# Asteroseismic model fitting using ε phases

ref. Roxburgh, I W, A&A 585A, 63R, 2016

**Goals:** To find stellar models whose interior structure is compatible with an observed frequency set ($\nu_{n\ell}$) of p-modes

To estimate mass and age

Refine stellar modelling

Observed frequencies $\mu$Hz

**How?** Structure of outer layers not well understood (convection, diffusion, composition…)

Subtract off contribution of outer layers by comparing ε phases

$$\nu_{n\ell} = \Delta\,[n + \ell/2 + \varepsilon_{n\ell}] \quad \varepsilon_{n\ell} = \varepsilon_{\ell}(\nu)$$

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# ε and phase shifts

$\varepsilon_{n\ell} = \nu_{n\ell}/\Delta - n - \ell/2 \quad \varepsilon_{n\ell} = \varepsilon_\ell(\nu)$

Eigen frequency eqn*

$\nu_{n\ell} = \Delta\,[n + \ell/2 - \delta_\ell(\nu_{n\ell}) + \alpha_\ell(\nu_{n\ell})\,]$

$\delta_\ell(\nu)$ determined by inner layers

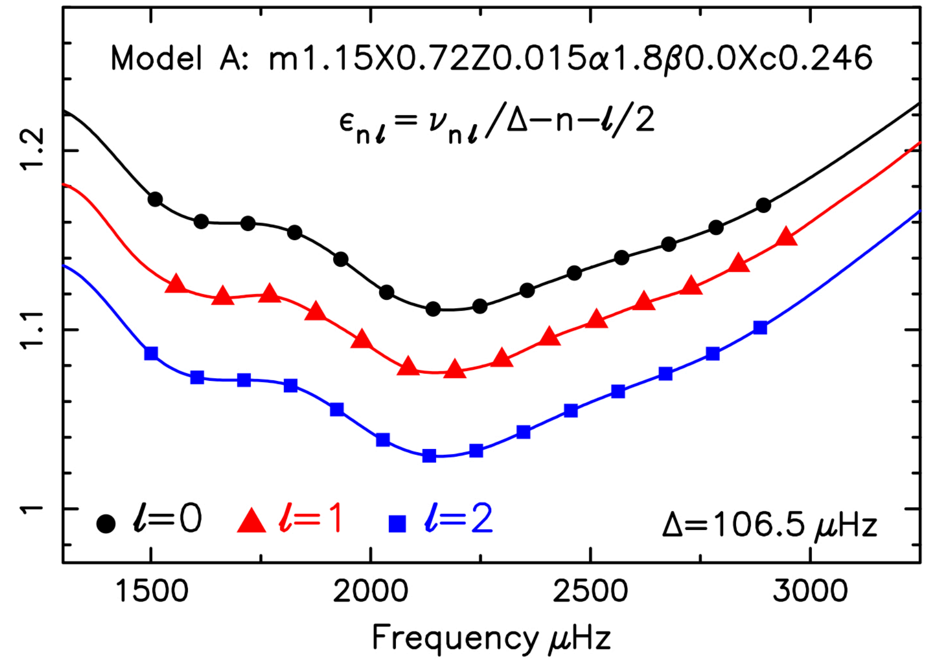
$\alpha_\ell(\nu)$ determined by outer layers

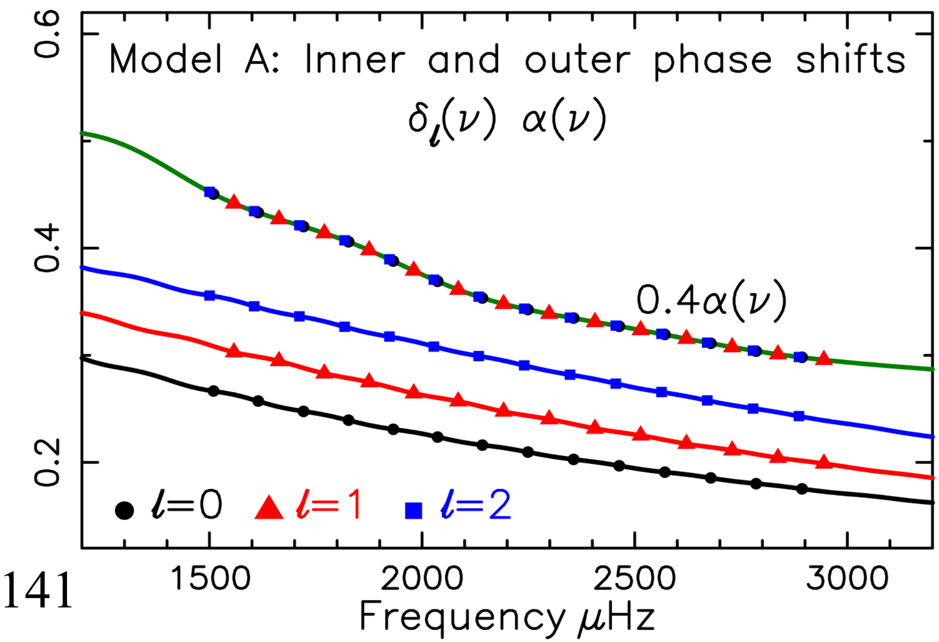
$\alpha_\ell(\nu)=\alpha(\nu)$ independent of $\ell$

$\varepsilon_{n\ell} = \alpha(\nu_{n\ell}) - \delta_\ell(\nu_{n\ell}) \quad \varepsilon_{n\ell} = \varepsilon_\ell(\nu)$

If star (o) and model (m) have same interior structure then same $\delta_\ell(\nu)$

$\varepsilon_\ell^{o}(\nu^{o}) - \varepsilon_\ell^{m}(\nu^{o})$ = function only of $\nu$



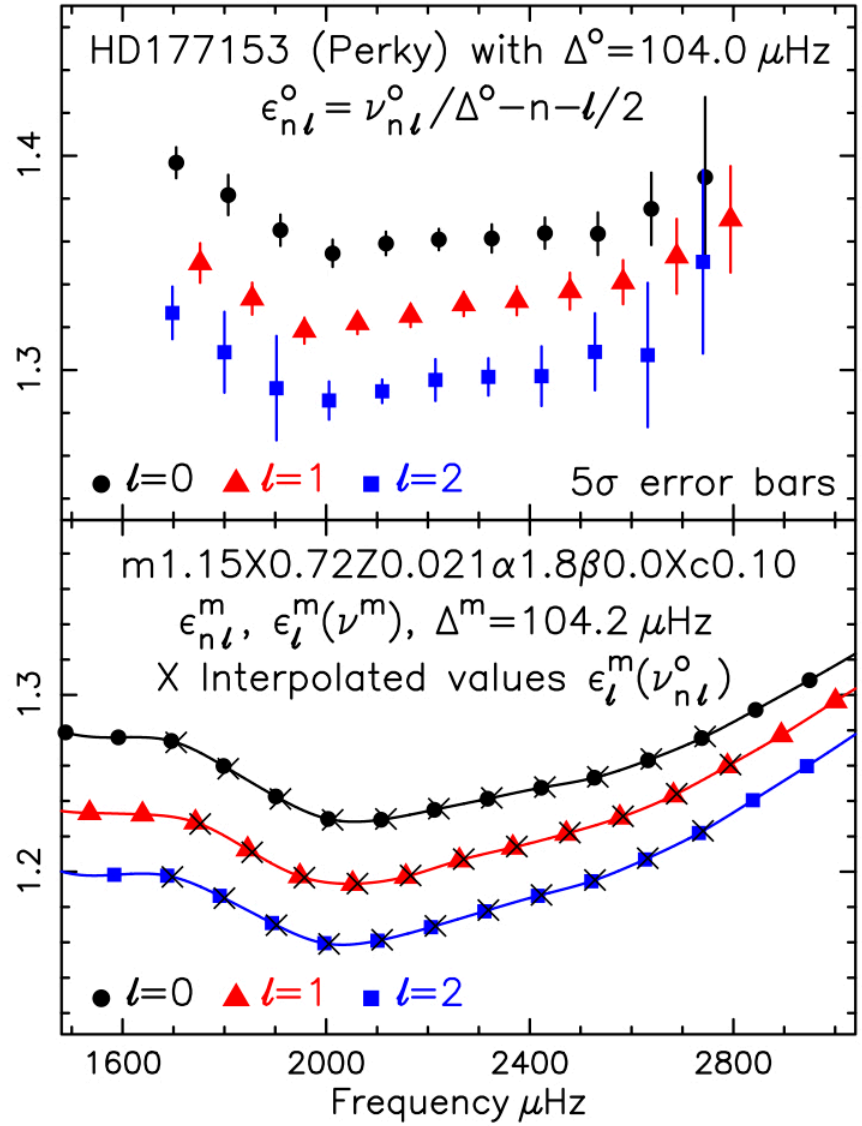

* Roxburgh & Vorontsov, 2000, MNRAS, 317,141

# Model fitting by epsilon matching

Errors uncorrelated - interpolate only in model frequencies

Roxburgh, I W, A&A 585A, 63R, 2016

# Fit to HD177153 (aka Perky)



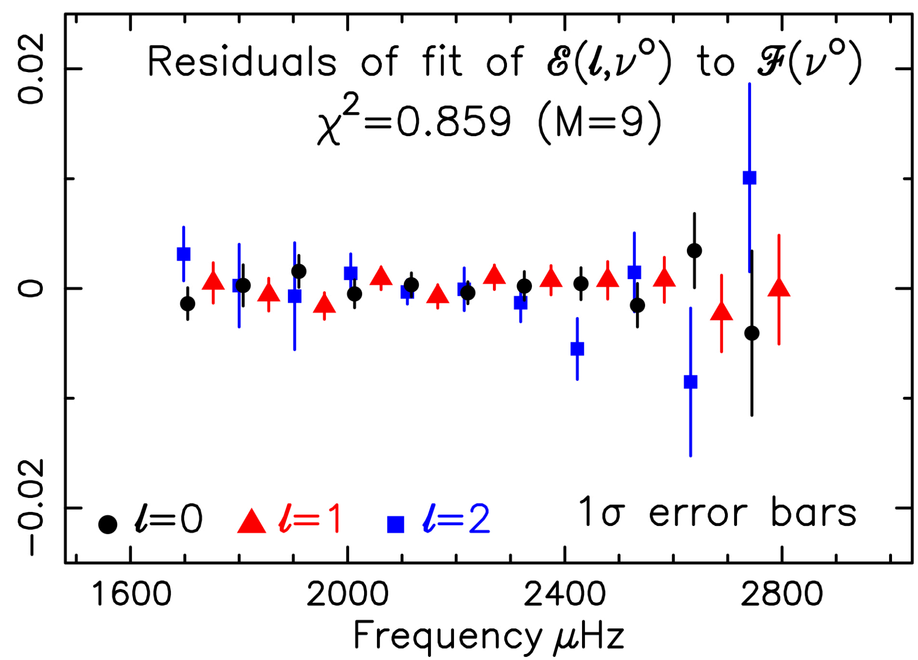

Observed star = HD177153 (SA 13)

11 ν each $l$=0,1,2; Δ=104±0.5μHz

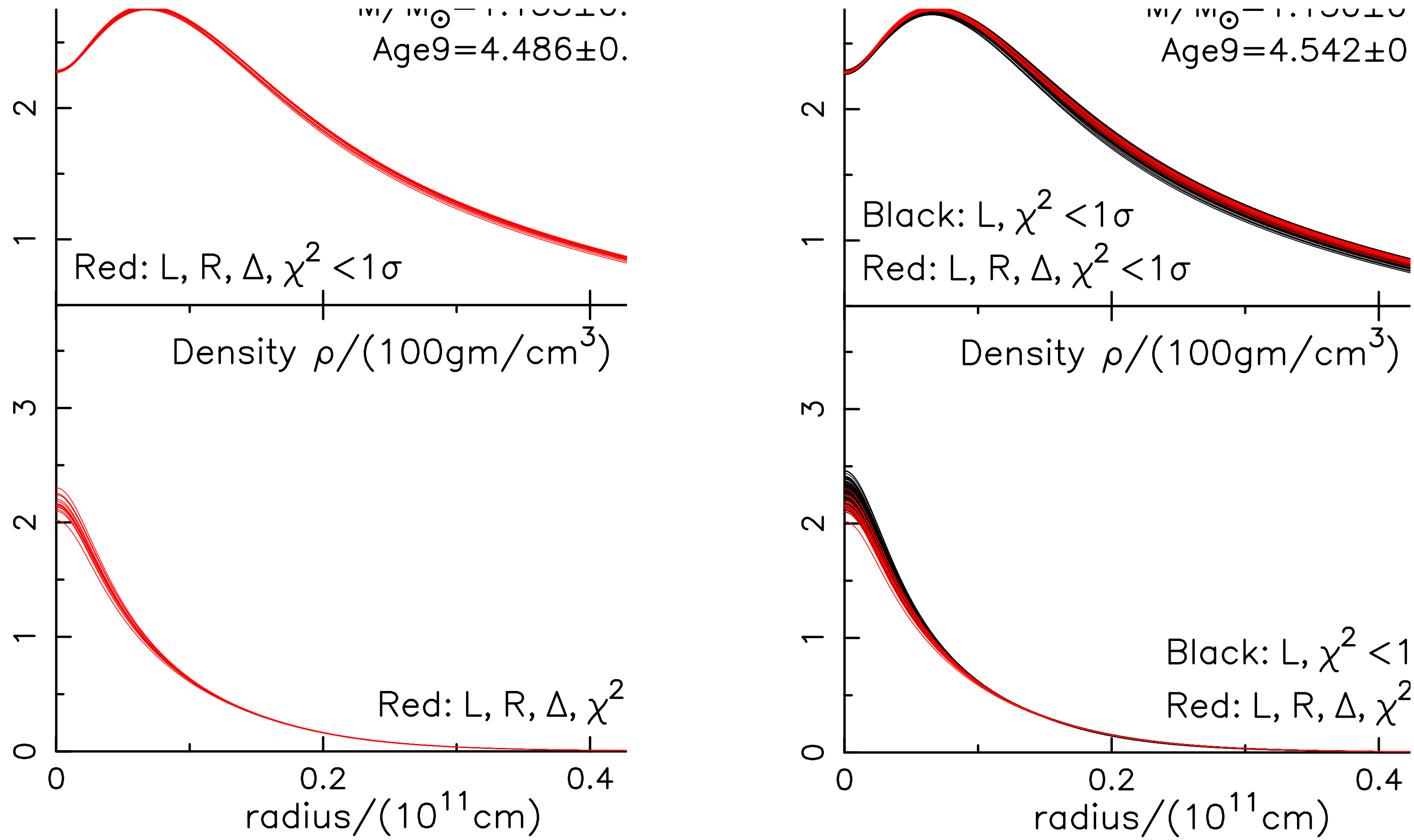
$L/L_{\odot}$=1.82±0.08 $R/R_{\odot}$=1.29±0.04

Model star = 1.15$M_{\odot}$ (GS98)

$L/L_{\odot}$=1.87, $R/R_{\odot}$=1.25, Δ=104.2μHz



# Internal structure of models fitting HD 177513, $\chi^2$ <1



# Application to HH2a

Data provided to fitters (*not the same as those of the input model*)

$1.941 \leq L/L_{\odot} \leq 2.228$, $5814 \leq T_{eff} \leq 5974$ °K, [Fe/H]=0.065 ± 0.051

Frequencies $\nu_{n\ell} \pm \sigma_{n\ell}$ $\ell=0$, n=16−26; $\ell=1$, n=15−25; $\ell=2$, n=16−23

Find models with lowest $\chi^2$ from ε matching. Model set: GS98, MLT, EOS5, OPAL+ Wichita opacities, NACRE reaction rates, no diffusion.

ε matching subtracts off effect of the outer layers-no constraint on [Fe/H].

Δ is strongly dependent on the structure of the outer layers* as are, to a lesser extent, R and $T_{eff}$ which are taken as supplementary constraints.

