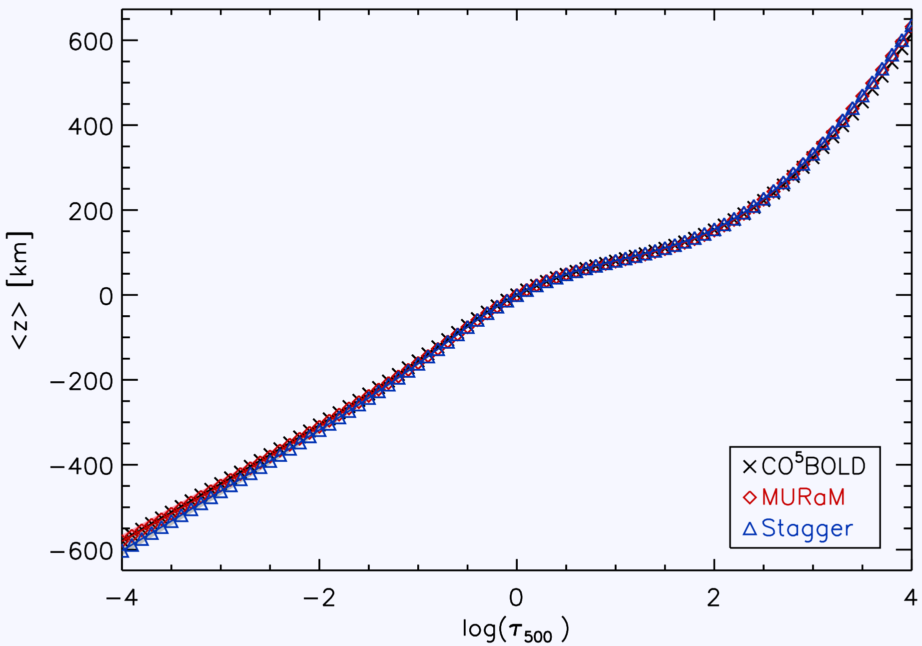
Mean Thermal Structure III

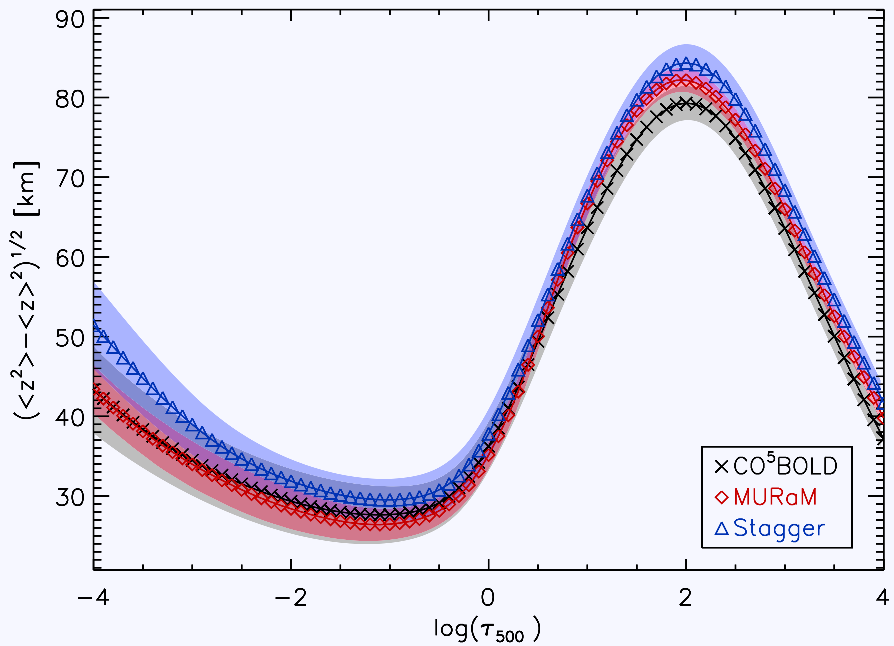
Solar photospheres: CO⁵BOLD vs. MURaM vs. STAGGER

Mean temperature now as a function of optical depth for solar parameters:

T_{eff} ~ 5777 K, g = 274 m sec^{-2} (log g = 4.4377), M = 1 $M_{\odot}$

3D simulations averaged over iso-$\tau_{500\ nm}$ surfaces and in time





Systematic differences start to increase at optical depths smaller than 0.01, otherwise differences are mostly around the superadiabatic peak (< 100 K).

(plot taken from Fig. 9 in Beeck et al. 2012, A&A 539, A121)

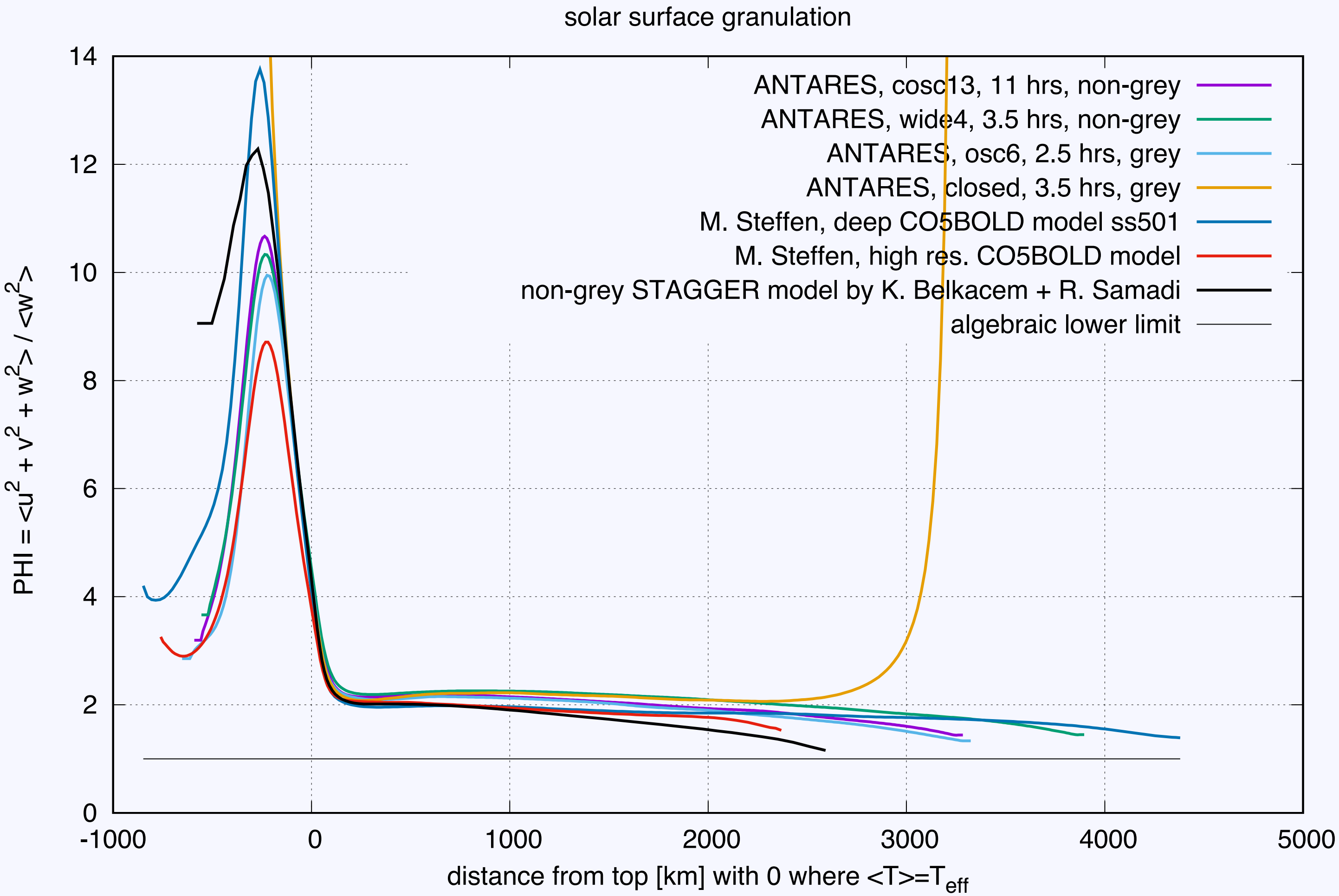
Kinetic Energy and Kurtosis I

Distribution of total to vertical kinetic energy in granulation simulations

PHI is needed by many semi-analytical models

$T_{eff} \sim 5777$ K,
g = 274 m sec^{-2}
(log g = 4.4377),
M = 1 $M_\odot$

3D simulations, ensemble averages



Notice: 1. Very large spread in the photosphere. 2. Systematic trend in all simulations towards (erratic) purely vertical flow due to influence of lower boundary conditions !

