

# The ESTER project: modelling fast rotating stars

**Michel Rieutord**

Institut de Recherche en Astrophysique et Planétologie,  
Toulouse, France

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# Why should we make 2D-models ?

## To deal properly with rotation !

Rotation means

- non spherical stars
- baroclinic flows in radiative region
- anisotropic convection

We note that

- 1D rotating models are valid when  $\Omega \rightarrow 0$
- A lot of physics is condensed inside adjustable (transport) coefficients
- 1D models are not usable in asteroseismology of rapid rotators
- New data from optical/IR interferometry require a 2D view...

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# Interferometry : Achernar

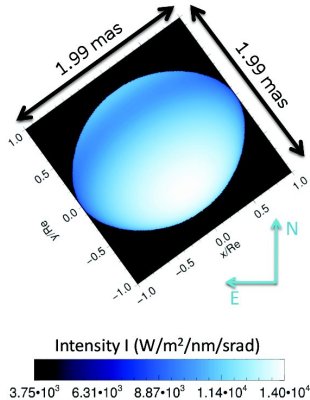
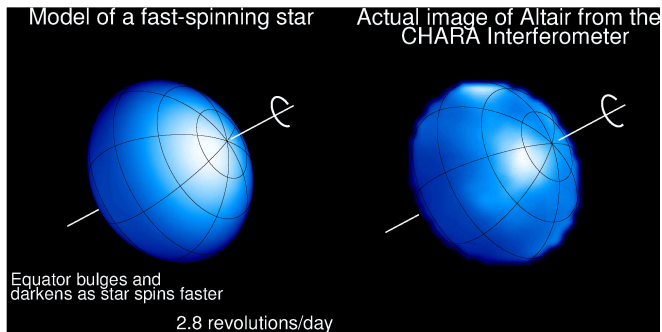


FIGURE : Achernar with VLTI (Domiciano de Souza et al. 2014, AA 569)

# Interferometry : Altair



**FIGURE :** Altair seen by CHARA (Monnier et al. 2007).

# An idealization/simplification

- We consider a lonely rotating star
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# The equations of the structure

PDE

$$\left\{ \begin{array}{l} \Delta\phi = 4\pi G\rho \\ \rho T\vec{v} \cdot \vec{\nabla} S = -\text{Div}\vec{F} + \varepsilon_* \\ \rho(2\vec{\Omega}_* \wedge \vec{v} + \vec{v} \cdot \vec{\nabla}\vec{v}) = -\vec{\nabla}P - \rho\vec{\nabla}(\phi - \frac{1}{2}\Omega_*^2 s^2) + \vec{F}_v \\ \text{Div}(\rho\vec{v}) = 0. \end{array} \right. \quad (1)$$

# The equations of the structure

## Microphysics

$$\left\{ \begin{array}{ll} P \equiv P(\rho, T) & \text{OPAL} \\ \kappa \equiv \kappa(\rho, T) & \text{OPAL} \\ \varepsilon_* \equiv \varepsilon_*(\rho, T) & \text{NACRE} \end{array} \right. \quad (2)$$

# The equations of the structure

## Turbulence

The energy flux

$$\vec{F} = -\chi_r \vec{\nabla} T - \frac{\chi_{\text{turb}} T}{\mathcal{R}_M} \vec{\nabla} S$$

The transport of momentum

$$\begin{aligned} \vec{F}_v = \mu \vec{\mathcal{F}}_\mu(\vec{v}) = & \mu \left[ \Delta \vec{v} + \frac{1}{3} \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) + 2 (\vec{\nabla} \ln \mu \cdot \vec{\nabla}) \vec{v} \right. \\ & \left. + \vec{\nabla} \ln \mu \times (\vec{\nabla} \times \vec{v}) - \frac{2}{3} (\vec{\nabla} \cdot \vec{v}) \vec{\nabla} \ln \mu \right]. \end{aligned}$$

**or any mean-field expression of the Reynolds stress.**

- On pressure

$$P_s = \frac{2}{3} \frac{\bar{g}}{\bar{\kappa}}$$

- On the velocity field

$$\vec{v} \cdot \vec{n} = 0 \quad \text{and} \quad ([\sigma] \vec{n}) \wedge \vec{n} = \vec{0}$$