L is determined by the inner structure and is the primary constraint

Initial Helium abundance is constrained ≥ 0.265

* Roxburgh, I W, A&A, 571, A88,2014

## Results: properties of models with $\chi^2 < 1$

Constraint on L only

154 models with

$1.17 < M/M_\odot < 1.25$, ,   $2.61 < Age < 3.24$,   $1.28 < R/R_\odot < 1.40$

Constraint on L and $T_{eff}$

85 models with

$1.17 < M/M_\odot < 1.24$, ,   $2.66 < Age < 3.24$,   $1.30 < R/R_\odot < 1.40$

Constraint on L and $T_{eff}$ and $\Delta$

35 models with

$1.17 < M/M_\odot < 1.23$, ,   $2.66 < Age < 3.07$,   $1.34 < R/R_\odot < 1.37$

**But 10 best fit models with lowest $\chi^2$ :**

**10 models with lowest $\chi^2$ which satisfy just the L constraint**
**All models except those marked * also satisfy the $T_{eff}$ constraint**

| $\chi^2$ | $M/M_\odot$ | Age | $R/R_\odot$ | $L/L_\odot$ | $T_{eff}$ | $\Delta$ | XH | Z |
|---|---|---|---|---|---|---|---|---|
| 0.770 | 1.210 | 3.059 | 1.330 | 1.945 | 5917 | 98.28 | 0.710 | 0.025 |
| 0.798 | 1.220 | 2.917 | 1.328 | 2.008 | 5967 | 98.74 | 0.710 | 0.025 |
| *0.803 | 1.210 | 2.931 | 1.320 | 1.999 | 5980 | 99.32 | 0.710 | 0.024 |
| 0.806 | 1.210 | 3.052 | 1.317 | 1.945 | 5944 | 99.61 | 0.710 | 0.025 |
| 0.810 | 1.220 | 2.884 | 1.326 | 2.001 | 5968 | 99.02 | 0.710 | 0.025 |
| *0.811 | 1.210 | 2.991 | 1.324 | 2.011 | 5979 | 98.83 | 0.710 | 0.024 |
| 0.816 | 1.200 | 2.921 | 1.358 | 1.985 | 5885 | 95.14 | 0.710 | 0.023 |
| 0.821 | 1.210 | 2.937 | 1.370 | 1.998 | 5869 | 94.33 | 0.710 | 0.024 |
| 0.821 | 1.210 | 2.874 | 1.365 | 1.985 | 5870 | 94.84 | 0.710 | 0.024 |
| 0.821 | 1.220 | 2.971 | 1.333 | 2.019 | 5966 | 98.30 | 0.710 | 0.025 |

**$M/M_\odot = 1.21 \pm 0.01$, Age $= 2.97 \pm 0.09$ Gyr, $R/R_\odot = 1.34 \pm 0.03$**

# Concluding observation

The results give above were obtained using constraints which were not exactly those of the input model, which were

| | | |
|---|---|---|
| $M/M_\odot = 1.182$ | 10% | $1.0638 < M/M_\odot < 1.3002$ |
| $R/R_\odot = 1.335$ | 1% | $1.32165 < R/R_\odot < 1.35835$ |
| Age=3.2162 | 10% | $2.8946 < \text{Age} < 3.5378$ |
| $\text{Log}(L/L_\odot) = 0.3032 \pm 0.030$ | | $1.8759 < L/L_\odot < 2.0100$ |
| $\text{Teff} = 5954 \pm 80$ | | $5874 < \text{Teff} < 6043$ |

Of the 10 models in previous table:

4 satisfy these constraints including the 2 best fit models with $\chi^2 < 0.8$

8 satisfy the M, age, L constraints but not the 1% requirement on R
the remaining 2 are < 1% outside range for age

This is not surprising as ε matching compares interior structure and R (and Δ) are not outer layer independent.

# Inner and outer solutions of the oscillations equations

Define $\psi=2\pi\nu\, S/(dS/dt)$ where $S(t)=r\,\delta p_E/(\rho c)^{1/2}$ $t=\int dr/c$

For modes of angular degree $\ell=0,1$, $\psi_\ell$ satisfy the 1st order equation

$$d\psi_\ell/dt = 2\pi\nu + Q_\ell\,\psi_\ell + (4\pi^2\nu^2 - V_\ell)\,\psi_\ell^2/(2\pi\nu)$$

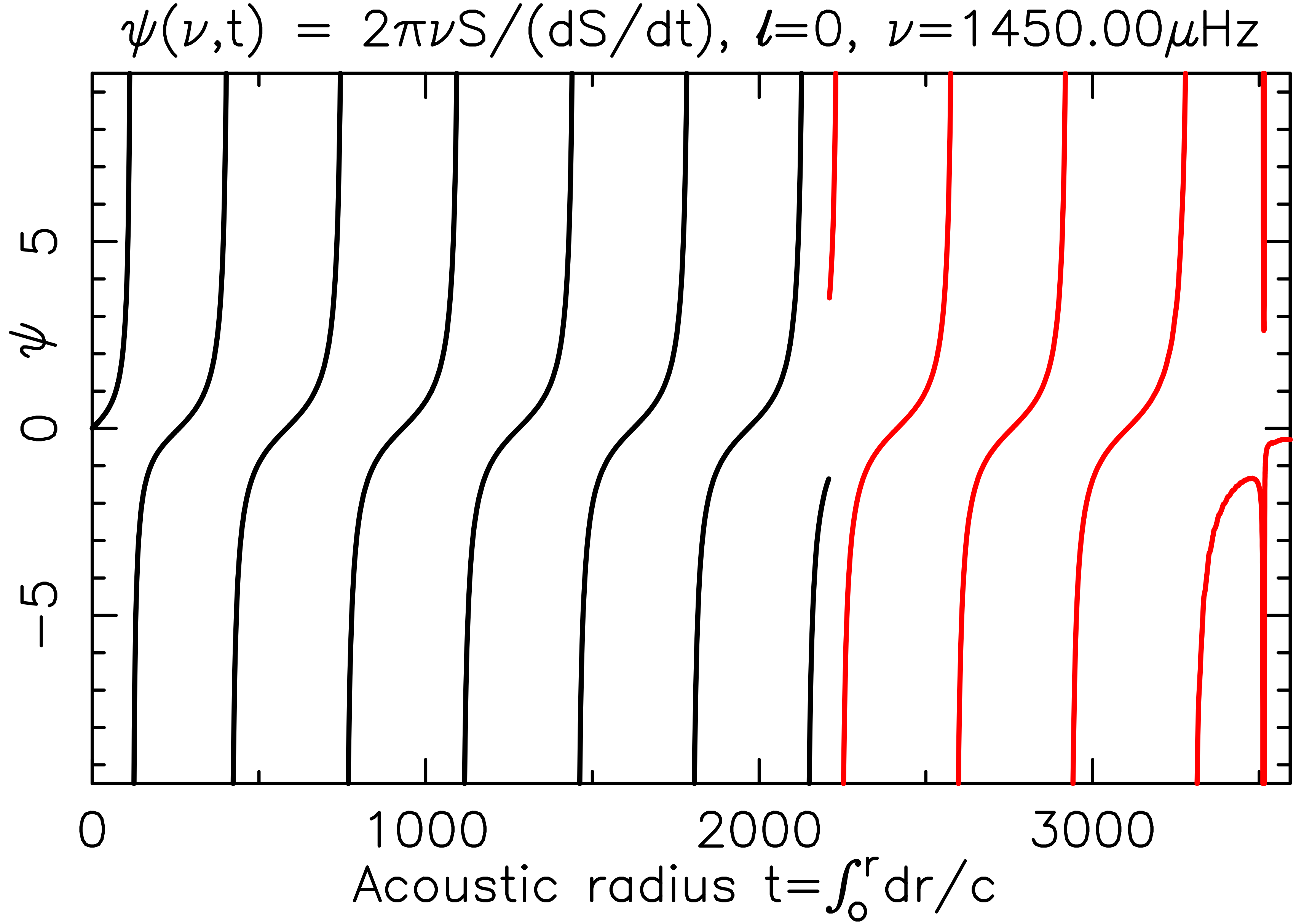
where the "potentials" $Q_\ell(\nu,t)$, $V_\ell(\nu,t)$ depend on $\ell$, $\nu$ and structure

So solution $\psi_{out}$ to $t_1$ depends only on $\ell$, $\nu$ and the structure below $t_1$

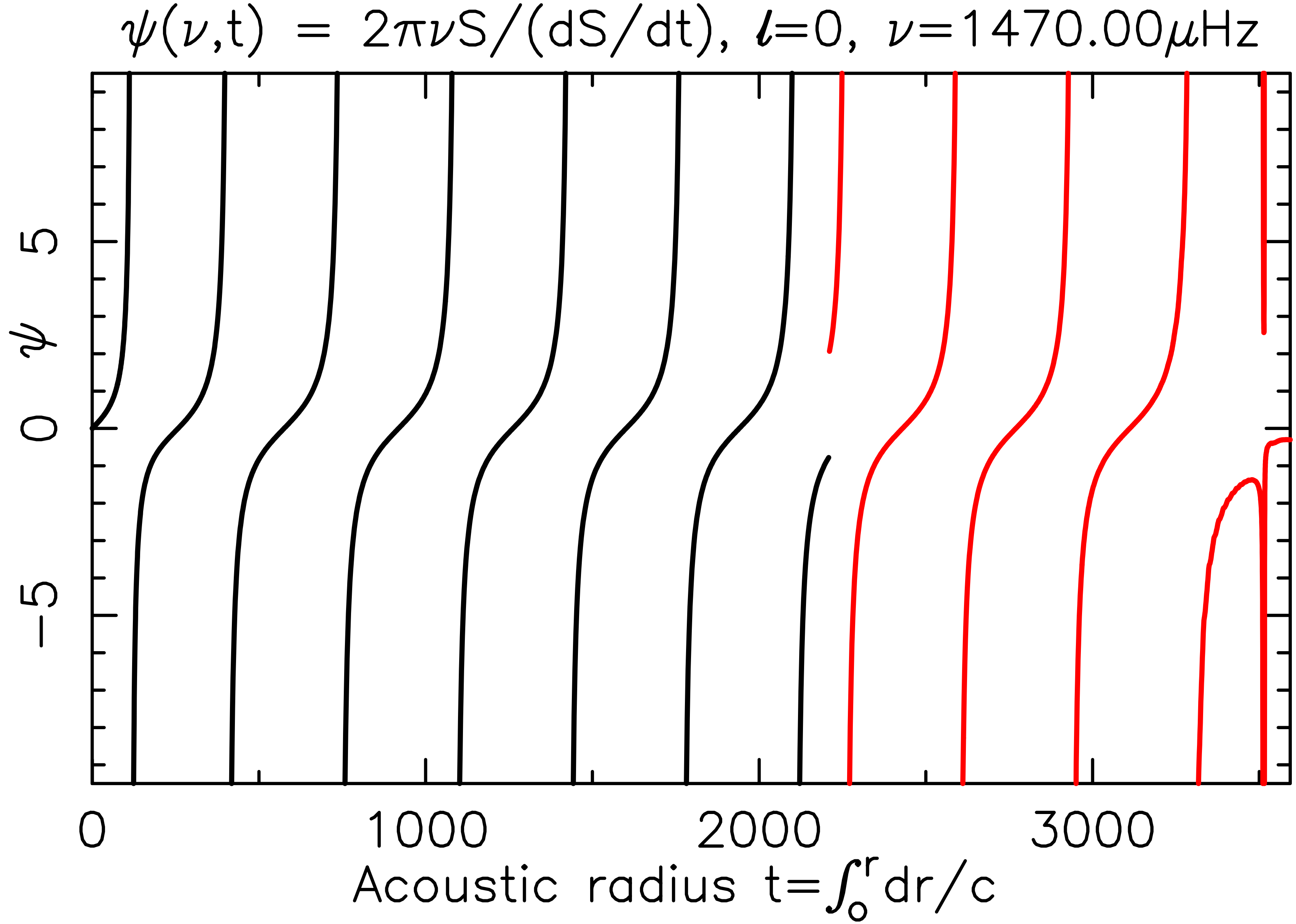
and solution $\psi_{in}$ to $t_1$ depends only on $\ell$, $\nu$ and the structure above $t_1$

eigenvalue when $\psi_{out}=\psi_{in}$ at any $t_1$

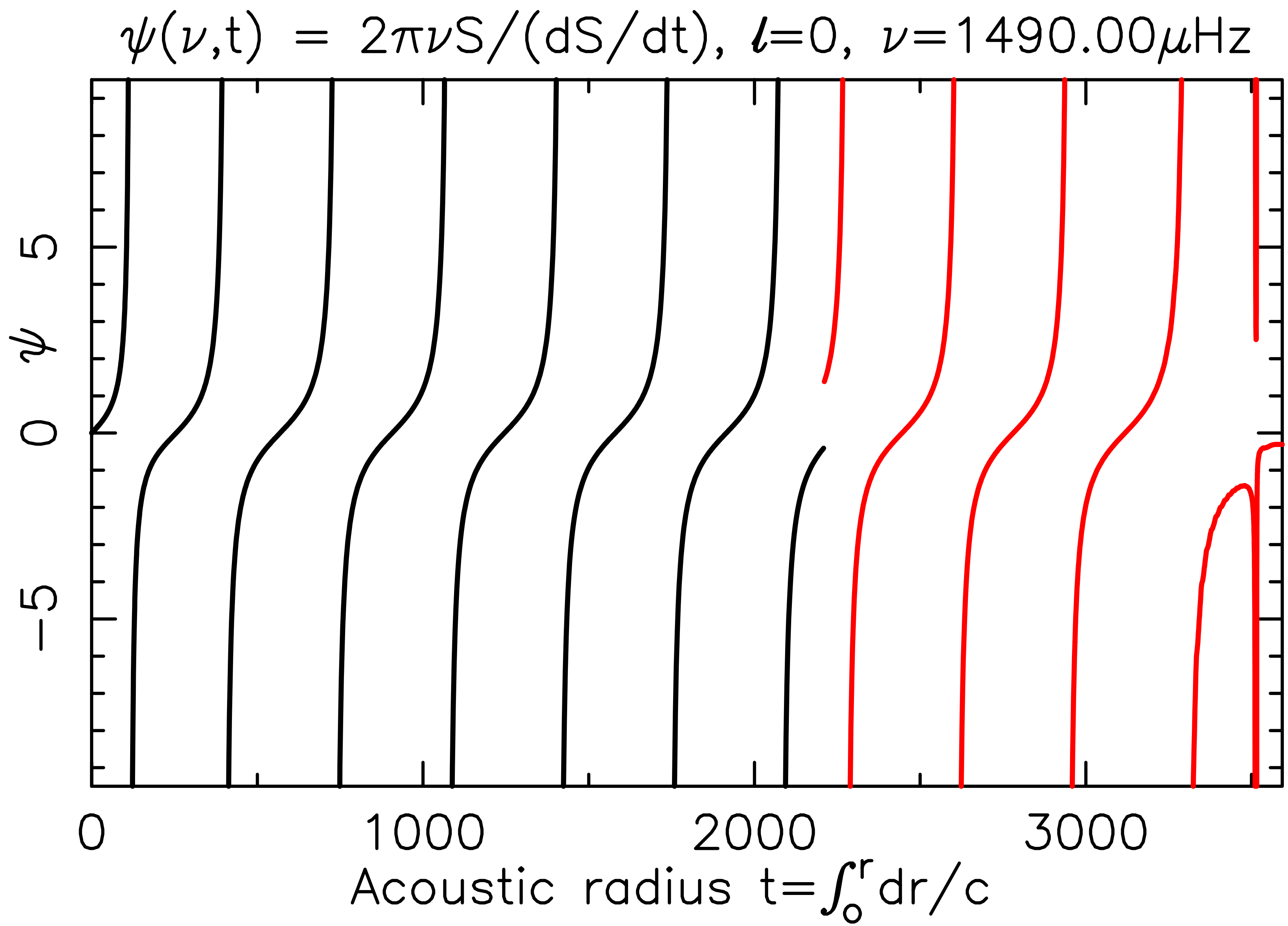
For modes $\ell \geq 2$ it is a little more complicated but same applies to a very good approximation if $t_1$ is in the outer layers (eg $r/R=0.95$)