Kinetic Energy and Kurtosis II

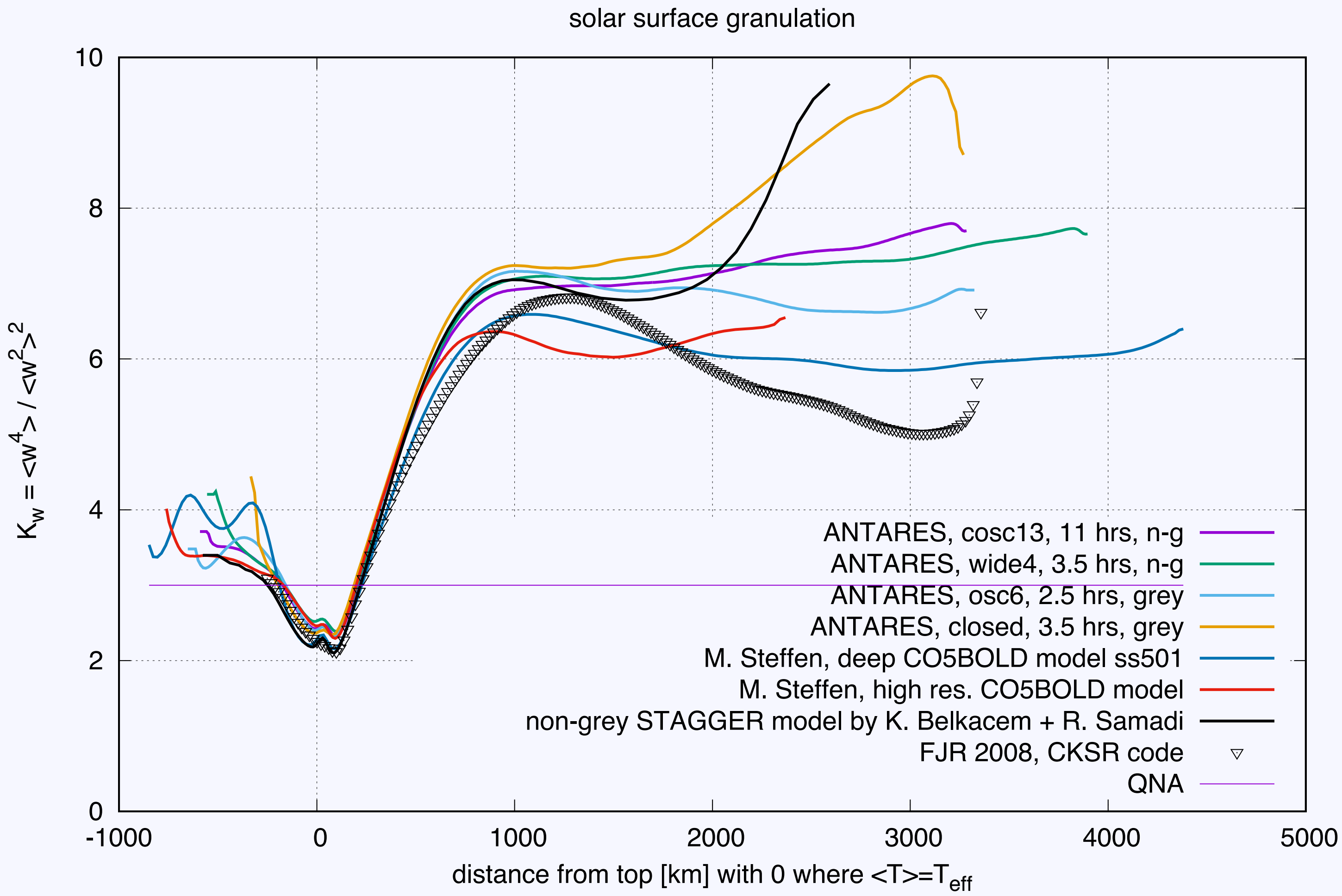
Distribution of kurtosis of vertical kinetic energy in granulation simulations

Related quantities are needed in semi-analytical models of p-mode driving

$T_{eff} \sim 5777$ K, g=274 m sec^{-2} (log g = 4.4377),

M = 1 M$_\odot$

3D simulations, ensemble averages



Agreement only within and just underneath the superadiabatic layer. Within a region of up to three pressure scale heights very sensitive to boundary conditions (esp. lower ones).

Implications for Modelling I

- **Constructing 1D models based on 3D simulations**
  - "Universality" of thermal equilibrium structure in 3D simulations of surface convection with different numerical codes supports
  - the construction of T-τ laws, i.e. surface boundary conditions, for stellar pulsation and stellar evolution calculations based on 3D simulations.

Implications for Modelling I

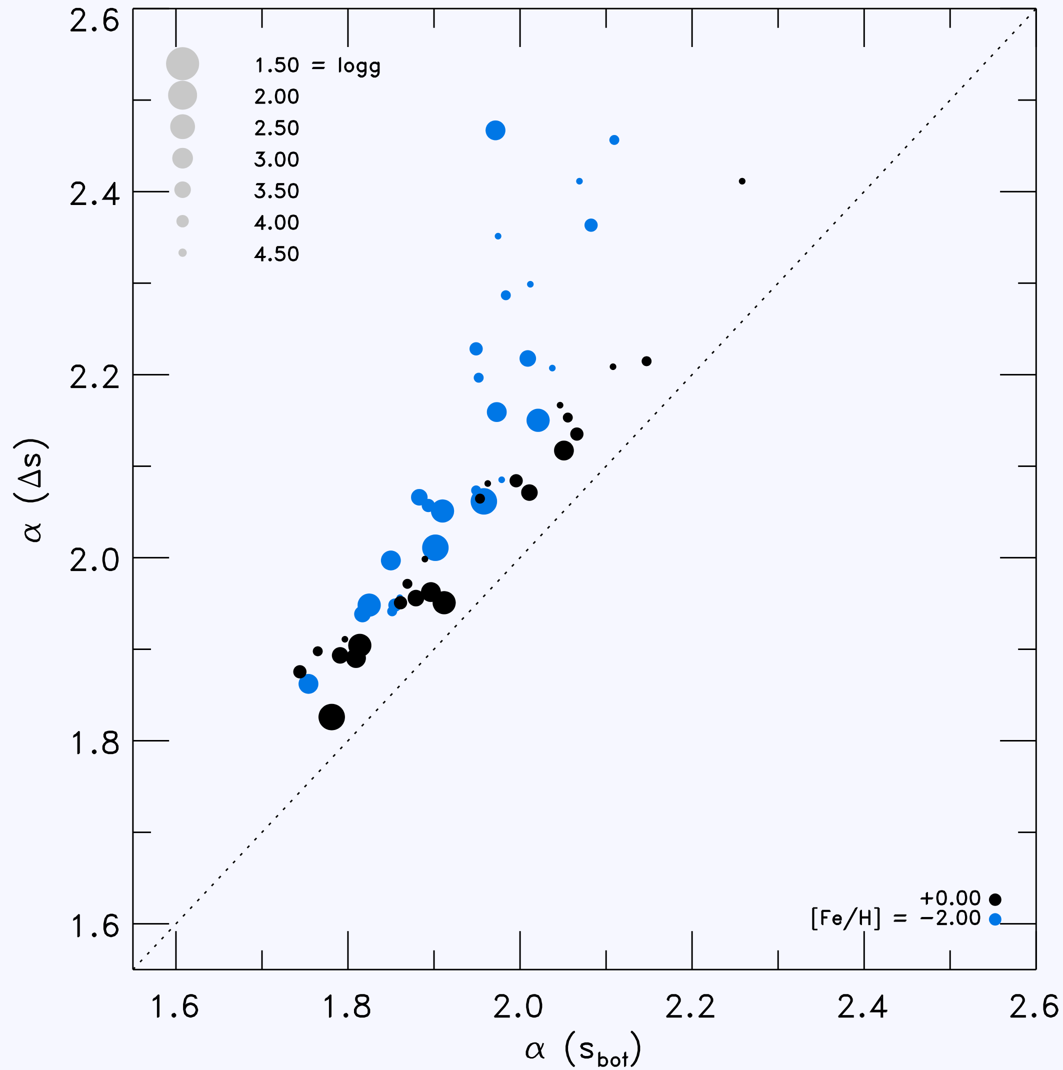
- **Constructing 1D models based on 3D simulations**
  - "Universality" of thermal equilibrium structure in 3D simulations of surface convection with different numerical codes supports
  - the construction of T-τ laws, i.e. surface boundary conditions, for stellar pulsation and stellar evolution calculations based on 3D simulations.
- **Methods**
  - Calibrate / tune MLT parameter α:
    - reproduce integral property (L) or local quantity (entropy jump Δs, s_{bot}, ...)
  - Scaling laws from 3D simulations: for quantities such as entropy as a function of depth for a (limited) range within the HRD
  - "Model patching": use 3D simulation average as upper boundary condition (relocated into stellar envelope, below superadiabatic layer)

Implications for Modelling II

The optimum fit parameter α is depth dependent.