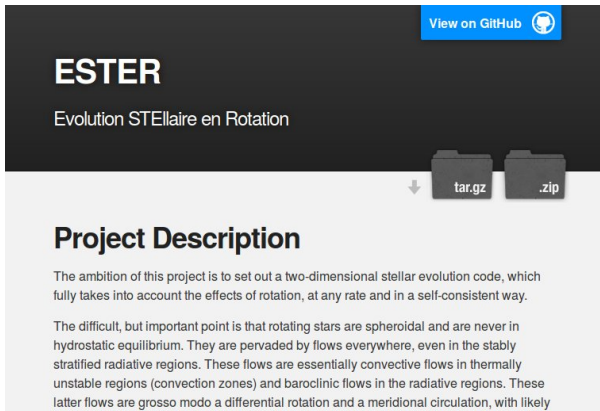
- On temperature (black body radiation)

$$\vec{n} \cdot \vec{\nabla} T + T/L_T = 0$$

$$\int_{(V)} r \sin \theta \rho u_{\varphi} dV = L$$

or

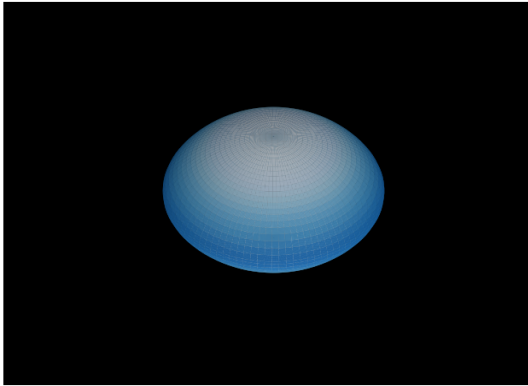
$$v_{\varphi}(r = R, \theta = \pi/2) = V_{\text{Eq}}$$



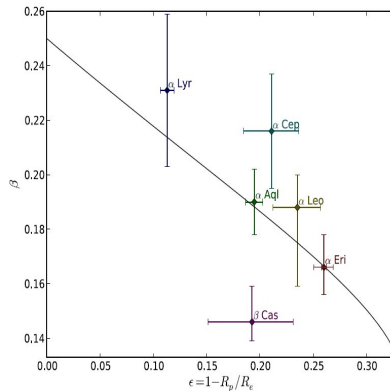
The screenshot shows the GitHub repository page for the ESTER project. At the top right, there is a blue button labeled "View on GitHub" with the GitHub logo. Below this, the repository name "ESTER" is displayed in large white letters, followed by the subtitle "Evolution STEllaire en Rotation" in smaller white text. In the center, there are two dark gray buttons labeled "tar.gz" and ".zip" with a downward arrow icon to their left. Below these buttons, the section "Project Description" is visible, containing two paragraphs of text. The first paragraph states: "The ambition of this project is to set out a two-dimensional stellar evolution code, which fully takes into account the effects of rotation, at any rate and in a self-consistent way." The second paragraph states: "The difficult, but important point is that rotating stars are spheroidal and are never in hydrostatic equilibrium. They are pervaded by flows everywhere, even in the stably stratified radiative regions. These flows are essentially convective flows in thermally unstable regions (convection zones) and baroclinic flows in the radiative regions. These latter flows are grosso modo a differential rotation and a meridional circulation, with likely

**FIGURE :** Freely available on the www

# Gravity darkening of Achernar ( $\alpha$ Eri)



# Gravity darkening exponent : $T_{\text{eff}} \propto g_{\text{eff}}^{\beta}$



**FIGURE :** Observed values of  $\beta$  and a simple model of Espinosa Lara & Rieutord (2011).



# Models of nearby stars

We have modeled 8 stars of intermediate mass :

Star		M ( $M_{\odot}$ )	$V_{\text{eq}}$ (km/s)
Altair	$\alpha$ Aql	1.9	286
Alderamin	$\alpha$ Cep	1.9	265
Ras Alhague	$\alpha$ Oph	2.2	242
	$\delta_A$ Vel	2.27 & 2.43	150 & 143
Vega	$\alpha$ Lyr	2.4	205
Regulus	$\alpha$ Leo	4.1	335
Achernar	$\alpha$ Eri	6.5	339