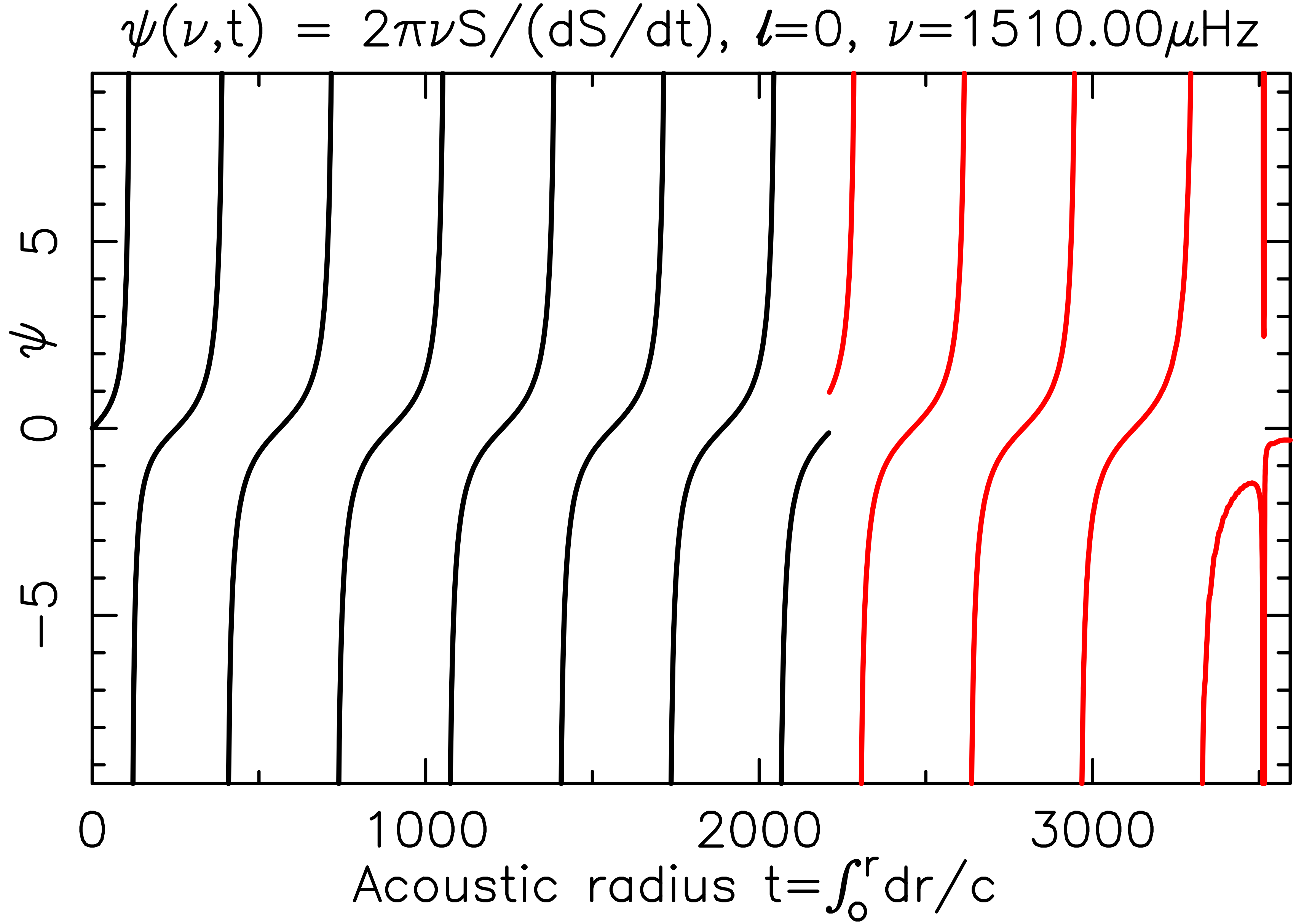
ψ(ν,t) = 2πνS/(dS/dt), ℓ=0, ν=1450.00μHz
ψ
5
0
−5
0
1000
2000
3000
Acoustic radius t=∫₀ʳdr/c



$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $\ell=0$, $\nu=1470.00\mu$Hz
5
0
−5
$\psi$
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l=0$, $\nu=1490.00\mu$Hz
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$


$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $\ell=0$, $\nu=1510.00\mu Hz$
$\psi$
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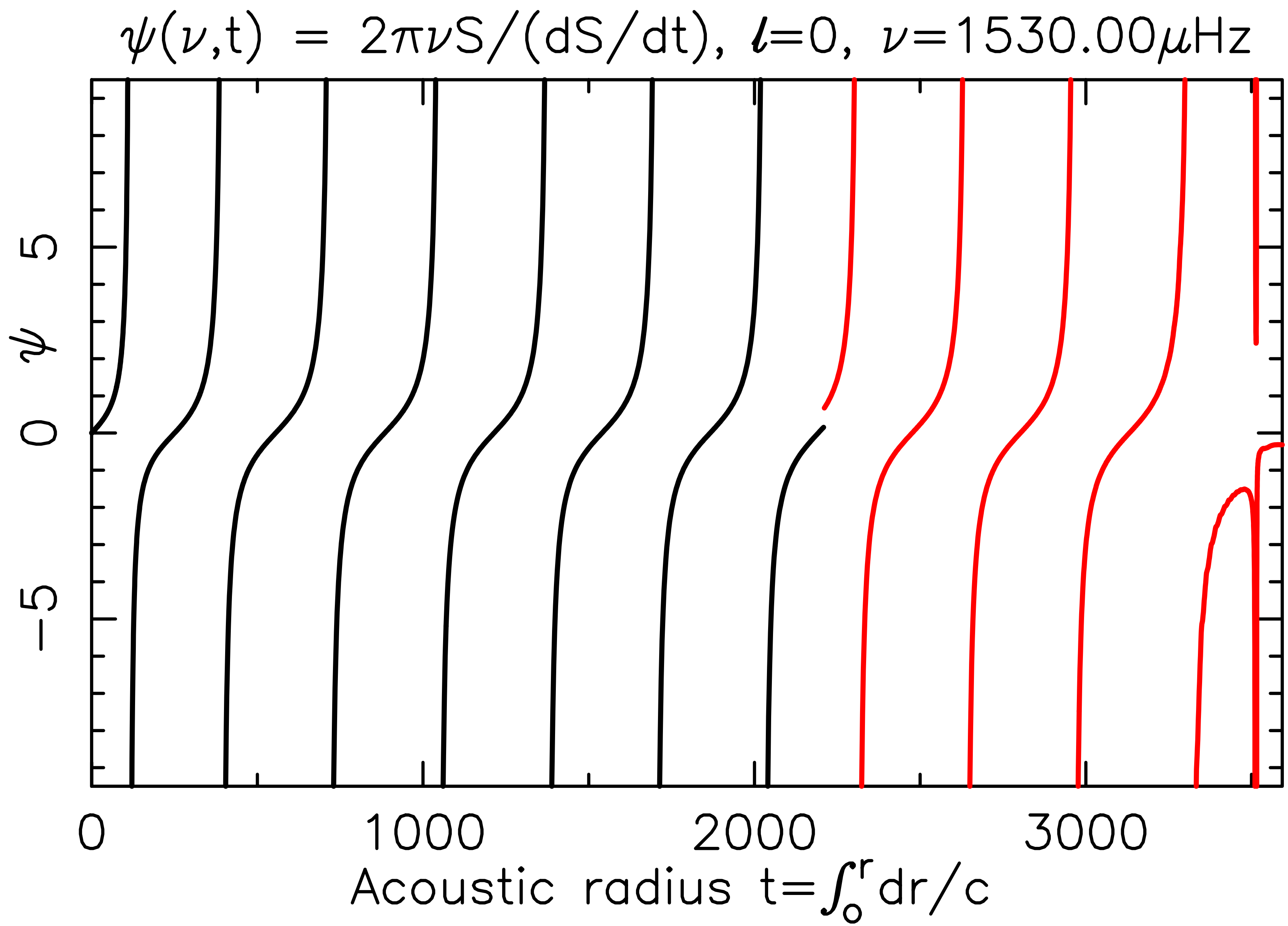
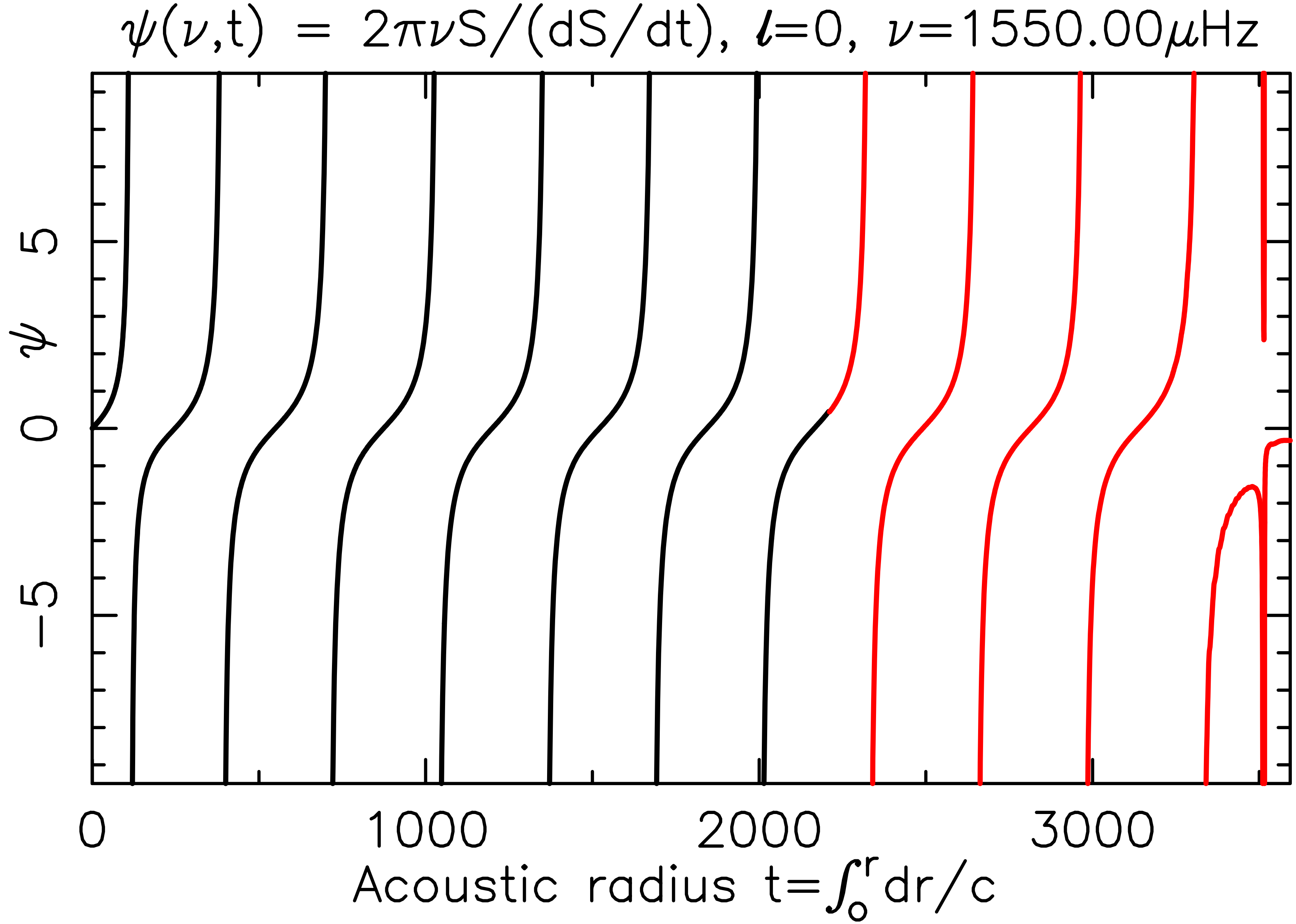
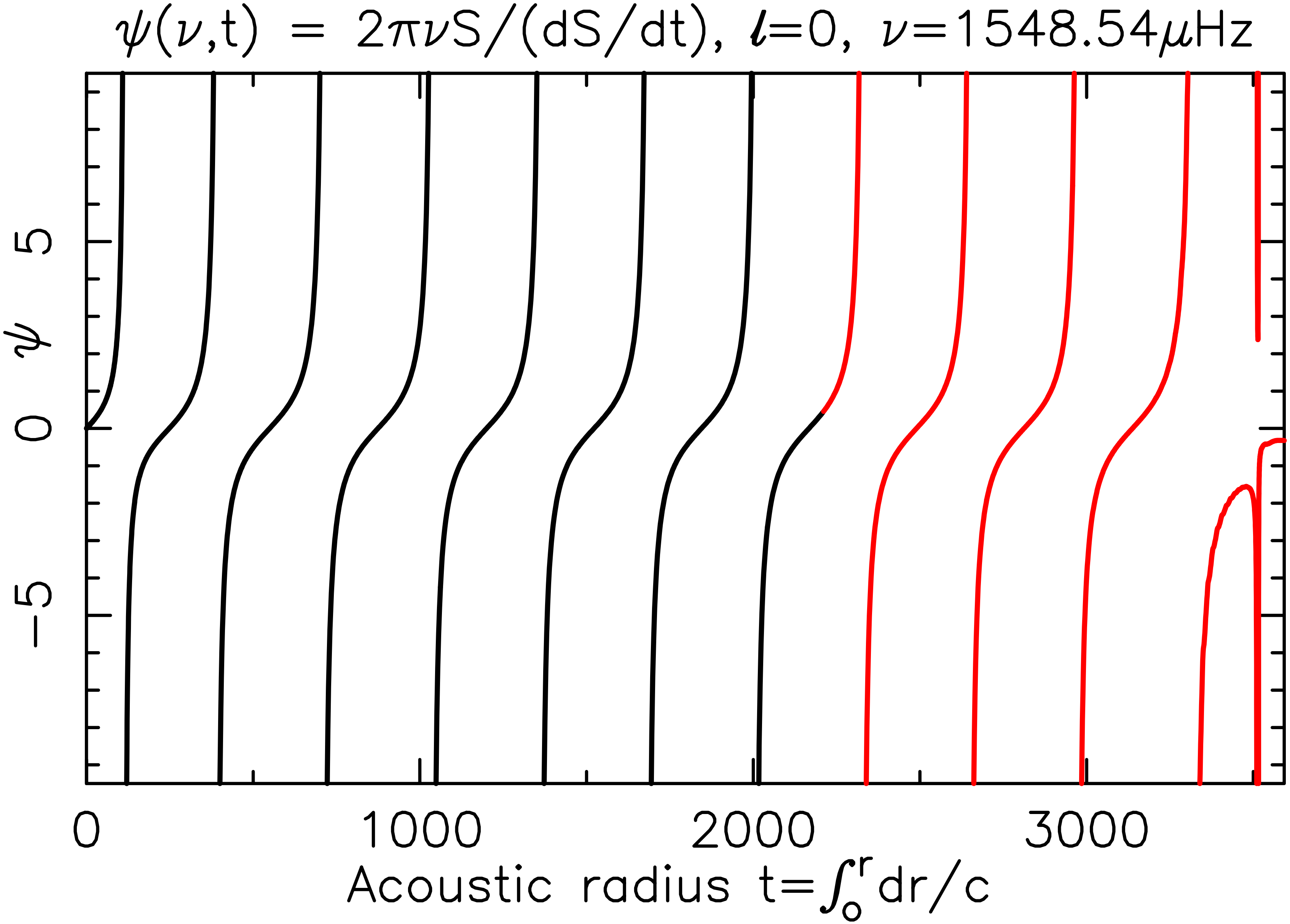
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