The value found for it depends on the exact choice of the dependent variable to be optimized.

Different optimizations hence yield different temperature structures.



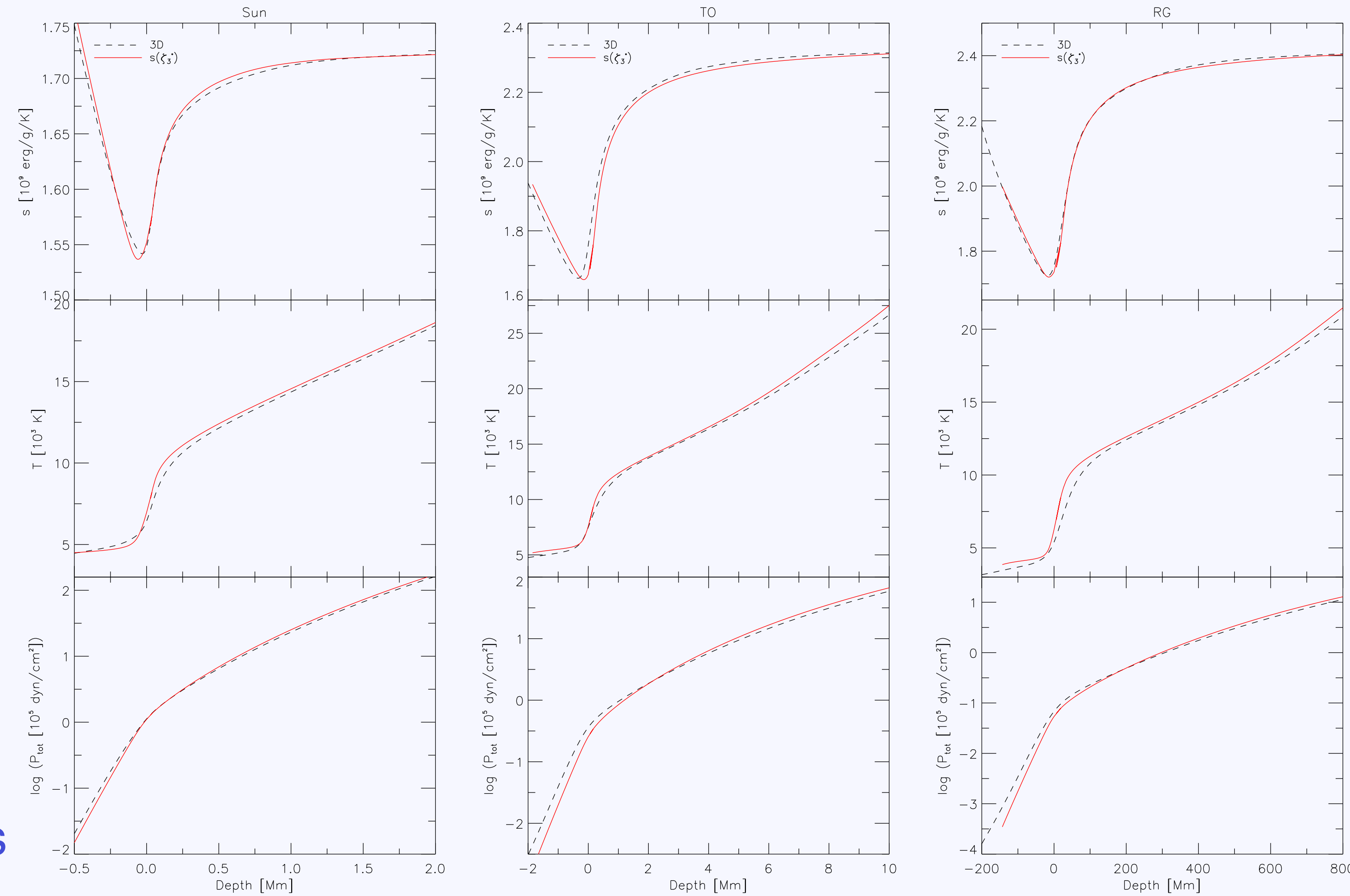
Optimum MLT values α for entropy jump Δs as a function of s_{bot} for the STAGGER grid by Magic et al. (2015), A&A 573, A89 (Fig. 5).

Implications for Modelling III

Entropy as function of depth for the Sun, a the turn off, and in the red giant phase.

Comparison of direct result (black dashes) with scaling formula (solid red line).

This recipe leads to systematic differences in temperature gradients & the pressure structure.



Test of a scaling law for the entropy based on the STAGGER grid by Magic et al. (2013 to 2015) derived by Magic (2016), A&A 586, A88 (Fig. 6).

Implications for Modelling IV

- **Conclusions**
  - The most promising approach appears to be the patching method:
    - 3D results carried over as much as possible
    - thermal / pressure structure of 3D simulations is on save grounds
    - Accuracy, if interpolation needed ? Simulation grid density ?
  - For the kinetics & dynamics of the velocity field one has to be much more careful: possible influence of boundary conditions !
  - Consider physically more complete models as target for optimizations through 3D simulations.
  - Take PLATO 2.0 as an incentive to develop a library of convection models of different complexity (including averaged 3D simulations).

- **Some more results provided in part II ...**

...THANK YOU FOR YOUR TIME !

Topics in Part II

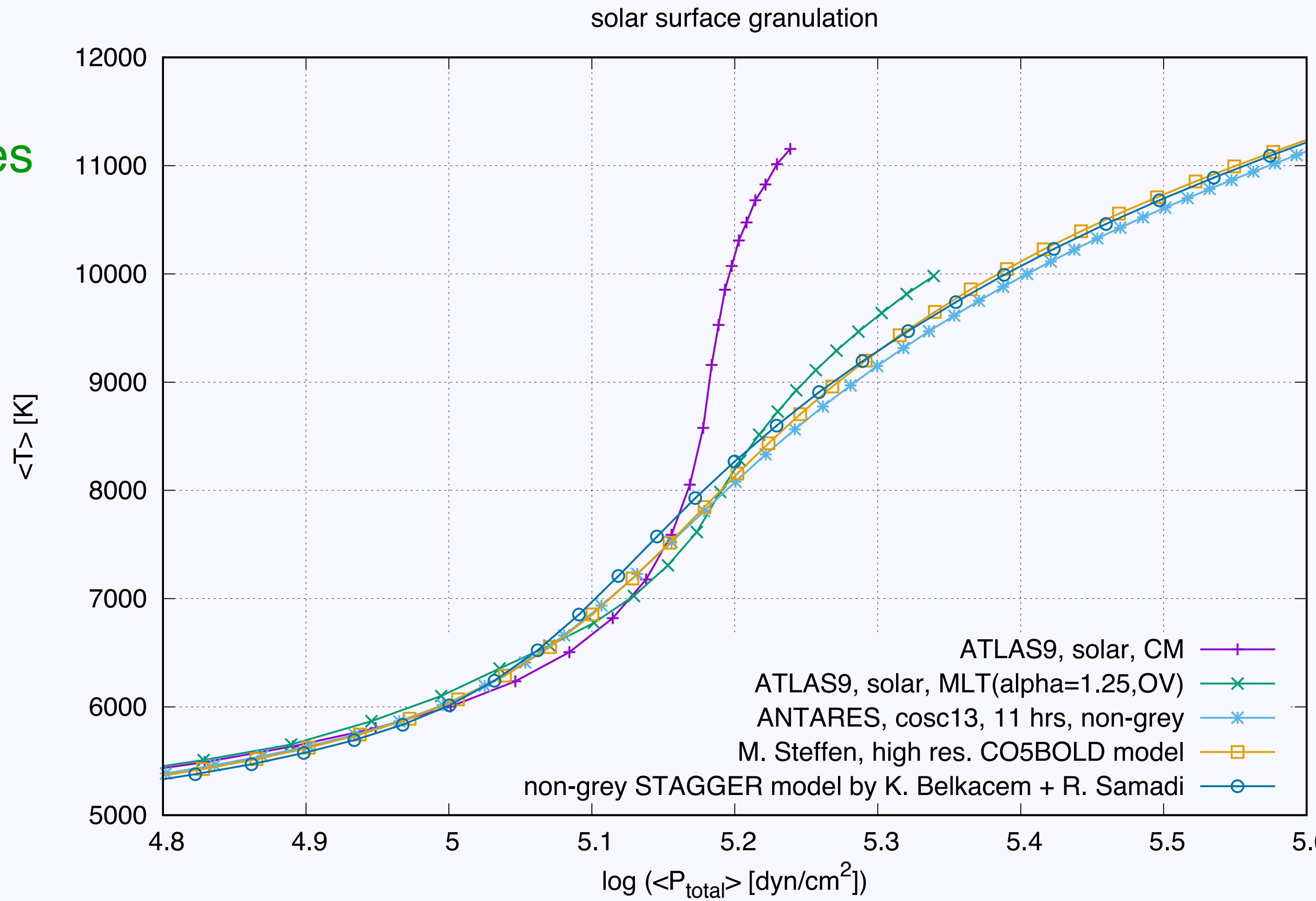
- 1D + 3D Mean Thermal Structures
- Velocity Fields and Boundaries
- Different Initial Conditions
- Implications for Modelling
- Lessons from DAs and Cepheids