# $\delta$ Vel seen by Kervella et al. 2013 at VLT with PIONIER

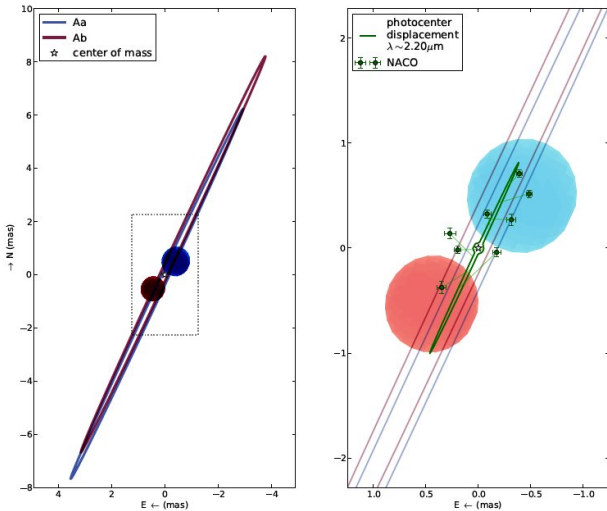


FIGURE : The orbit of delta vel (Kervella et al. 2013).

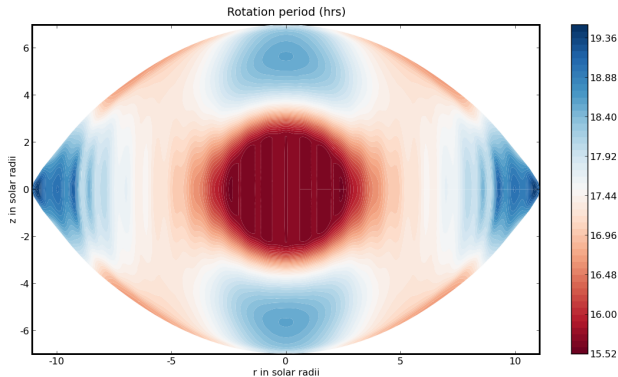
# $\delta$ Velorum A

An eclipsing binary made of A stars

Star	Delta Velorum Aa		Delta Velorum Ab	
	Obs.	Model	Obs.	Model
Mass ( $M_{\odot}$ )	$2.43 \pm 0.02$	<b>2.43</b>	$2.27 \pm 0.02$	<b>2.27</b>
$R_{\text{eq}}$ ( $R_{\odot}$ )	$2.97 \pm 0.02$	<b>2.95</b>	$2.52 \pm 0.03$	<b>2.52</b>
$R_{\text{pol}}$ ( $R_{\odot}$ )	$2.79 \pm 0.04$	<b>2.77</b>	$2.37 \pm 0.02$	<b>2.36</b>
$T_{\text{eq}}$ (K)	9450	<b>9440</b>	9560	<b>9477</b>
$T_{\text{pol}}$ (K)	10100	<b>10044</b>	10120	<b>10115</b>
$L$ ( $L_{\odot}$ )	$67 \pm 3$	<b>65.2</b>	$51 \pm 2$	<b>48.5</b>
$V_{\text{eq}}$ (km/s)	143	<b>143</b>	150	<b>153</b>
$P_{\text{eq}}$ (days)		<b>1.045</b>		<b>0.832</b>
$P_{\text{pol}}$ (days)		<b>1.084</b>		<b>0.924</b>
$X_{\text{env.}}$		<b>0.70</b>		<b>0.70</b>
$X_{\text{core}}/X_{\text{env.}}$		<b>0.10</b>		<b>0.30</b>
Z		<b>0.011</b>		<b>0.011</b>