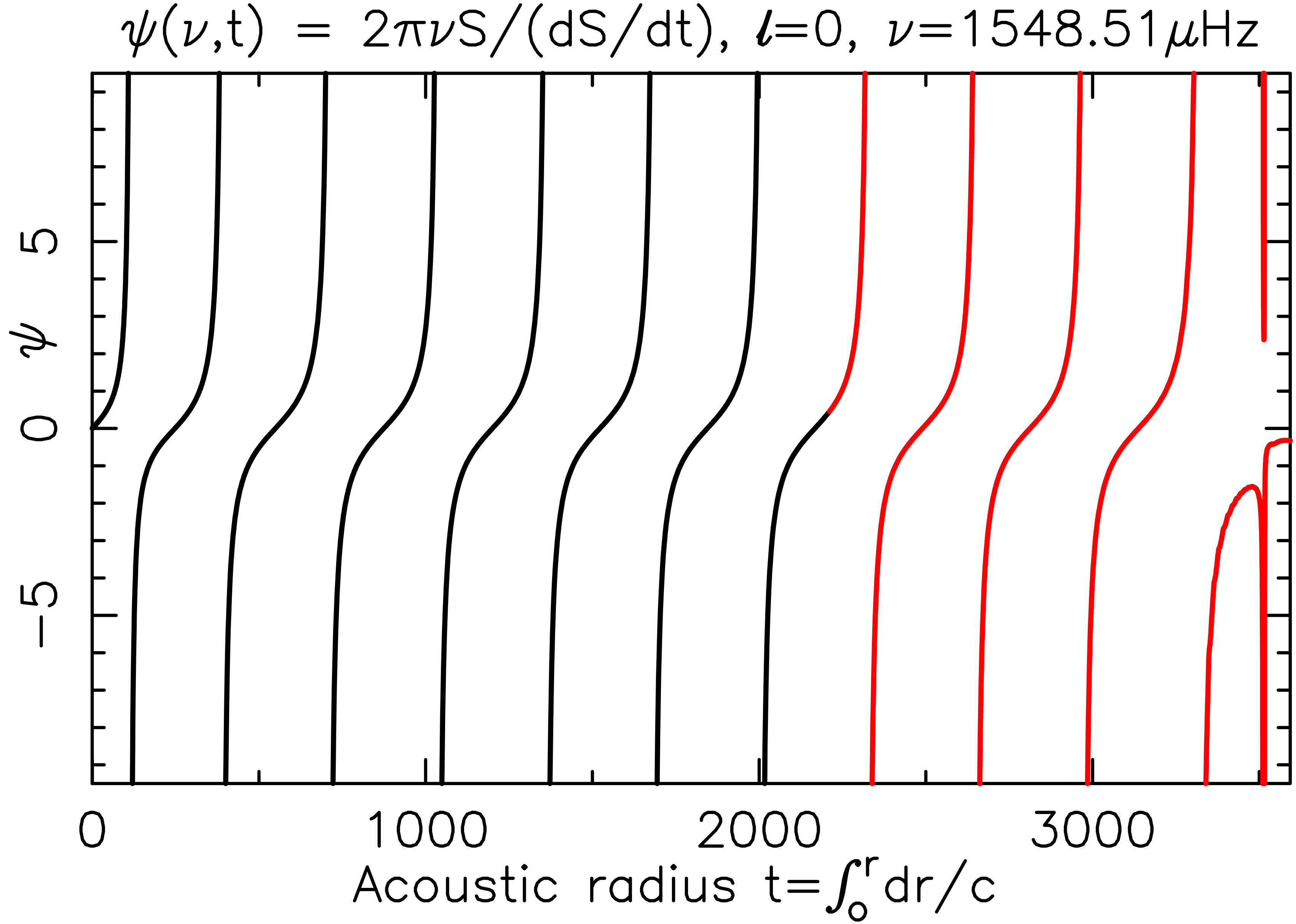
$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l=0$, $\nu=1530.00\mu Hz$
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



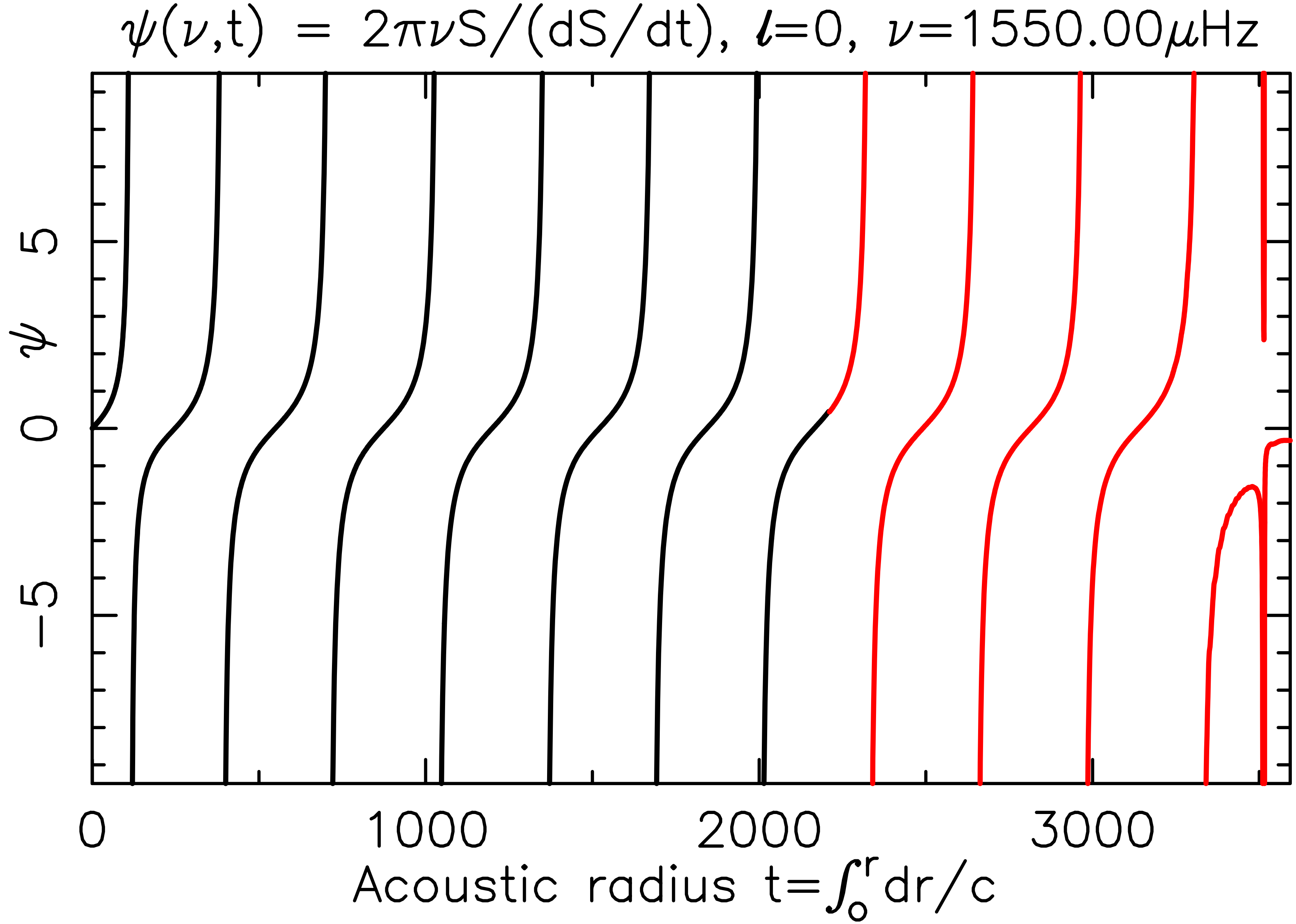
$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l$=0, $\nu$=1550.00$\mu$Hz
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$

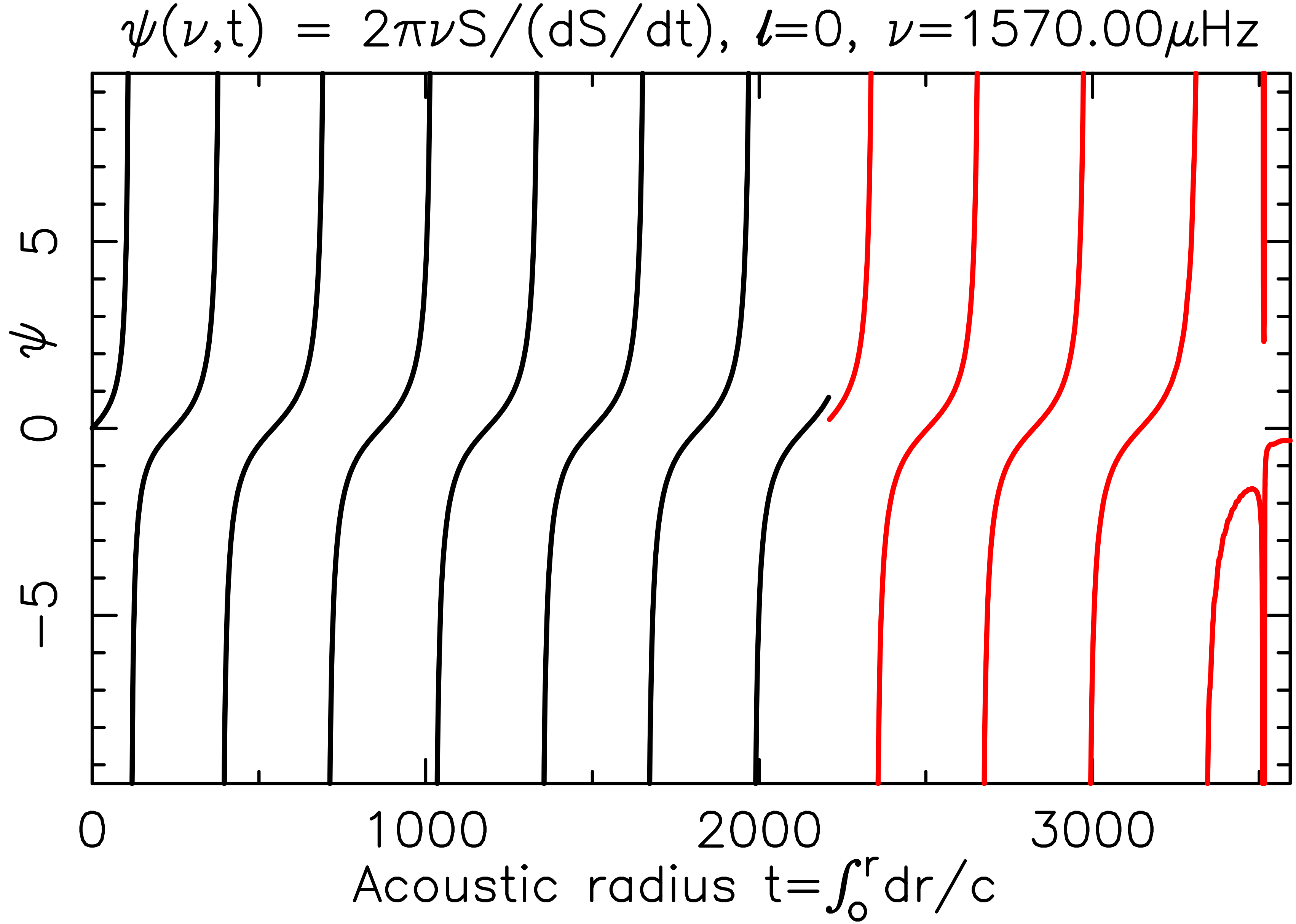


$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $\ell=0$, $\nu=1548.54\mu$Hz
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$

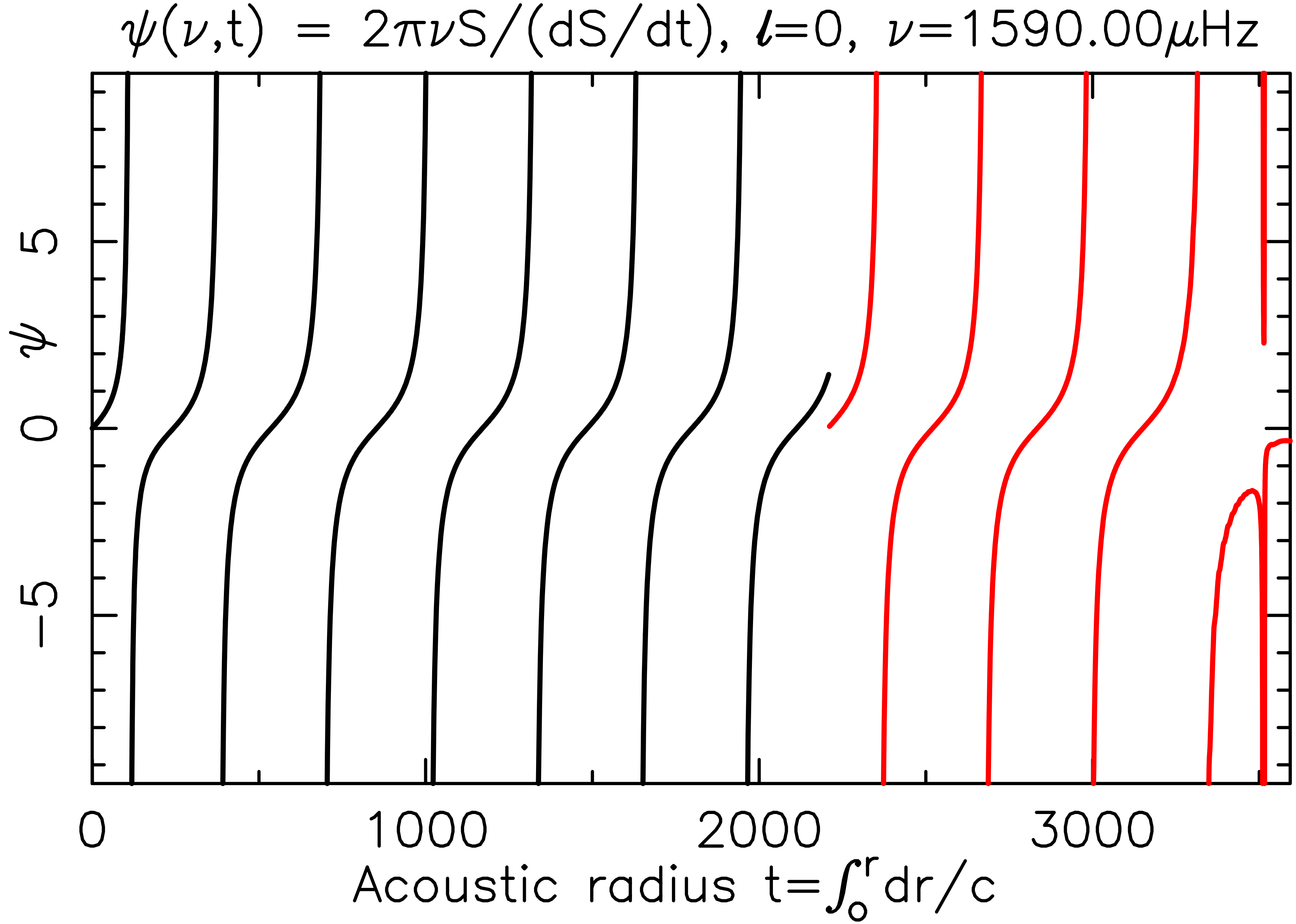


ψ(ν,t) = 2πνS/(dS/dt), l=0, ν=1548.51μHz
ψ
5
0
−5
0
1000
2000
3000
Acoustic radius t=∫ₒʳdr/c

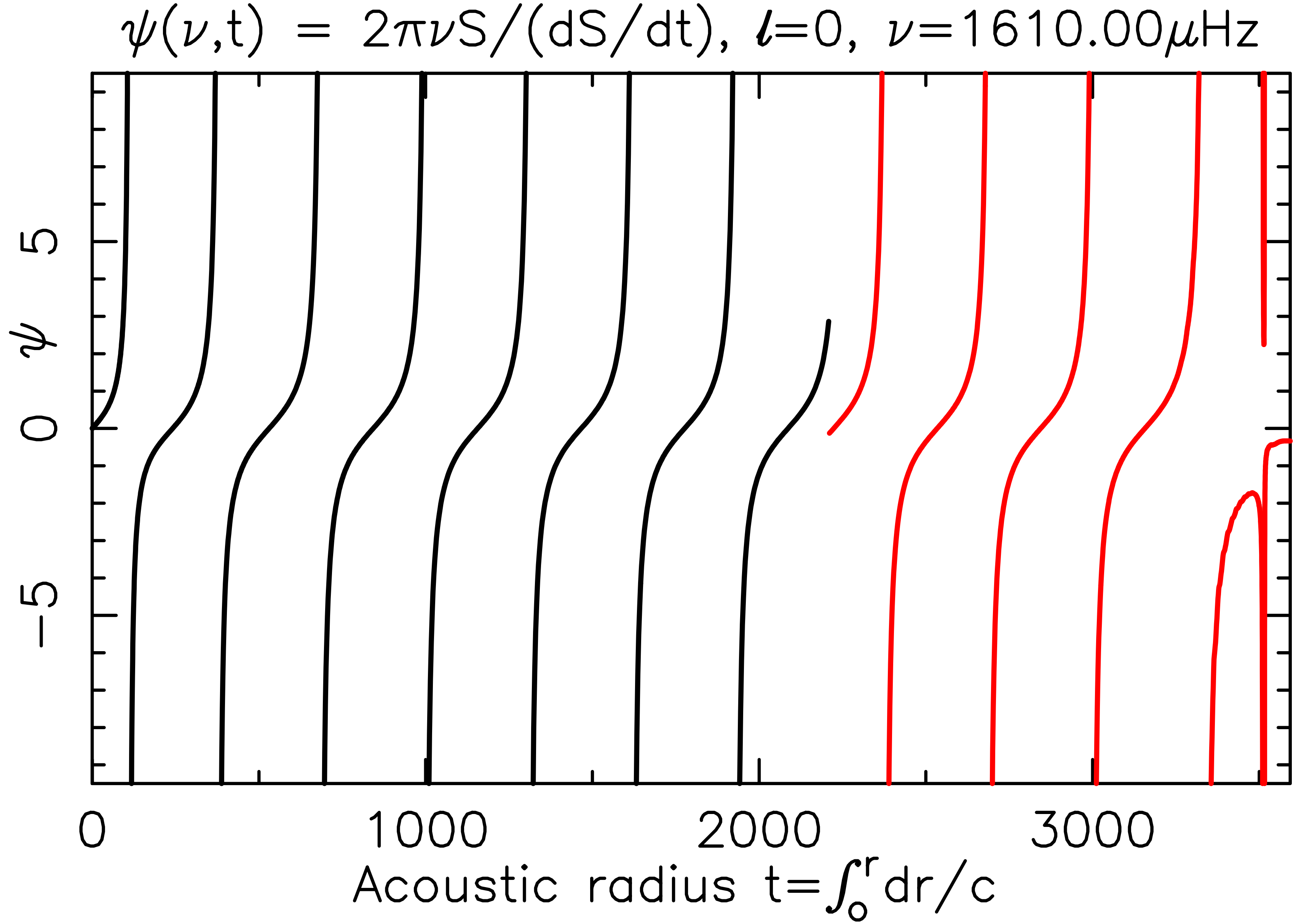

$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l=0$, $\nu=1550.00\mu Hz$
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