1D + 3D Mean Thermal Structures I

Temperature profile:
ATLAS9 1D atmospheres
vs. 3D simulations

mean temperature vs.
mean total pressure

T_{eff} ~ 5777 K,
g = 274 m sec^{-2}
(log g = 4.4377),
M = 1 M$_\odot$



3D simulations agree while 1D models show systematic differences. Gradient of CM model typical for average over updrafts only, for MLT model it depends on α.

1D + 3D Mean Thermal Structures II

Temperature profile:
ATLAS9 1D atmospheres
vs. 3D simulations

mean temperature vs.
mean total pressure

$T_{eff} \sim 5777$ K,
g = 274 m sec^{-2}
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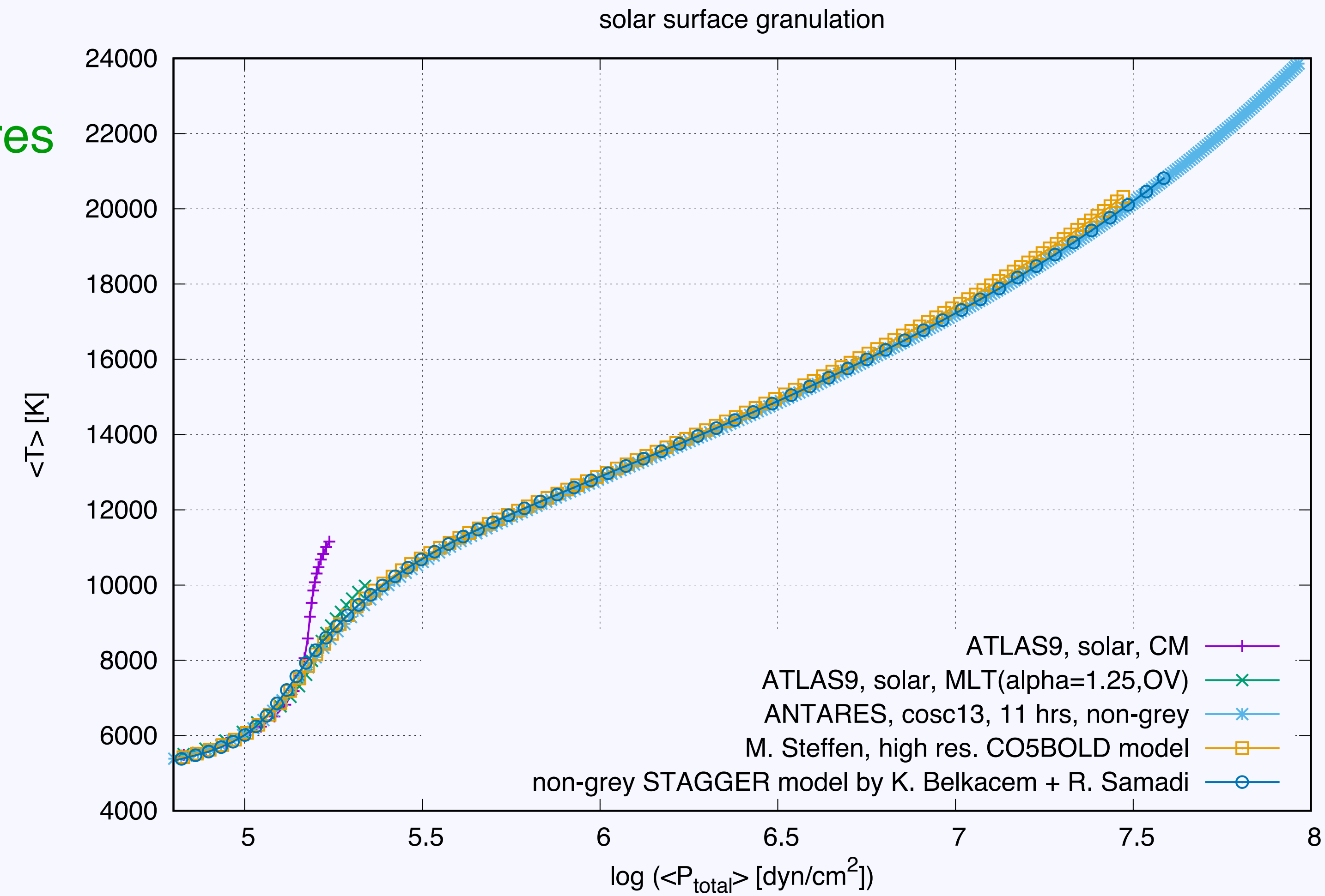


For deeper layers only differences between the simulations appear (as they are based on different EOS, among others. The CM model mainly changes the surface layers (the adiabatic temperature gradient is reached underneath those included in ATLAS9 models).