# Inside the stars : internal differential rotation

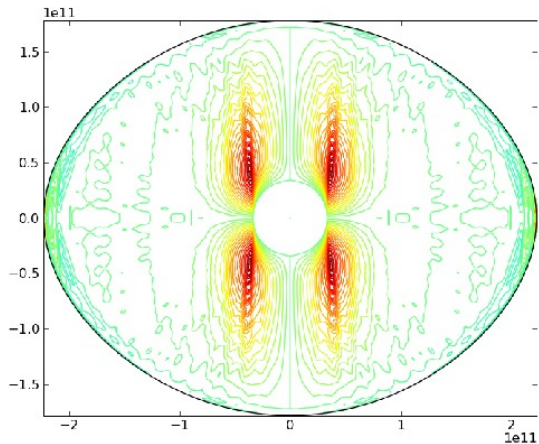
$M=30M_{\odot}$  at 98% of critical angular velocity



Espinosa Lara & Rieutord (2013) *A&A*, **552**, A35

# Inside the stars : meridional circulation

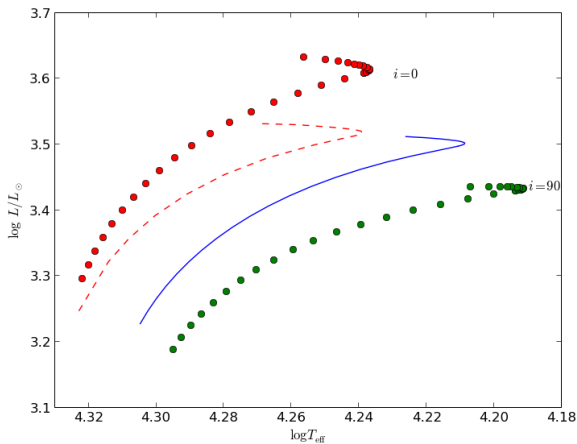
$M=5M_{\odot}$  at 70% of critical angular velocity



Espinosa Lara & Rieutord (2013)

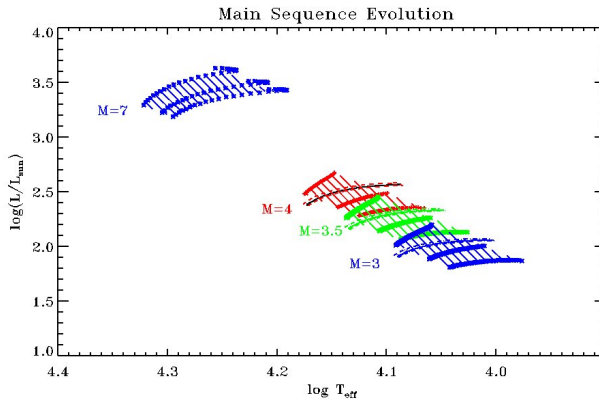
# Towards evolution

HR diagram track of a  $7M_{\odot}$  star of constant angular momentum, starting at  $\Omega/\Omega_k = 0.5$ .

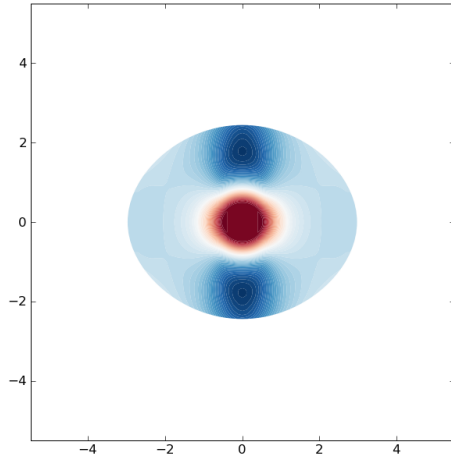


# Towards evolution

HR diagram tracks at constant angular momentum



# Evolution of a $5M_{\odot}$ star at constant angular momentum : heading to the Be state





# Last developments and road map

- Portability improved, github management
- Documentation strongly improved (93 pages)
- Low mass stellar models under construction

## Next :

- 1 Implement nuclear evolution on MS
- 2 Implement thermal evolution (PMS and post-MS)

## Points to take away :

- 1 ESTER 2D models are ripe to face observational data in
  - asteroseismology (coupled with TOP)
  - interferometry (coupled with CHARRON)for early-type stars.
- 2 low-mass fast rotating stellar model should come soon...

# Some references

- Rieutord, Espinosa Lara & Putigny (2016), J. Comput. Phys. 318, 277
- Espinosa Lara & Rieutord (2013), A&A, **552**, A35
- ESTER website : <http://ester-project.github.io/ester/>