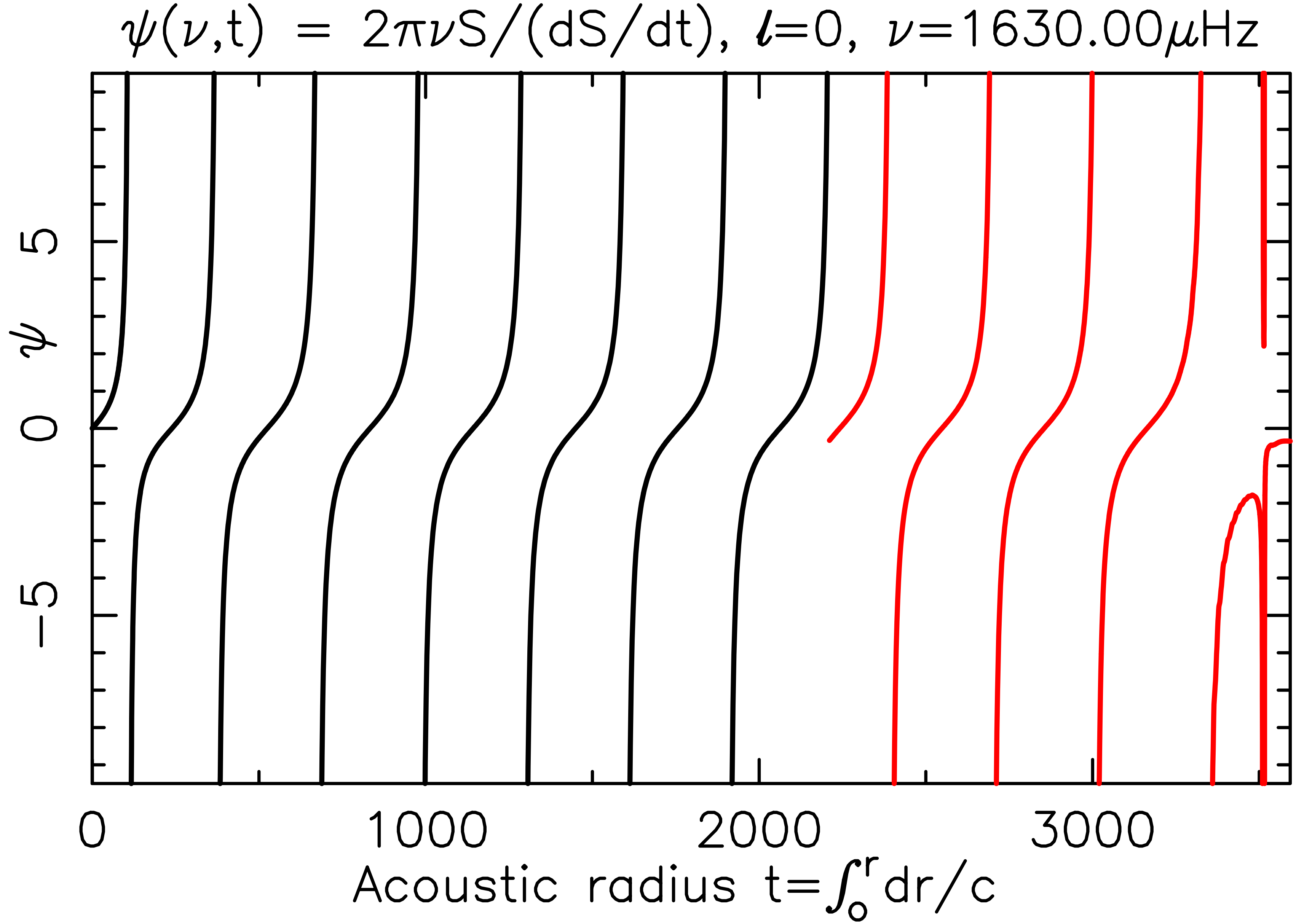
ψ(ν,t) = 2πνS/(dS/dt), l=0, ν=1570.00μHz
ψ
5
0
−5
0
1000
2000
3000
Acoustic radius t=∫₀ʳdr/c



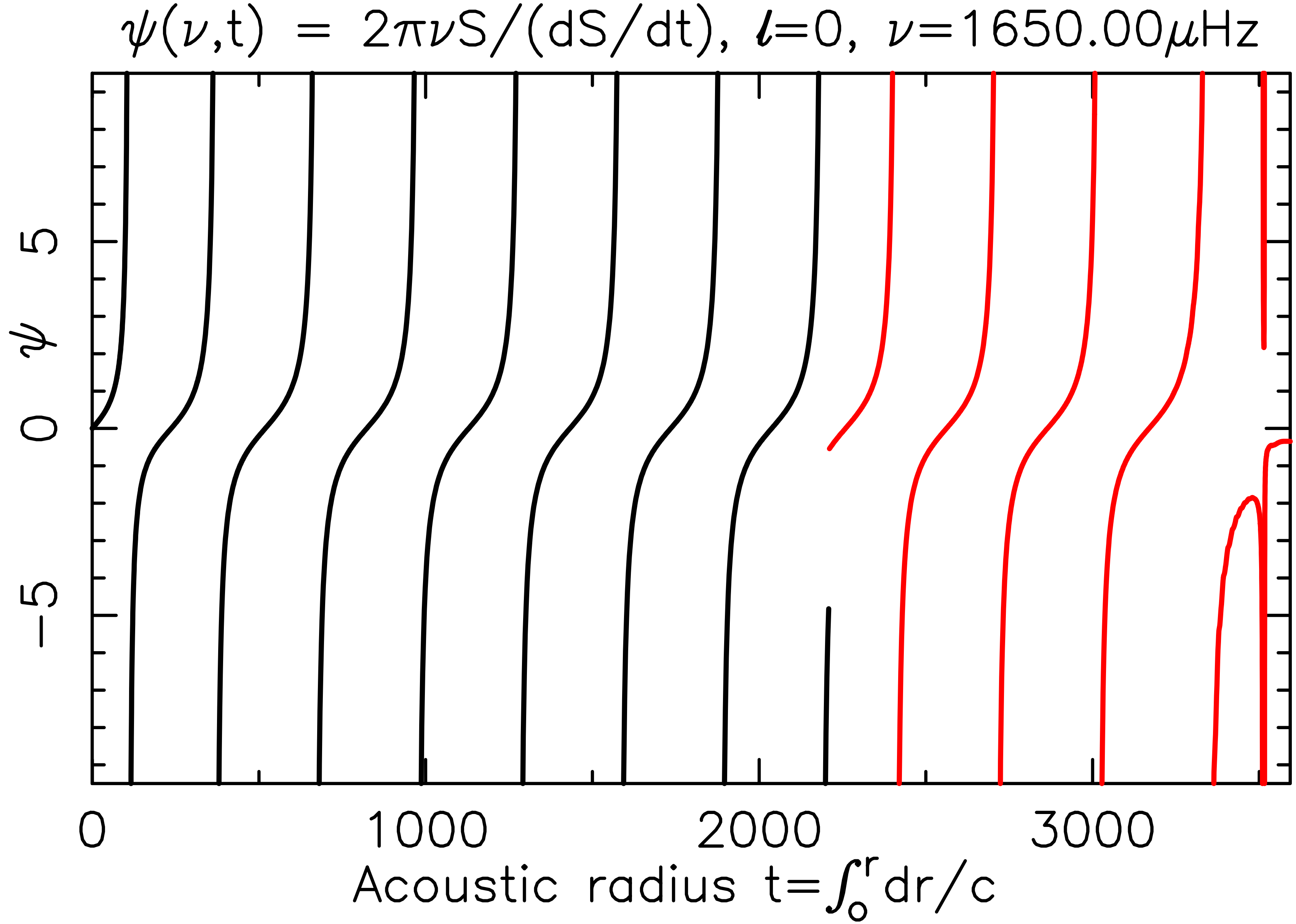
$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l=0$, $\nu=1590.00\mu Hz$
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



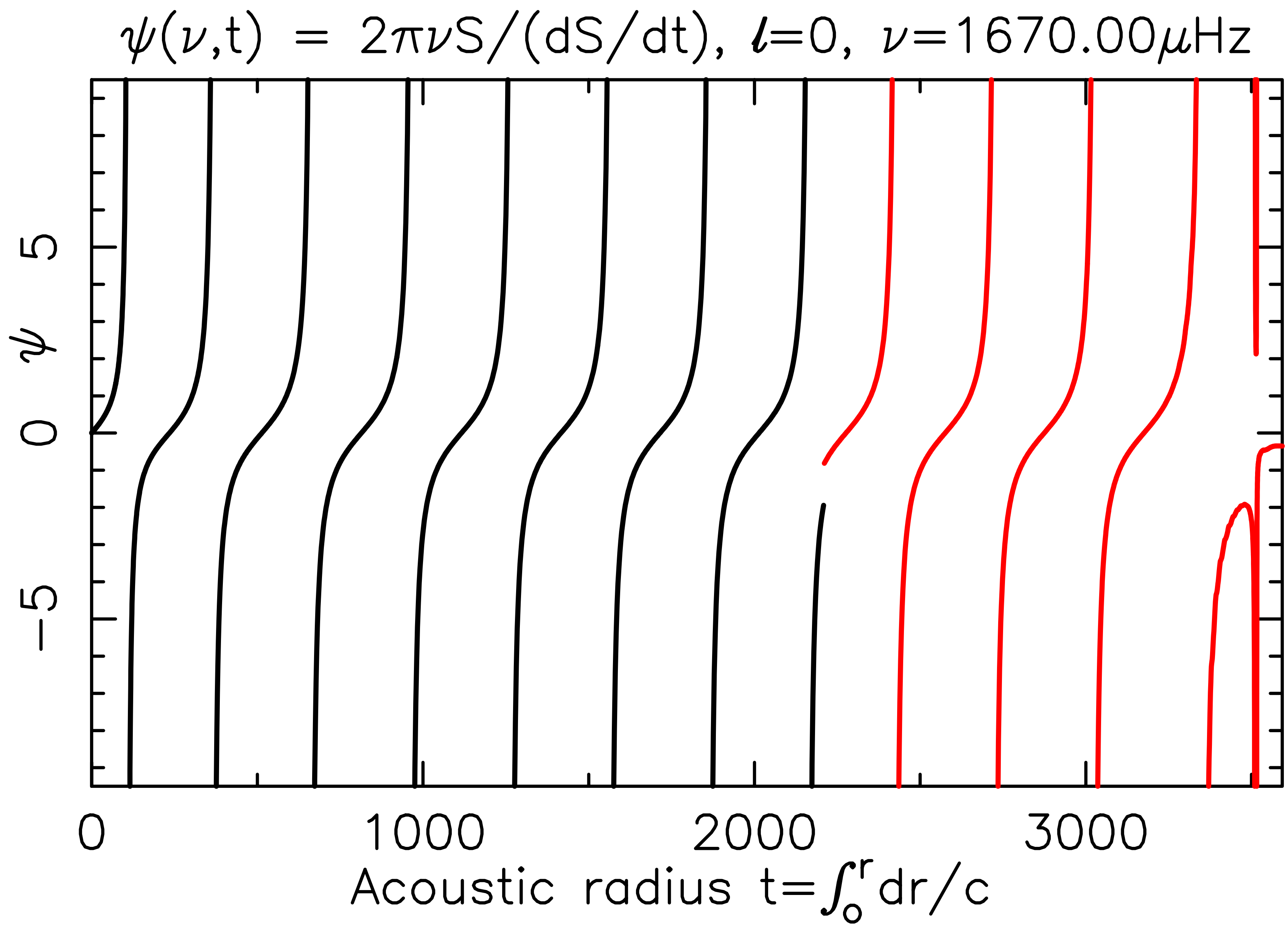
ψ(ν,t) = 2πνS/(dS/dt), l=0, ν=1610.00μHz
ψ
5
0
−5
0
1000
2000
3000
Acoustic radius t=∫₀ʳdr/c



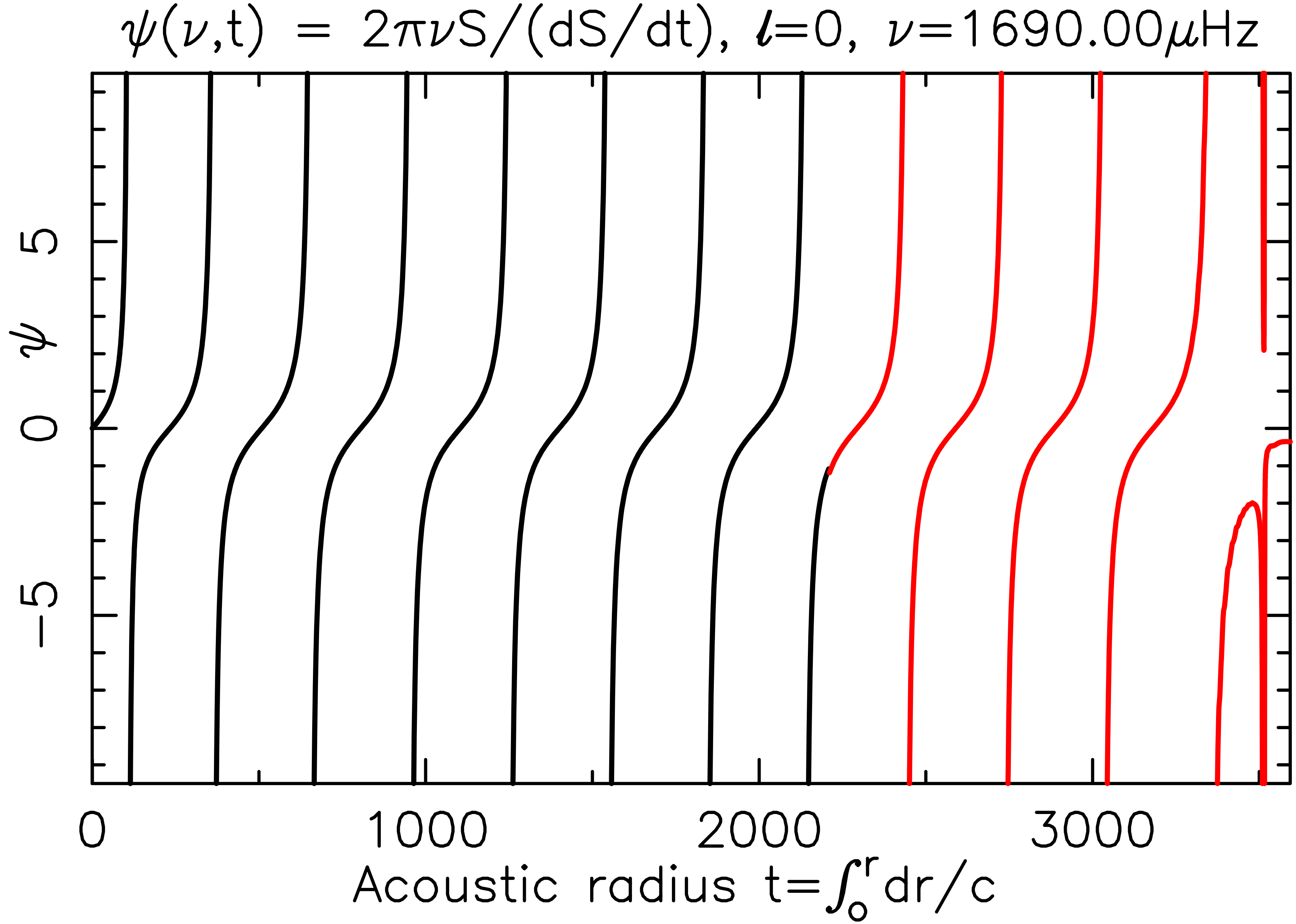
$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l=0$, $\nu=1630.00\mu Hz$
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