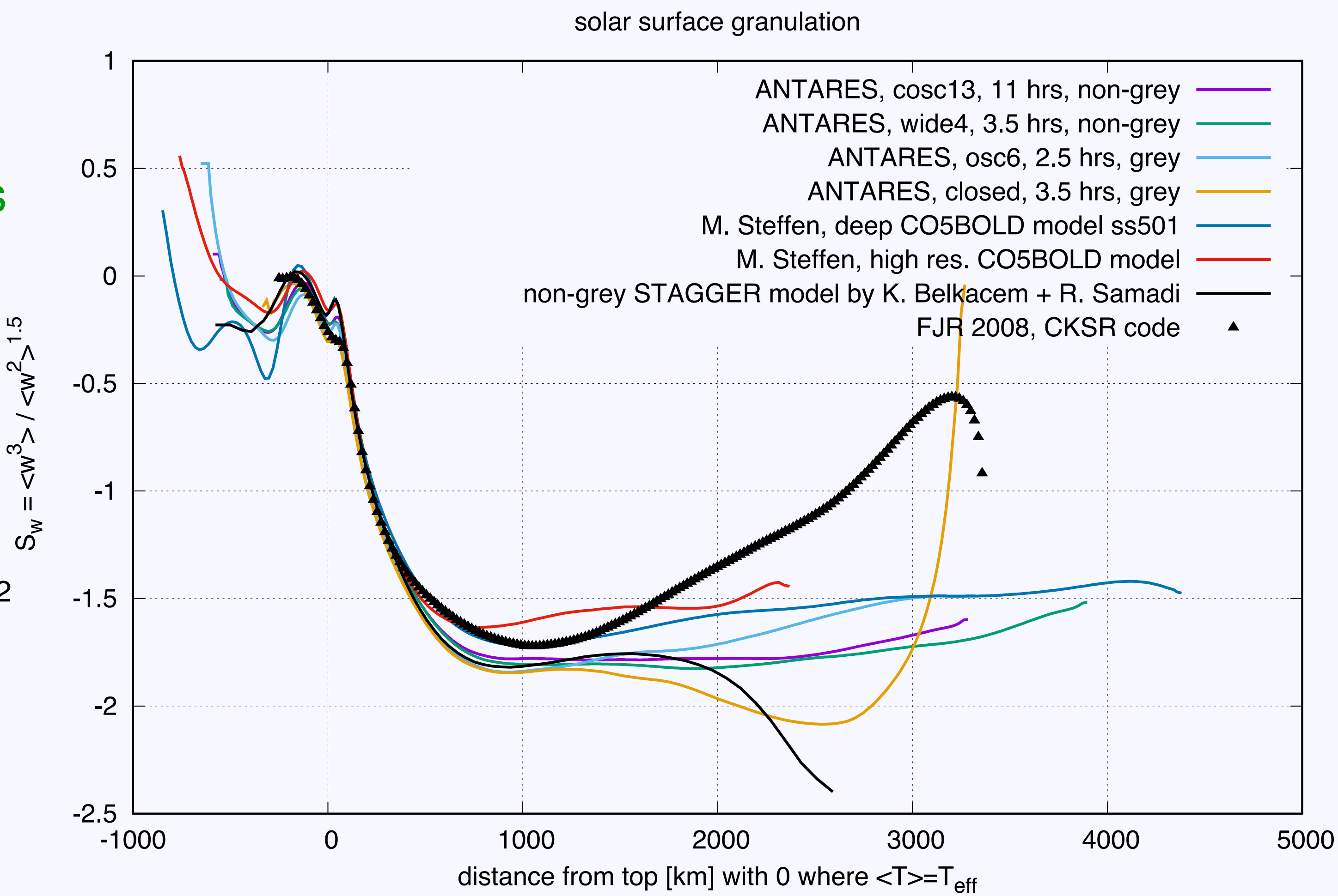
Velocity Fields and Boundaries I

Distribution of skewness of vertical kinetic energy in granulation simulations

Skewness measures the asymmetry between up- and downflows

$T_{eff} \sim 5777$ K, g=274 m sec^{-2} (log g = 4.4377), M = 1 $M_\odot$

3D simulations, ensemble averages



Agreement only within and just underneath the superadiabatic layer. Very sensitive to boundary conditions. Note: contrary to some claims, the non-local Reynolds stress models by Canuto (1992, etc.) and Xiong (1985, etc.) all account for skewness.

Velocity Fields and Boundaries II

Specifying inflowing internal energy / entropy at the bottom

$$\epsilon_{\mathrm{inflow}}^{(n+1)} = \epsilon_{\mathrm{inflow}}^{(n)} \cdot \left(1.0 + \frac{\tau}{\tau_{\mathrm{KH}}}\left(1.0 - \frac{F_{\mathrm{rad}}^{\mathrm{top}}}{F_{\star}}\right)\right) \tag{1}$$

$$S_{\mathrm{inflow}}^{(n+1)} = S_{\mathrm{inflow}}^{(n)} \cdot \left(1.0 + \frac{\tau}{\tau_{\mathrm{S}}}\left(1.0 - \frac{F_{\mathrm{rad}}^{\mathrm{top}}}{F_{\star}}\right)\right) \tag{2}$$

$$S_{\mathrm{inflow}}^{(n+1)} = S_{\mathrm{inflow}}^{(n)} \cdot \left(1.0 + \frac{\tau}{\tau_{\mathrm{S}}}\left(1.0 - \frac{F_{\mathrm{tot}}^{\mathrm{bot}}}{F_{\star}}\right)\right) \tag{3}$$

Model 1: eq. (1), τ_{KH} = 550 h, entropy gradient zero in outflow
(similar to Vögler et al. 2005, A&A 429, 335; MuRAM code)

Model 2: eq. (3), τ_{S} = 100 h, c_{Pchange} = 0.1 (eq. 5, 6)

Model 3: eq. (2), τ_{S} = 1000 h, c_{Pchange} = 1.0 (eq. 5, 6)

(from Grimm-Strele et al. 2015, New Astron. 34, 278)

Velocity Fields and Boundaries III

Determining density + entropy but avoid generating shocks

$$S(\rho^{(1)}, \epsilon^{(1)}) = S_{\text{inflow}} \tag{4}$$

$$\rho^{(2)} = \rho^{(1)} + c_{\text{Pchange}} \frac{\tau}{t_{\text{char}}} \frac{1}{v_{\text{snd}}^2} (< p > - p) \tag{5}$$

$$\epsilon^{(2)} = \epsilon^{(1)} + c_{\text{Pchange}} \frac{\tau}{t_{\text{char}}} \frac{1}{\Gamma_1 \rho} (< p > - p) \tag{6}$$

For models 2 and 3 the density and vertical velocity are modified once more to ensure mass conservation (Eq. 4 to 6).

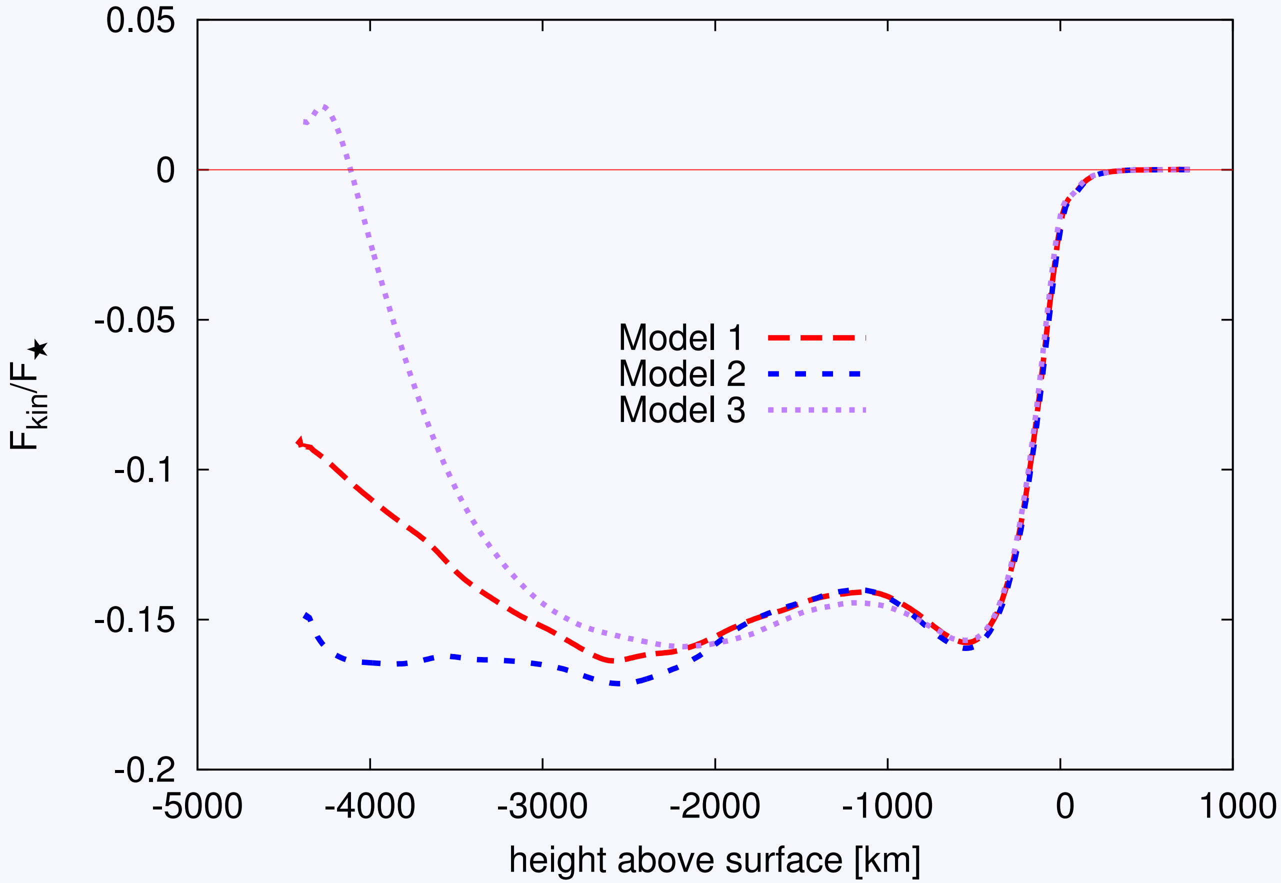
This is very similar to Freytag et al. (2012), J. Comp. Phys. 231, 919, who, however, specify a fixed inflow entropy which may vary on a time scale set by another adjustable parameter, c_{Schange}.

For the three velocity components, gradients are set to zero for all models.