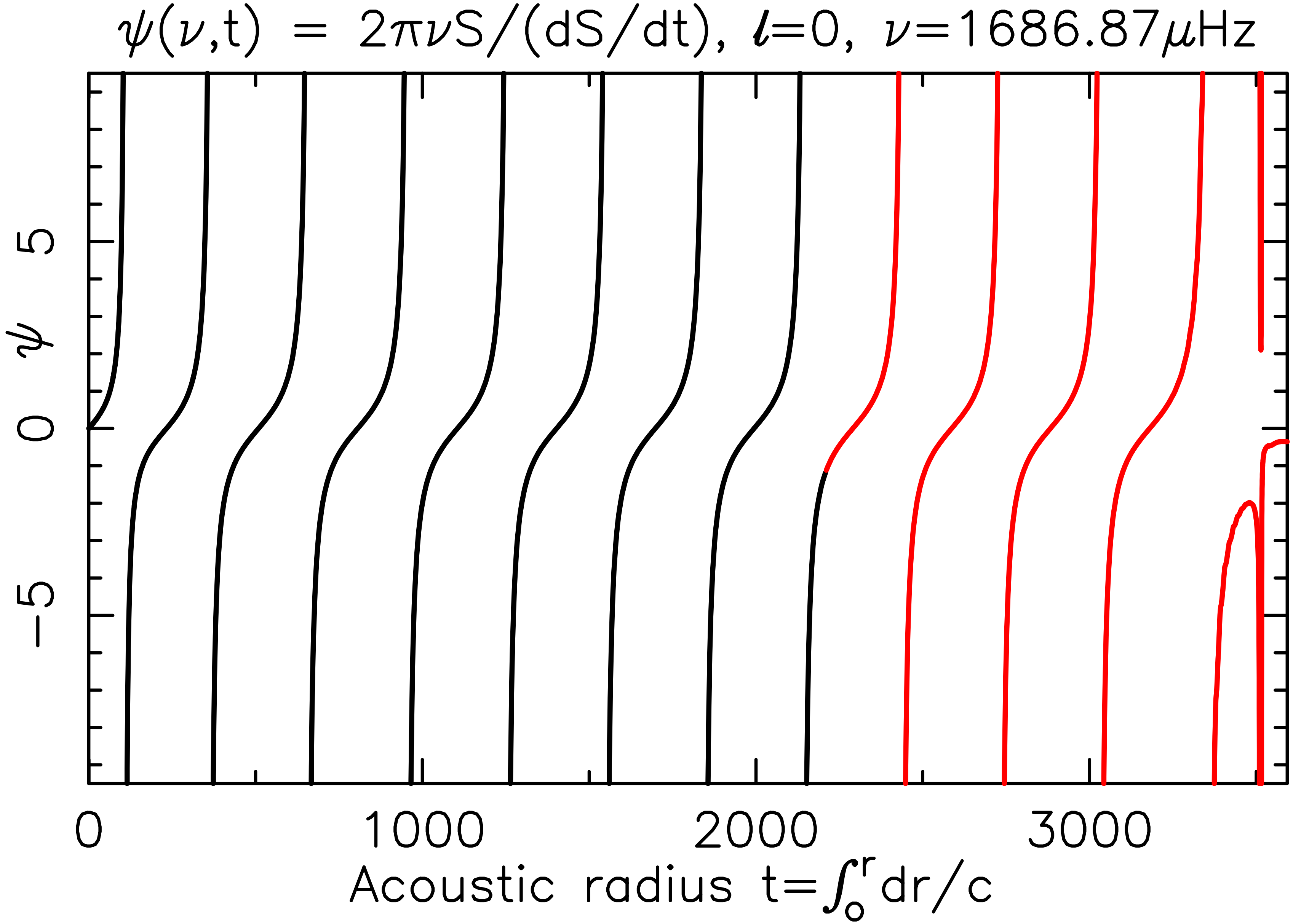
$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l=0$, $\nu=1650.00\mu Hz$
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



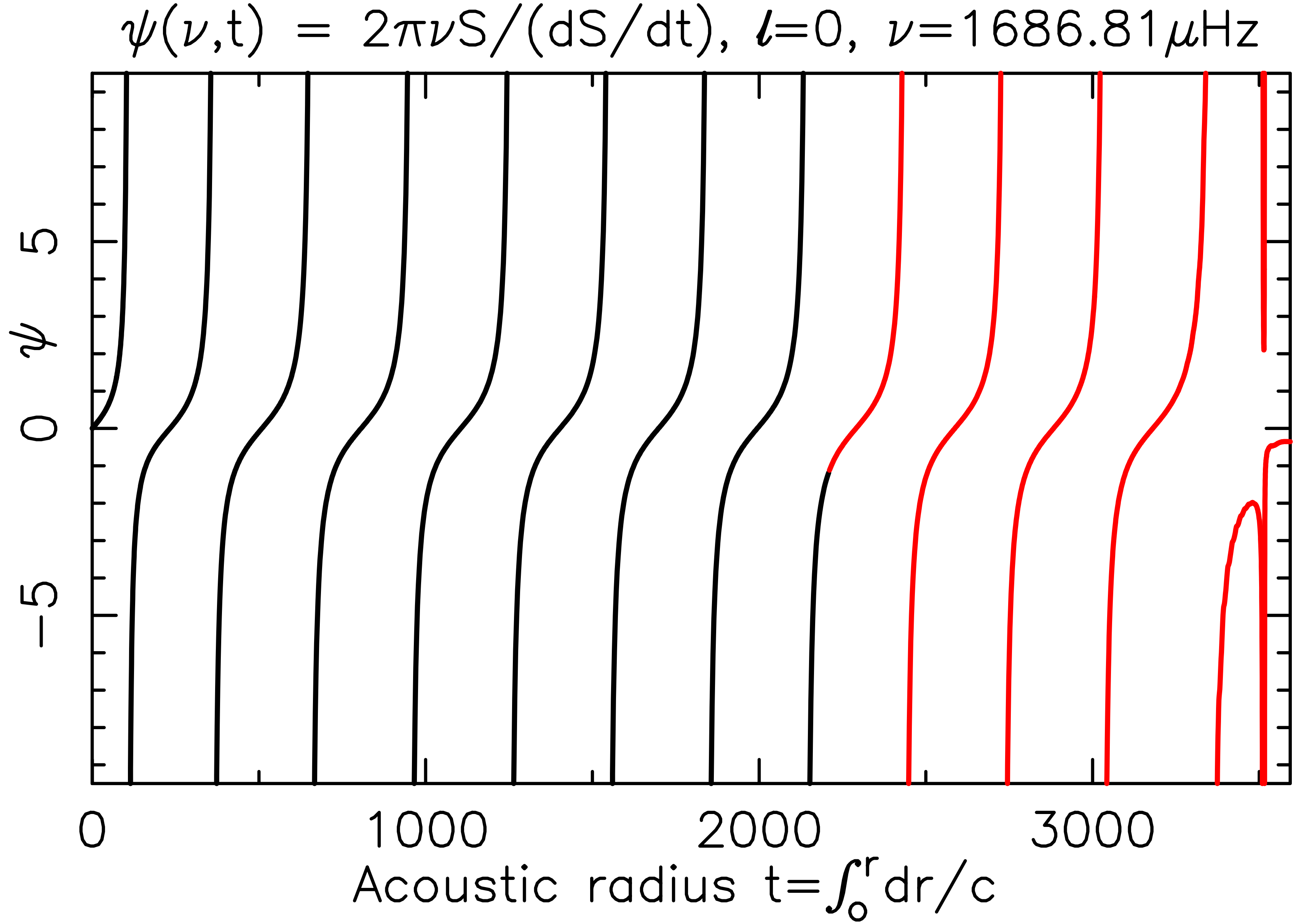
$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $\ell=0$, $\nu=1670.00\mu$Hz
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



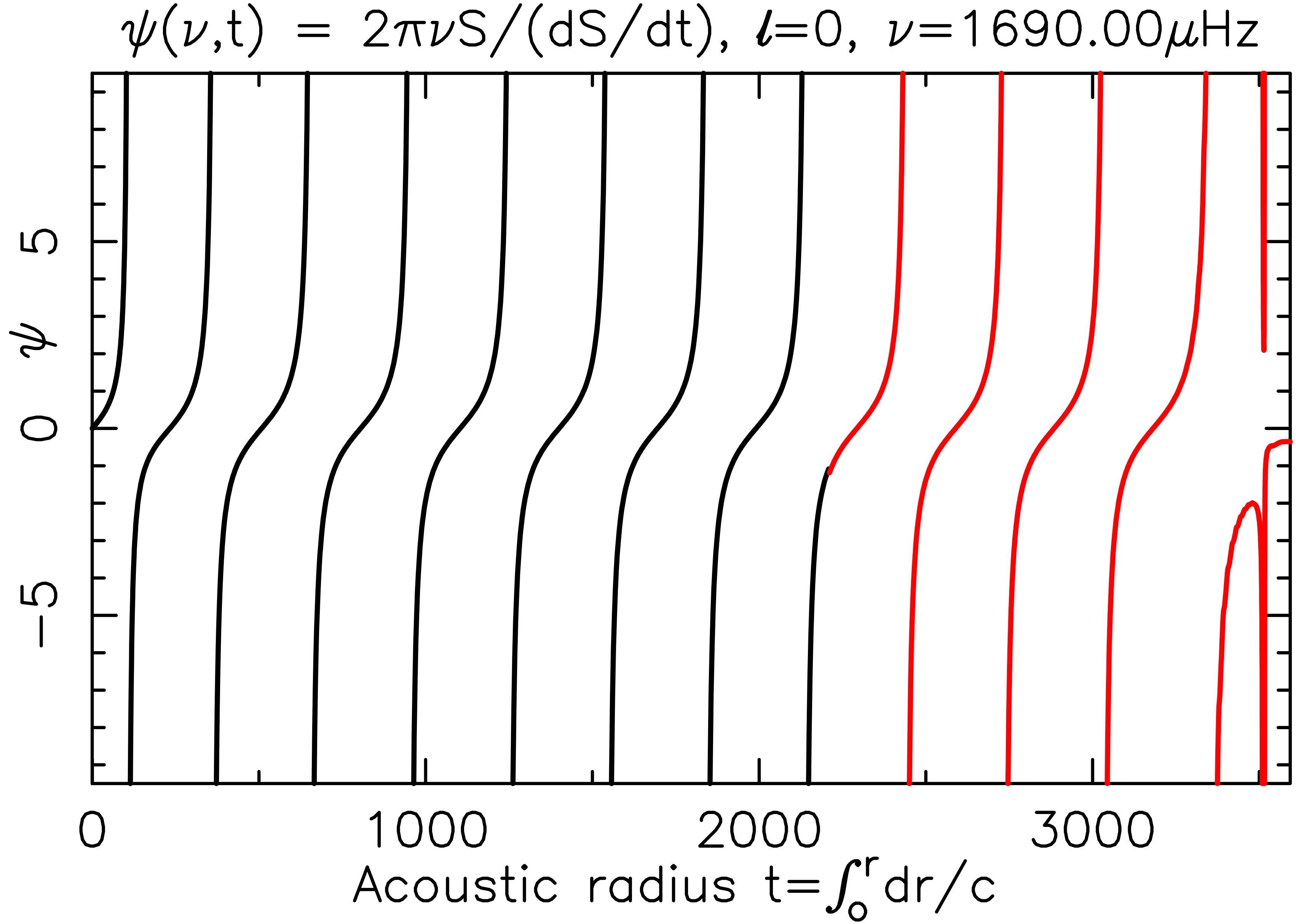
ψ(ν,t) = 2πνS/(dS/dt), l=0, ν=1690.00μHz
ψ
5
0
−5
0
1000
2000
3000
Acoustic radius t=∫₀ʳdr/c



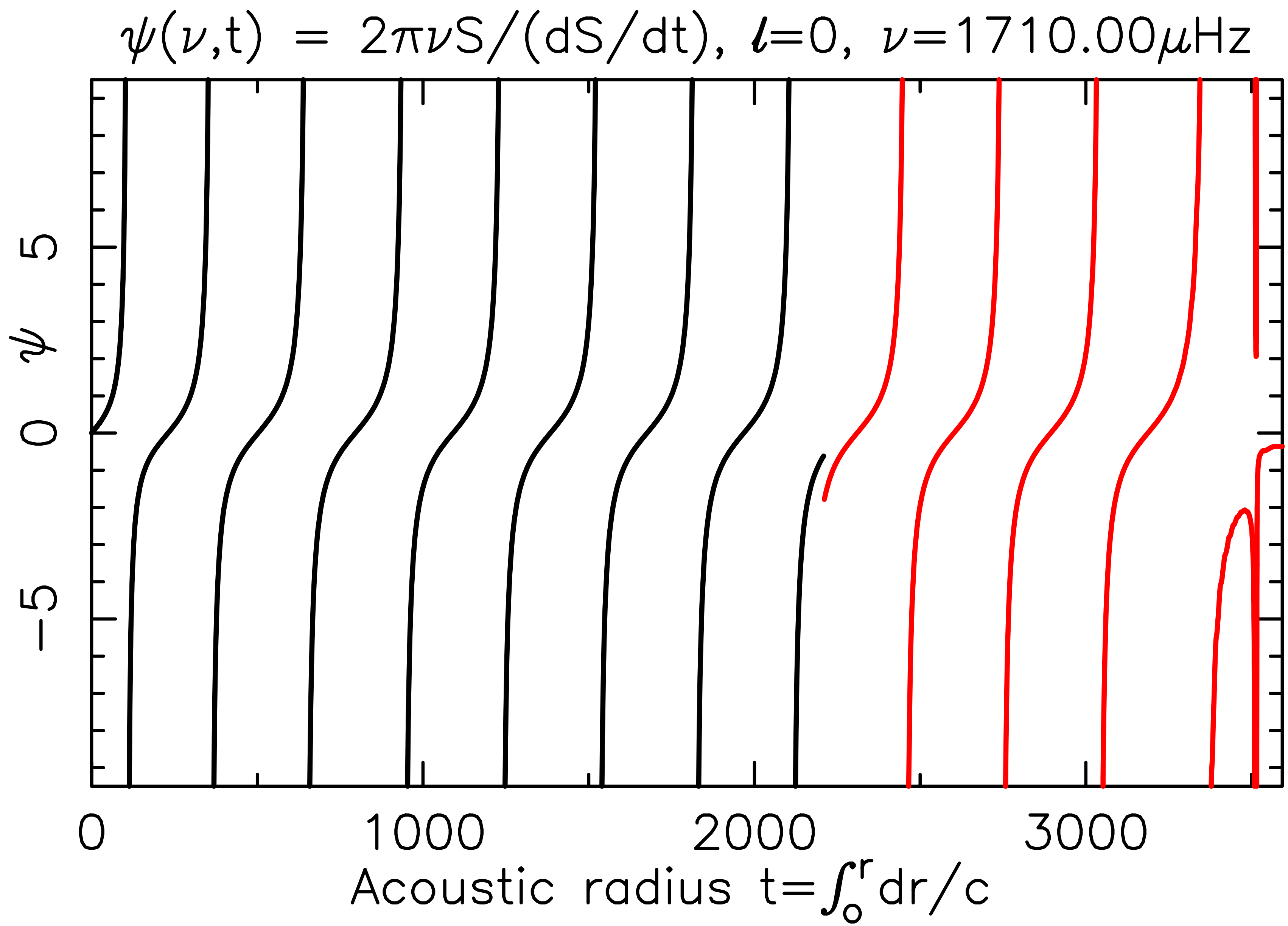
$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l=0$, $\nu=1686.87\mu$Hz
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$