(from Grimm-Strele et al. 2015, New Astron. 34, 278)

Velocity Fields and Boundaries IV

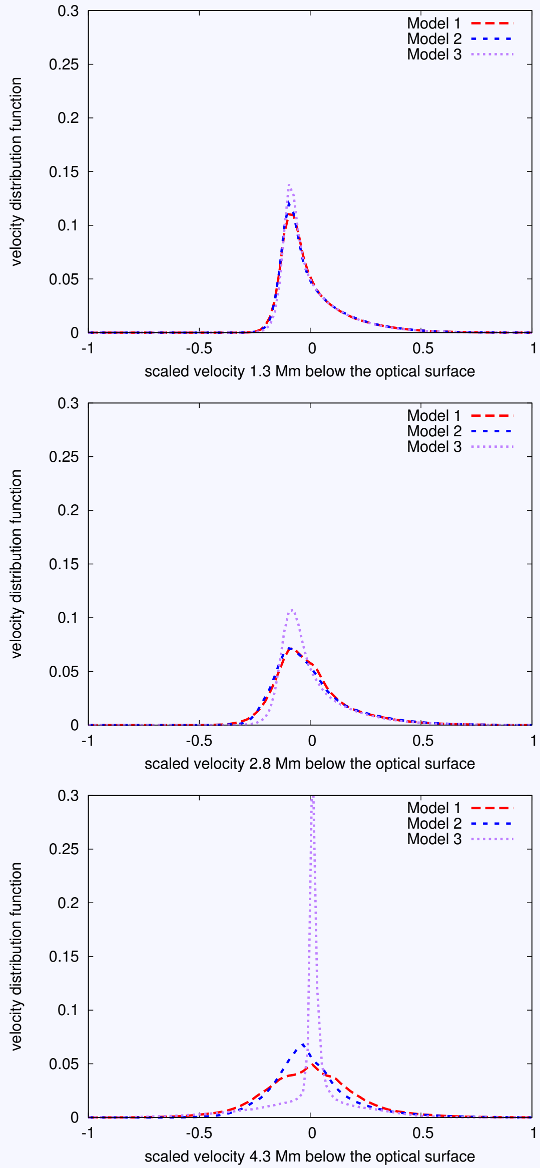
Different open bottom boundary conditions



Effects caused by changing inflow according to flux at top on kinetic energy flux: up to two pressure scale heights are modified (as with closed b.c.s).

(from Grimm-Strele et al. 2015, New Astron. 34, 278)

Velocity Fields and Boundaries V



Effect of different open bottom boundary conditions on the flow inside the domain

Scaled velocity distribution at 1.3 Mm, 2.8 Mm and 4.3 Mm below the optical surface (from top to bottom).

Velocities in each of the layers were grouped into 96 equal–sized bins and then scaled by the maximum velocity in this layer.

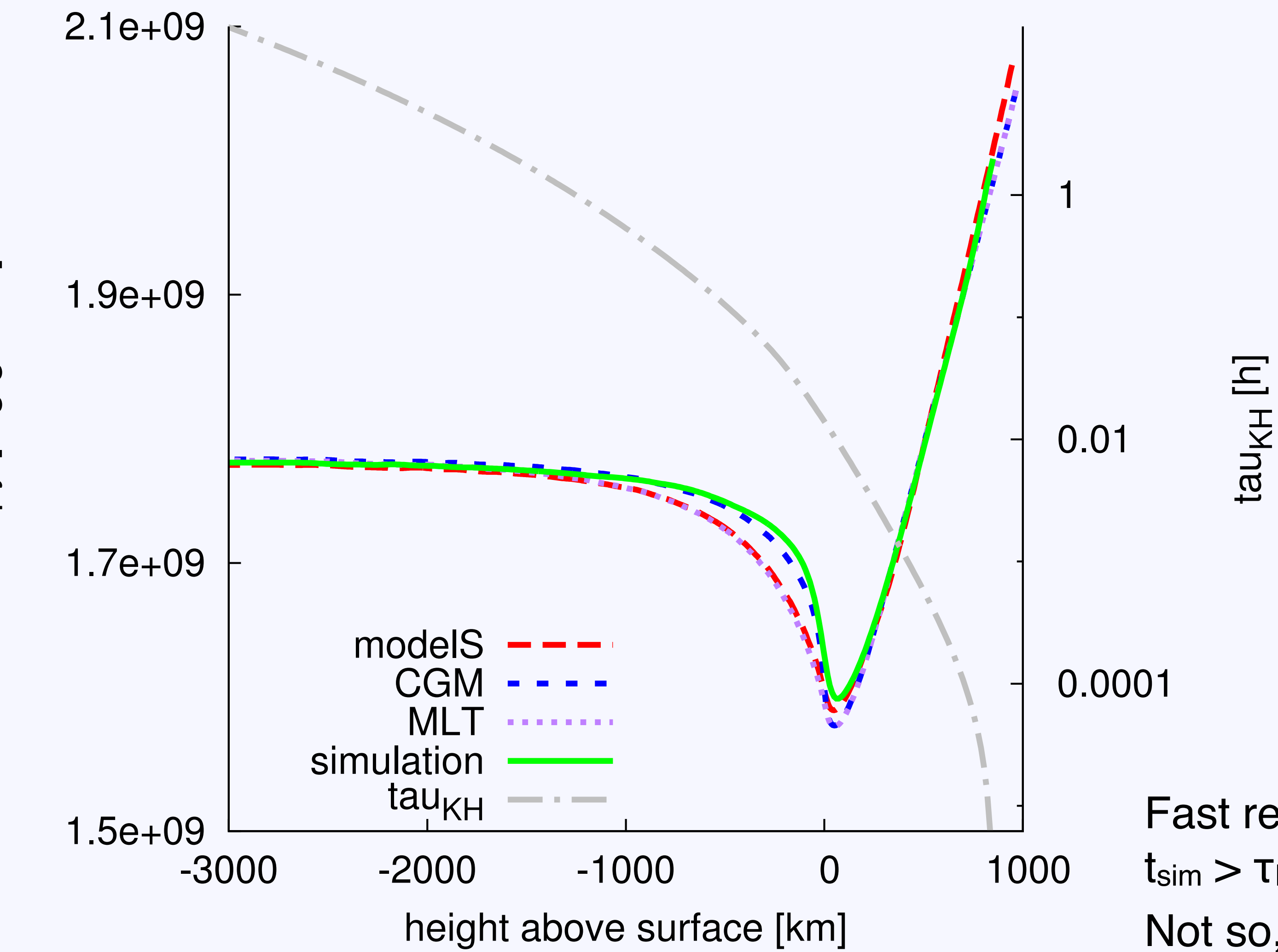
Finally, the distribution function was normalised by the number of nodes and time steps such that the integral from –1 to 1 yields 1.

Due to the coordinate system used in ANTARES upflows have $u(x, y, z) < 0$.

(from Grimm-Strele et al. 2015, New Astron. 34, 278)

Different Initial Conditions I

Different 1D solar structure models as initial conditions



Entropy of three initial models and a relaxed simulation as well as τ_{KH} as a function of depth.

The data for the initial models are obtained from an average over the first 0.6 s of each 3D model simulation (→ 1D model).

Each of the simulations relaxes to the same profile, thus just one case shown (green line).

(from Grimm-Strele et al. 2015, New Astron. 34, 278)

Fast relaxation near surface since $t_{sim} > \tau_{KH} \sim 1$h.

Not so, if the entropy at the bottom is incompatible ...

Implications for Modelling I

- **Constructing 1D models based on 3D simulations**
  - Calibrate / tune MLT parameter α:
    - reproduce integral property (L) or local target quantity (entropy jump Δs, s_{bot})
  - Scaling laws from 3D simulations: for entropy as a function of depth, ...
  - “Model patching”: use 3D simulation as upper boundary condition

Implications for Modelling I

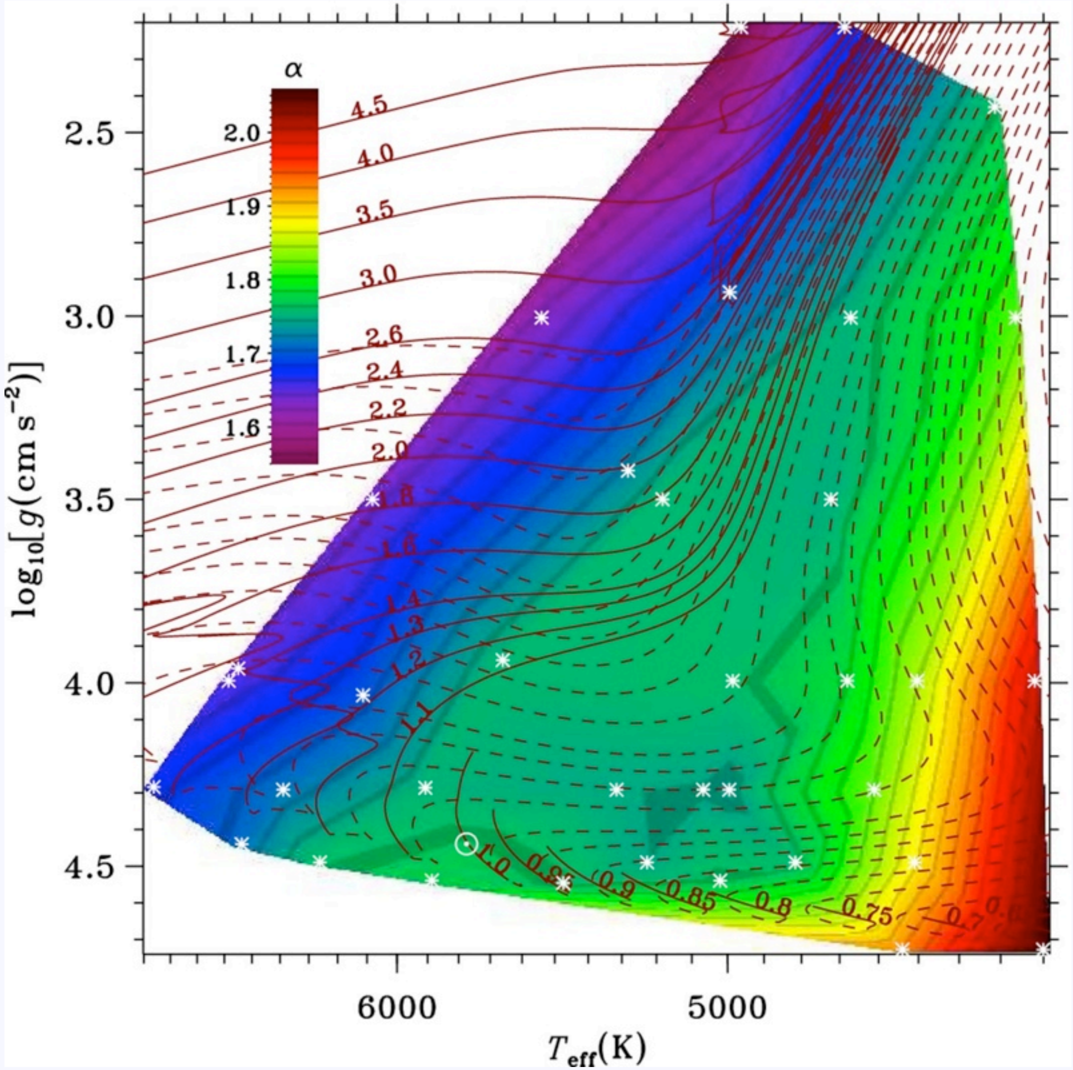
- **Constructing 1D models based on 3D simulations**
  - Calibrate / tune MLT parameter α:
    - reproduce integral property (L) or local target quantity (entropy jump Δs, s_{bot})
  - Scaling laws from 3D simulations: for entropy as a function of depth, ...
  - “Model patching”: use 3D simulation as upper boundary condition
- **Advantages & disadvantages**
  - Calibrate / tune of MLT parameter α:
    - most popolar, simple, no changes in codes, but different properties of the surface layers need different values of α, even as function of depth
  - Scaling laws: “easy”... but accurate ones difficult to derive in practice.
  - Model patching:
    - carries over results from 3D simulations to 1D models as much as possible, but no reliable interpolation algorithm available / known (model grids)

Implications for Modelling II

The optimum fit parameter α to reproduce the entropy jump Δs.

The value found for α is sensitive to the exact choice of the dependent variable to be optimized.

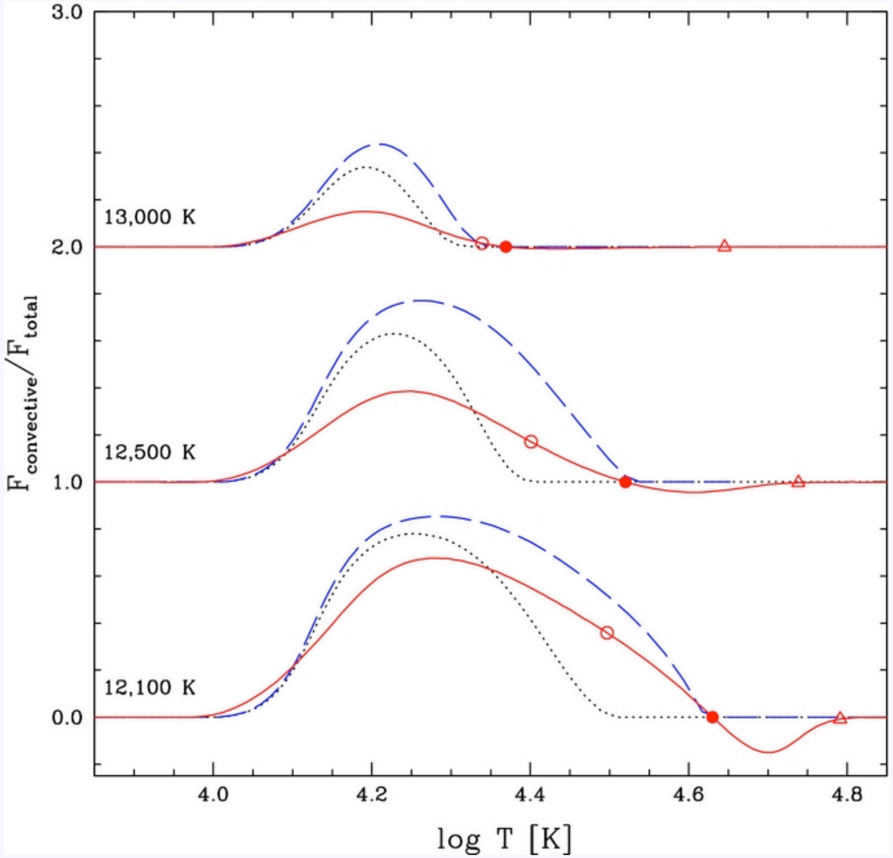
Different optimizations hence yield different temperature structures.

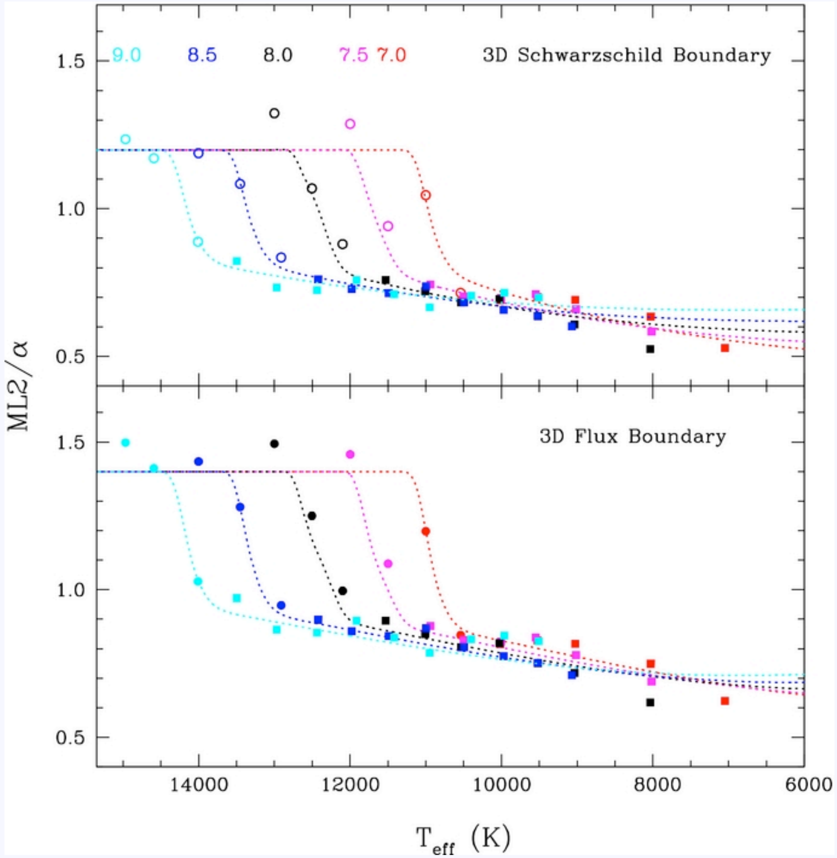


Optimum MLT values α for entropy jump Δs throughout the lower part of the HRD for the STAGGER grid by Trampedach et al. (2014), MNRAS 445, 4366 (Fig. 4).