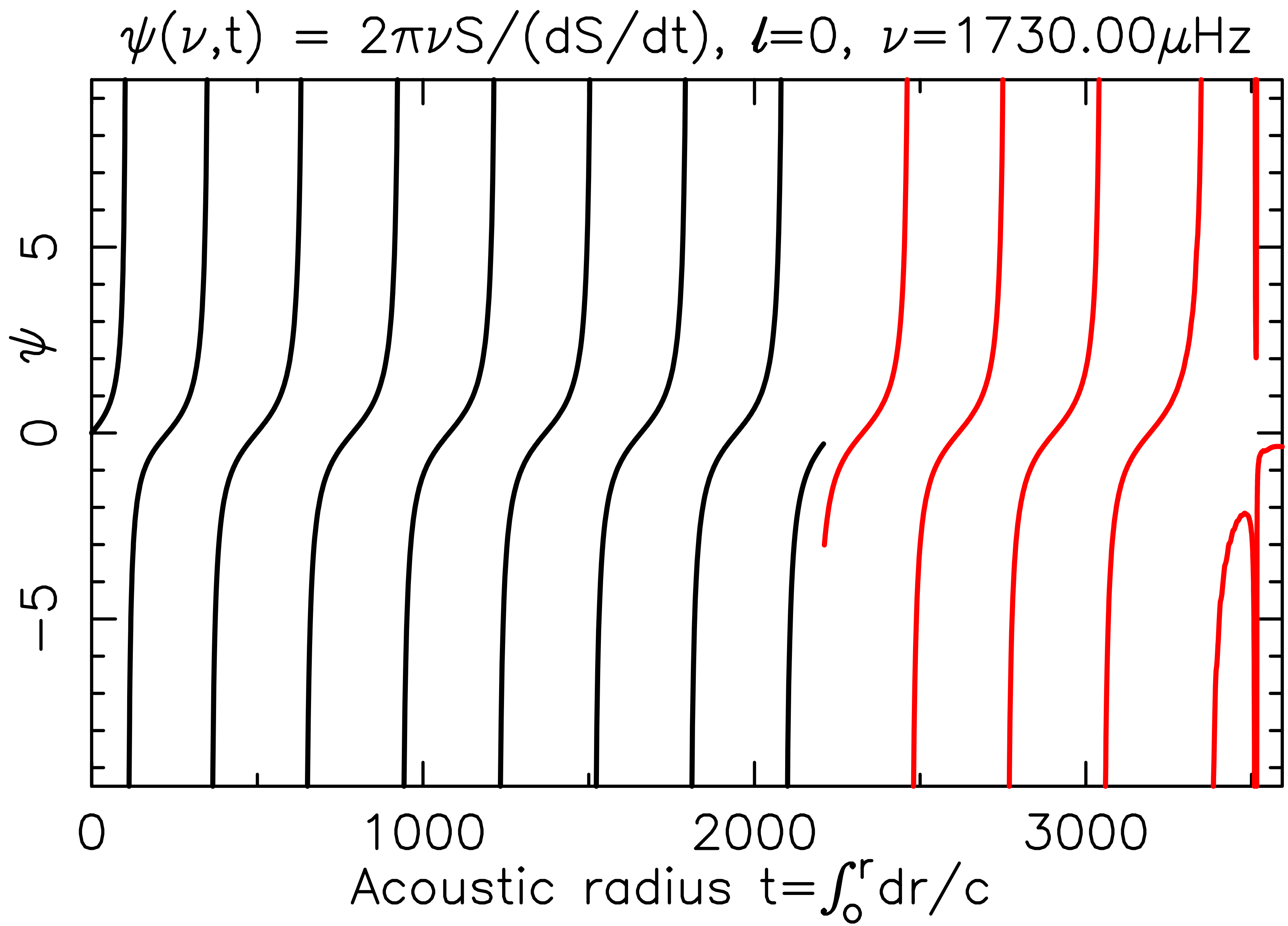


$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $\ell=0$, $\nu=1686.81\mu$Hz
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



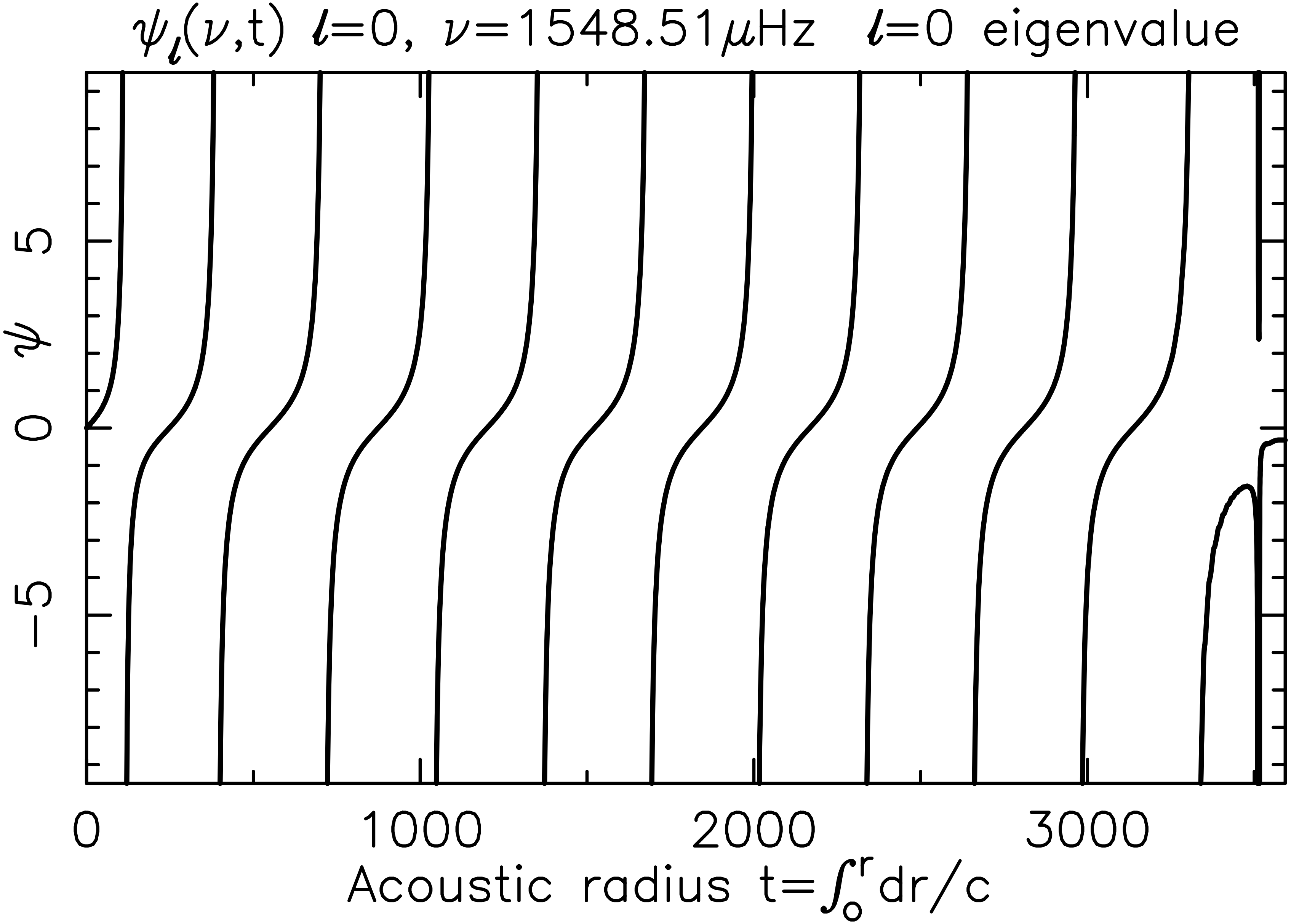
$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $l=0$, $\nu=1690.00\mu Hz$
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$


ψ(ν,t) = 2πνS/(dS/dt), l=0, ν=1710.00μHz
ψ
5
0
−5
0
1000
2000
3000
Acoustic radius t=∫₀ʳdr/c

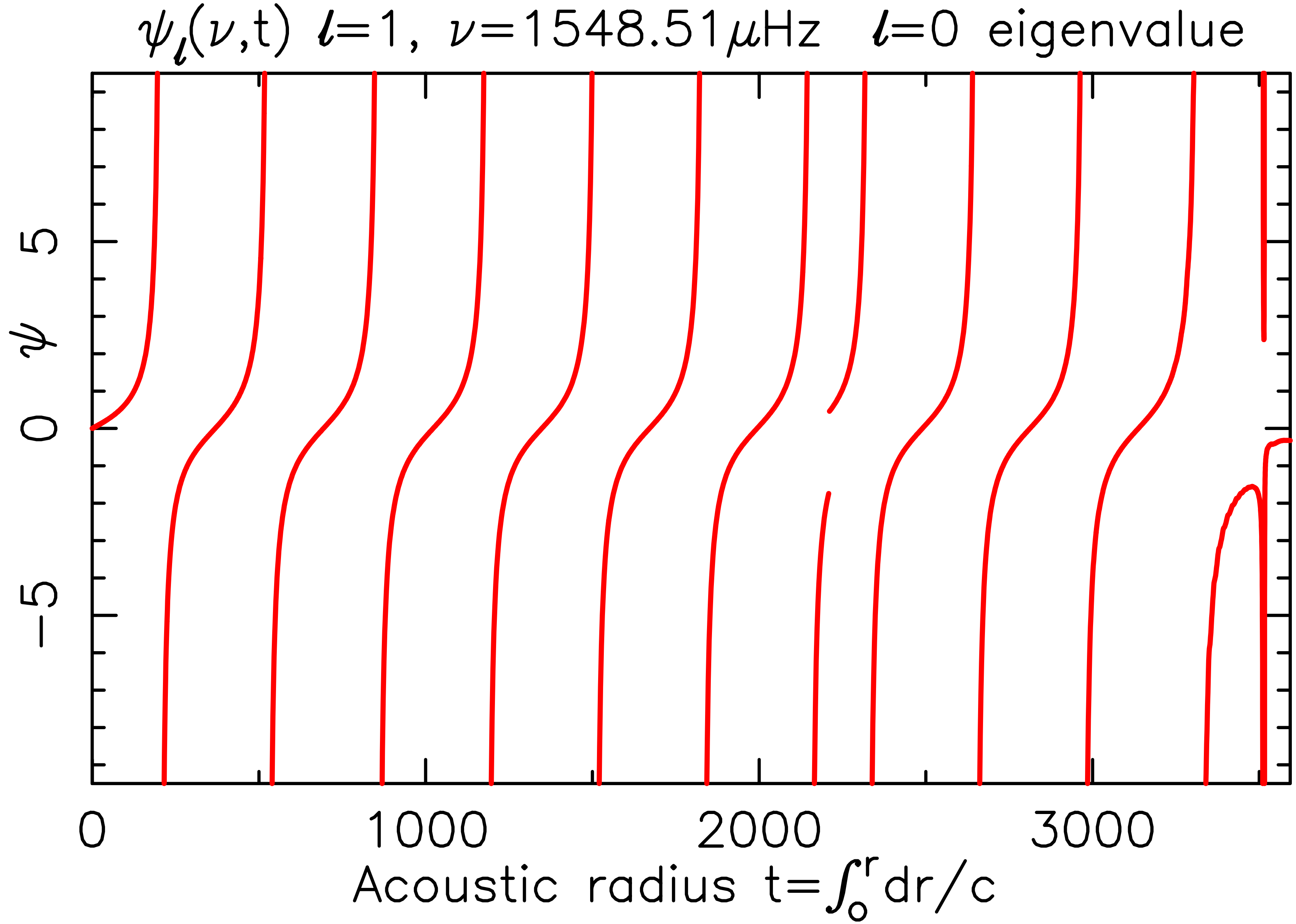


$\psi(\nu,t) = 2\pi\nu S/(dS/dt)$, $\ell=0$, $\nu=1730.00\mu Hz$
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$

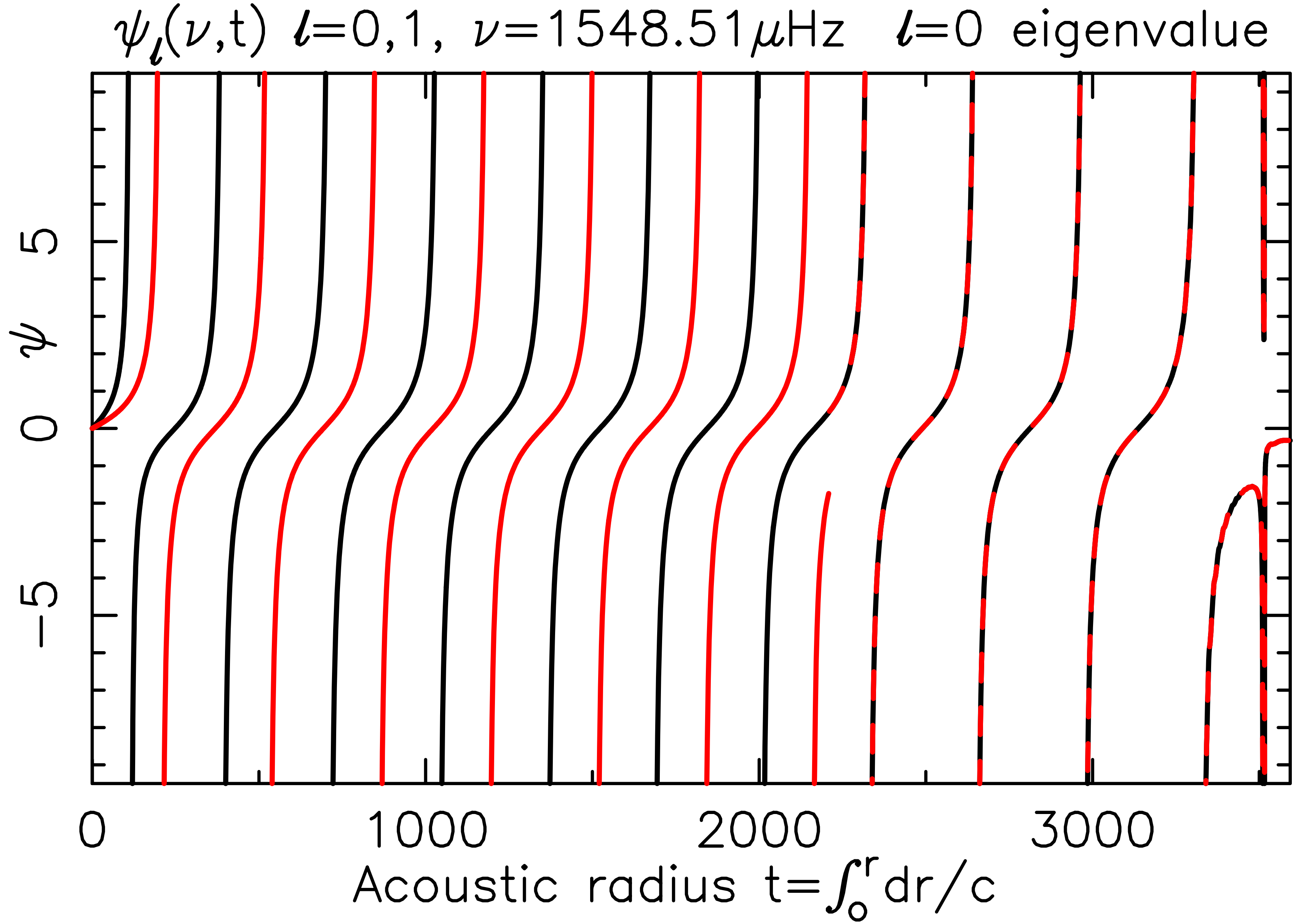
Compare solutions $\psi_\ell(\nu,t)$ for $\ell$=0,1,2,3