Lessons from DAs and Cepheids I

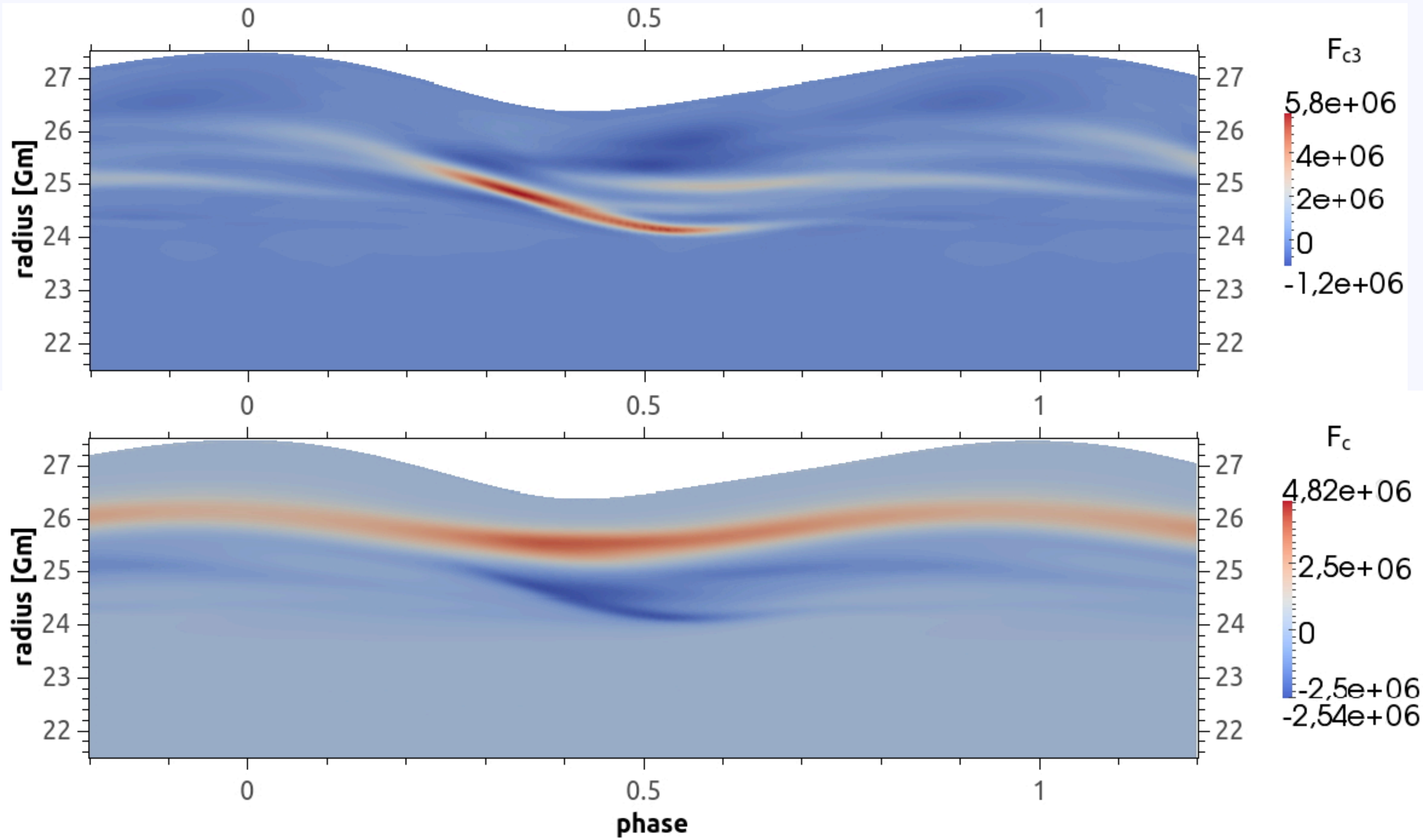
Determining the MLT parameter α for shallow and for deep convection zones in DA type white dwarfs with CO[5]BOLD (Fig. 5 & 11, Tremblay et al. 2015, ApJ 799, 142).





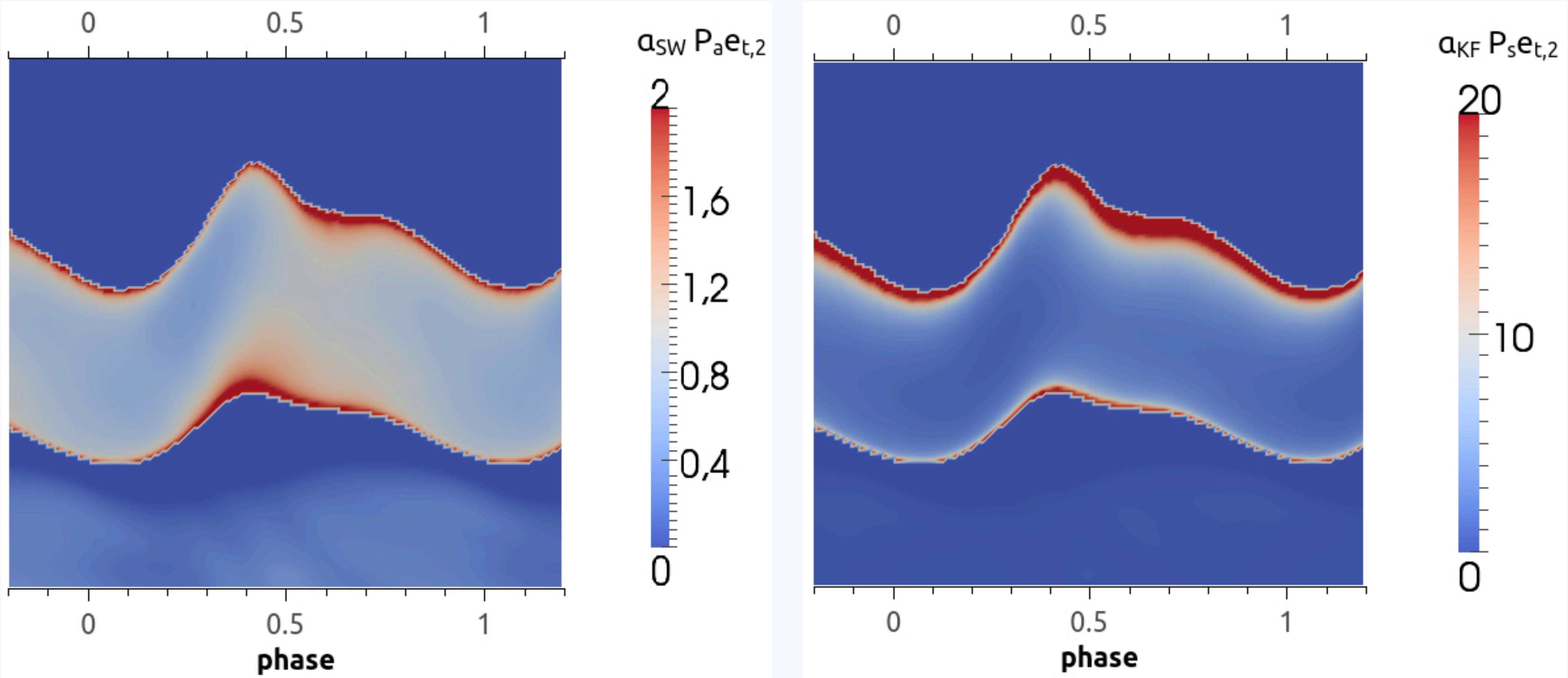
To match the point where layers become stable according to the Schwarzschild criterion (open circles) and where F_{conv} changes sign (closed circles) requires different values.

Lessons from DAs and Cepheids II



2D simulation of the top 42% of a Cepheid (P = 4 d, T_{eff} = 5125 K, log(g) ~ 1.97, R ~ 38.5 $R_\odot$, L ~ 913 $L_\odot$, M = 5 $M_\odot$). F_{conv} as function of radius and phase (10 phases average normalized to [0,1]). Upper/lower panel: without/with mean radial motion included. Mundprecht et al. 2015, MNRAS 449, 2539 (Fig. 1).

Lessons from DAs and Cepheids III



Convective flux parameter α required to match the convective flux of the simulations for Stellingwerf's model (SW, left panel) and the simplified Kuhfuß model (KF, right panel) as a function of phase (abscissa) and radius (ordinate), averaged over 10 phases.

An optimal α would have to change as a function of local stability (convective & overshooting zone), phase, and radius. From the same simulation of Mundprecht et al. 2015, MNRAS 449, 2539 (Fig. 3) as shown on the previous slide.

Conclusions

- **Conclusions**
  - The most promising approach appears to be the patching method:
    - 3D results carried over as much as possible
    - thermal / pressure structure of 3D simulations is on save grounds
  - For the kinetics & dynamics of the velocity field one has to be much more careful: possible influence of boundary conditions
  - Meanwhile one can substitute those approaches with 3D-simulation based calibrations of MLT models, but be aware of their limitations
  - Consider physically more complete models as target for optimizations through 3D simulations
  - As an alternative work out interpolation algorithms for 3D averaged simulation data
  - Take PLATO 2.0 as an incentive to develop a library of convection models of different complexity (including averaged 3D simulations)

...THANK YOU FOR YOUR TIME !