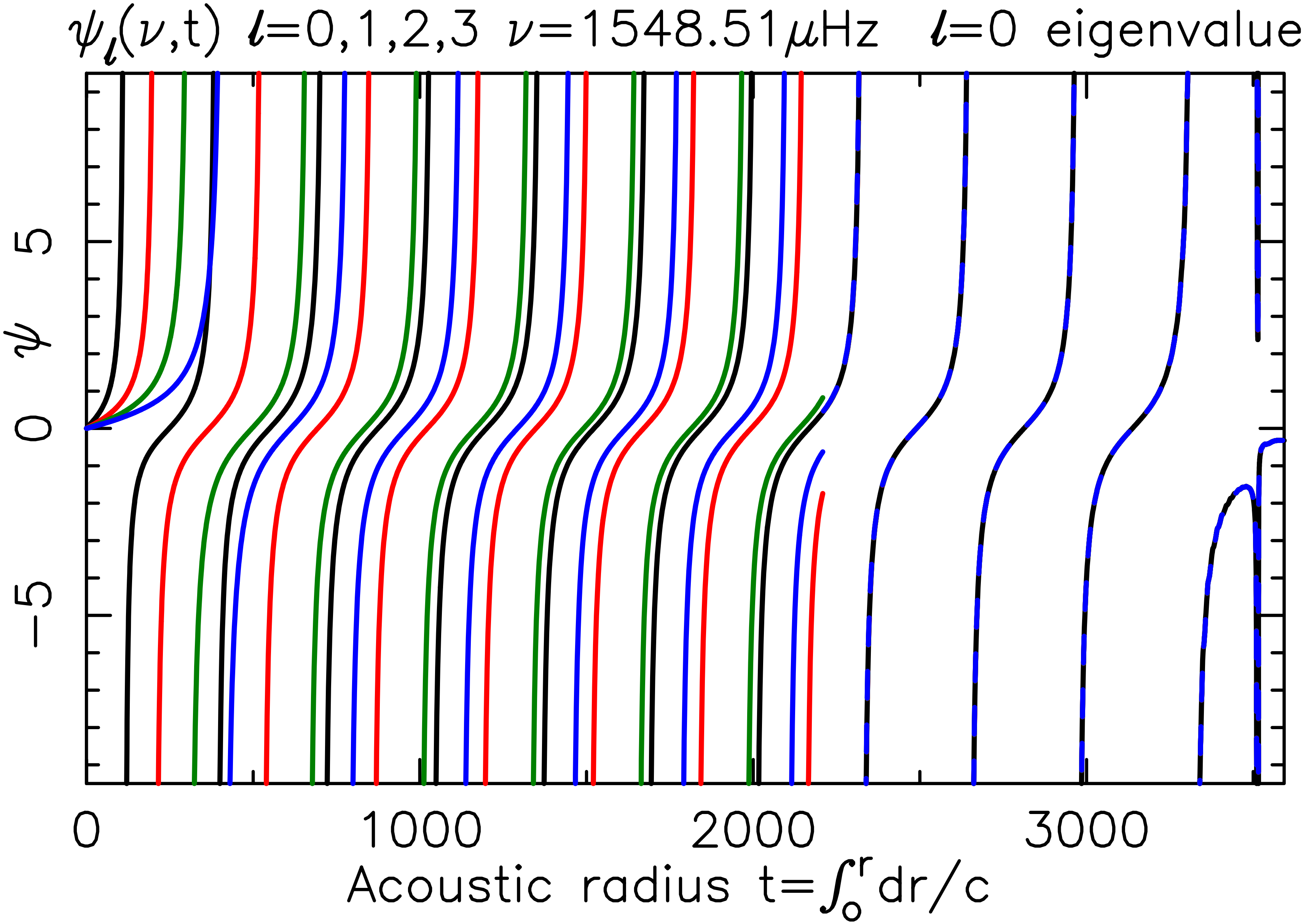
ψ_ℓ(ν,t) ℓ=0, ν=1548.51μHz ℓ=0 eigenvalue
ψ
5
0
−5
0
1000
2000
3000
Acoustic radius t=∫₀^r dr/c



$\psi_\ell(\nu,t)$ $\ell$=1, $\nu$=1548.51$\mu$Hz $\ell$=0 eigenvalue
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



$\psi_l(\nu,t)$ $l$=0,1, $\nu$=1548.51$\mu$Hz $l$=0 eigenvalue
$\psi$
5
0
−5
0
1000
2000
3000
Acoustic radius $t=\int_0^r dr/c$



ψ_l(ν,t) l=0,1,2,3 ν=1548.51μHz l=0 eigenvalue
ψ
5
0
−5
0
1000
2000
3000
Acoustic radius t=∫_o^r dr/c

# Properties of phase shifts $\alpha_\ell(\nu)$, $\delta_\ell(\nu)$, $\alpha_\ell(\nu)=\alpha(\nu)$

x=0.95

# Application to HH2a

Data provided to fitters (this is not the same as that of the input model)

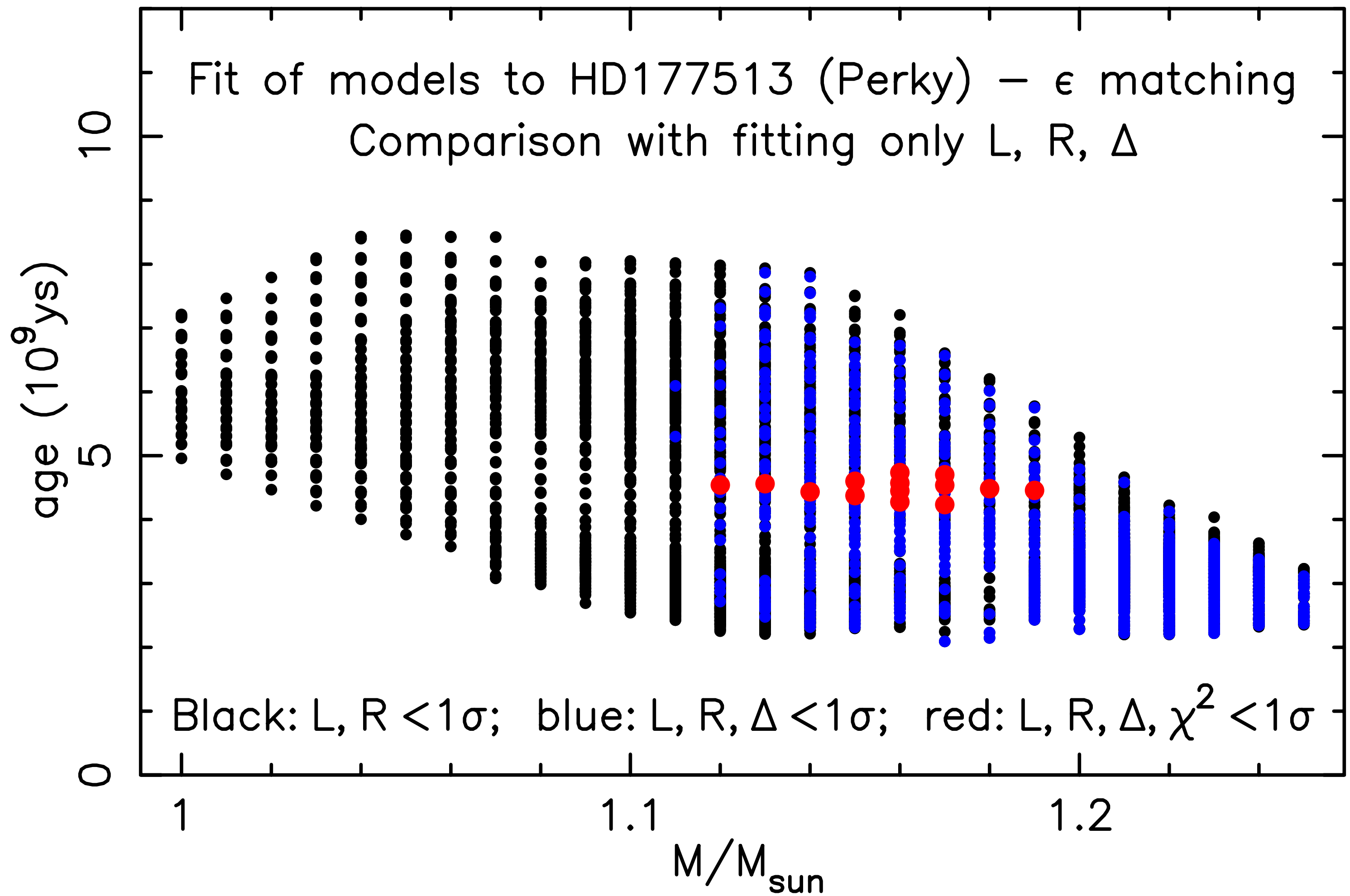
$1.941 < L/L_{\odot} < 2.228$, $5814 < T_{eff} < 5974$ °K, [Fe/H]=0.065 ± 0.051

Frequency set $\nu_{n\ell} \pm \sigma_{n\ell}$ 1578−2585 μHz

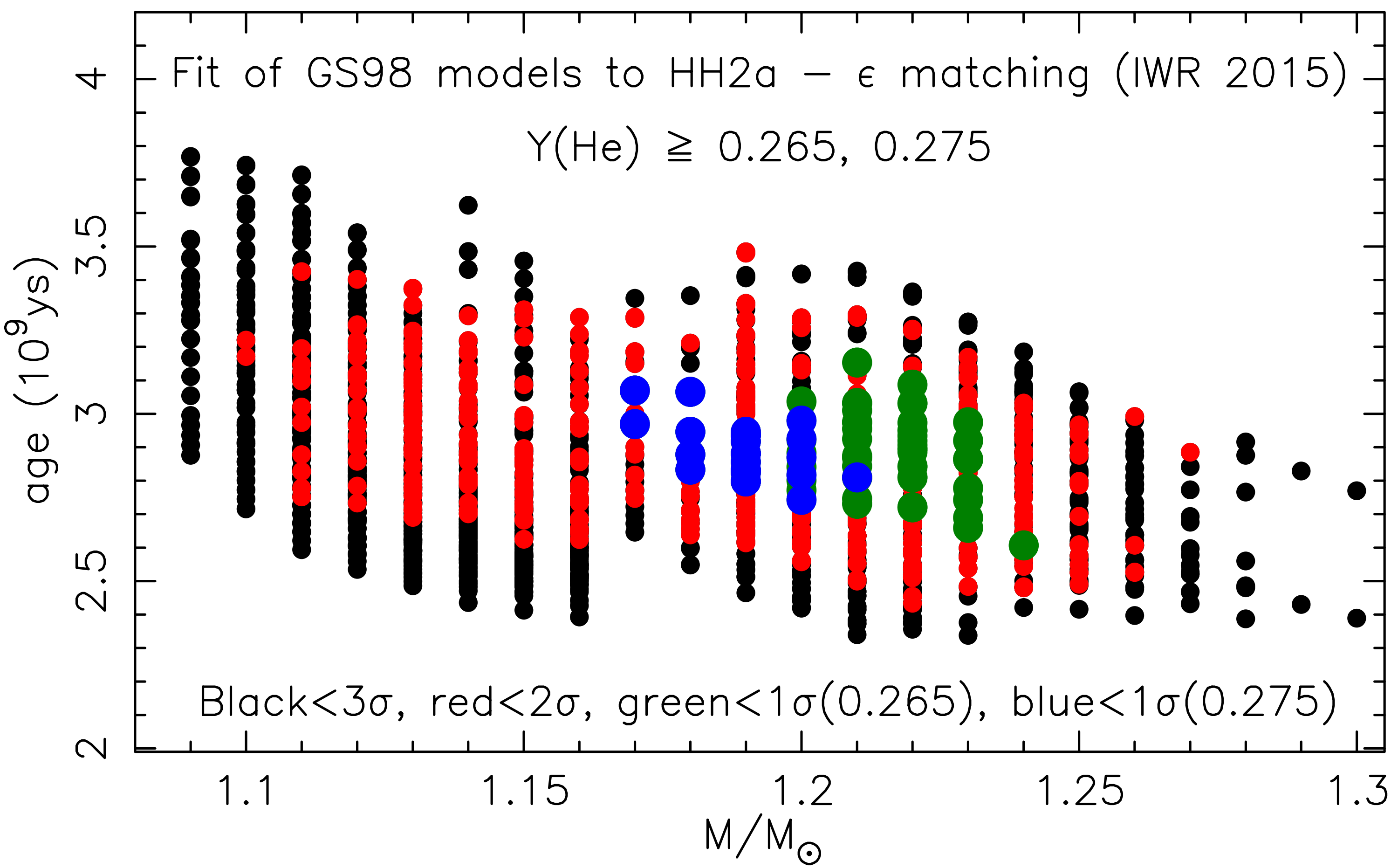
$\ell$=0, n=16−26, $\ell$=1, n=15−25, $\ell$=2, n=16−23

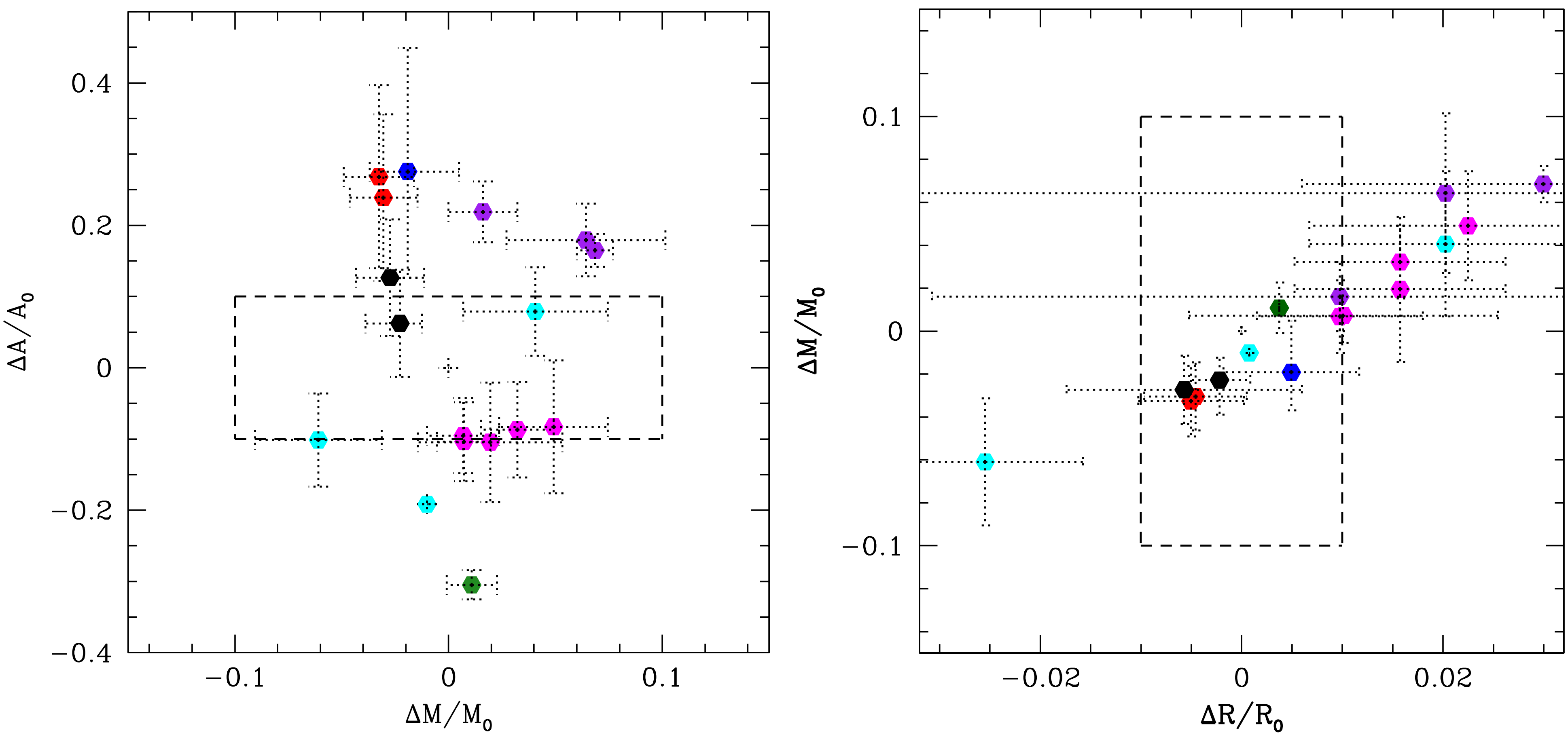
I searched through model sets with GS98 relative abundances, MLT convection, EOS5 equation of state, OPAL+ Wichita opacities, NACRE reaction rates, to find best fit models with lowest $\chi^2$ from ε matching

Since the whole point of epsilon matching is to subtract off the effect of uncertainties of the outer layers I did not impose any constraint on surface composition and gave less weight to $T_{eff}$ (and hence R), and to the large separation Δ. However L is determined by the inner structure and was the primary non asteroseismic constraint. I also imposed a constraint on the initial Helium abundance Y > 0.265



Fit of models to HD177513 (Perky) − ϵ matching
Comparison with fitting only L, R, Δ
Black: L, R <1σ; blue: L, R, Δ <1σ; red: L, R, Δ, χ2 <1σ
age (10^9 ys)
0
5
10
1
1.1
1.2
M/Msun


Fit of GS98 models to HH2a − ϵ matching (IWR 2015)
Y(He) ≧ 0.265, 0.275
Black<3σ, red<2σ, green<1σ(0.265), blue<1σ(0.275)
age (10^9 ys)
M/M⊙
4
3.5
3
2.5
2
1.1
1.15
1.2
1.25
1.3



0.4
0.2
0
−0.2
−0.4
ΔA/A0
−0.1
0
0.1
ΔM/M0
0.1
0
−0.1
ΔM/M0
−0.02
0
0.02
ΔR/R0