THE SEARCH FOR G MODES

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ABSTRACT

The Phoebus group was set up about 5 years ago with the expressed purpose to detect the g modes predicted by helioseismology to occur in the sun. The current status will be reviewed including new approaches proposed by other groups in the field. Over the years, the upper limit to g-mode amplitude set by Appourchaux et al (2000) (10 mm/s at 10 σ) has been lowered due to a longer time series as well as new detection techniques. Not withstanding these efforts the chance of a positive detection still appears remote with the current upper limit still way above that predicted by helioseismology. New techniques in particular observations involving limb intensity and/or velocity cross-correlations against various observables, or even the detection of gravitational waves may offer the best hope for a future positive detection.

Key words: intensity - p modes - SOHO - Sun.

1. INTRODUCTION

Since the beginning of helioseismology, the detection of g modes has been the most challenging quest in our field. There were several claims of g-mode detection (e.g. Delache & Scherrer 1983; Thomson et al. 1995), none of which has ever been confirmed. Since the conception of the SOHO mission, one of the goals of this mission was to detect g modes. In 1997, following the lack of g-mode detection by SOHO experimenters, the Phoebus group was formed, with the aim of detecting g modes. The group set an upper limit to the g-mode amplitude of 10 mm/s at 200 μHz (Appourchaux et al. 2000). The Phoebus group also reported on its activity at the Tenerife (Appourchaux et al. 2001) and Boston meeting (Appourchaux 1998; Appourchaux et al. 1998a; Fröhlich et al. 1998). Since the last meeting in Tenerife, there has been new developments in detection techniques, either related to the Phoebus group or a collaboration with this group.

In the first section, I review the current techniques deployed for finding the modes. I will especially outline the new techniques developed over the past 2 years. In the second section, I state the current upper limit to g-mode amplitudes. In the third section, I explore new ways of lowering the limit that may lead to a positive detection. In the conclusion, I project myself in the future.

2. G-MODE DETECTION TECHNIQUES: AN AIDE MÉMOIRE

There is now a wealth of techniques that are available for detecting g modes. Any technique aiming at detecting g modes should be able to detect also low-frequency p modes. These modes can indeed mathematically mimic the g modes provided their lifetime is longer than the observing time.

The g-mode detection techniques can be categorized as follows:

- Spectrum estimators
- Mode masking
- Statistical testing
- Patterns
- Data combination

Each of these categories can be combined for providing different methodology for detecting g modes. For instance *Statistical testing* is required on any *Spectrum Estimators* derived from any *Mode masking.* This is an example of a possible combination and other combination are always possible. Nevertheless a *must-be-included* category is certainly *Statistical testing* for it provides a safeguard against over-interpreting the data.

2.1. Spectrum estimators

The estimation of the spectrum of the time series is of prime importance when one wants to detect long

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[*] on behalf of the Phoebus group

lived oscillations like the g modes. The following estimators are at our disposal:

- Fourier spectrum (power spectra)
- Multitapered spectrum
- Random Lag Singular (Cross) Spectrum analysis
- Frequency matching (oversampling and bin shifting)
- Time-frequency spectrum
- Monte-Carlo filtering

The use of Fourier spectrum estimation is widely developed in helioseismology. It has been used by almost any scholar in the field. Its properties are well known and quite often well understood (Bracewell 2000), and so is its statistics (Davenport & Root 1958).

Fourier spectrum estimation is slowly being replaced by multitapered spectra that are widely used in geophysics (For a review, see Thomson 1982); in helioseismology the use of these slepian tapers has been replaced by the more practical (but less accurate) sine tapers (Komm et al. 1999). The statistics of the resulting spectrum is also well known (Thomson 1982). Unfortunately, for long-lived modes, they tend to broaden too much the peaks as shown by Thomson (1982). Tapers should be used for what they were designed in the first place: mean spectrum estimation; therefore they are more useful for short-lived p modes.

Random Lag Singular (Cross) Spectrum analysis is an elaborate technique based on Singular Value Decomposition (RLSSA, RLSCSA: Varadi et al. 1999, 2000, respectively). The technique claimed successful detection of low-frequency p modes (Bertello et al. 2000) but still lacks a proper assessment of its statistical property, i.e. it cannot be excluded that the technique produces large peaks solely due to noise.

Frequency matching has also recently been developed by Gabriel et al. (2002) and by Chaplin et al. (2002) using zero padding and bin shifting, respectively. When one wants to detect signals from oscillators having a lifetime longer than the observing time, there is a fair chance that the frequency bin will not match the frequency of the oscillator. As a result, the peak could be reduced by up to 60% as shown by Gabriel et al. (2002). In order to alleviate this problem one can either oversample the data by using zero padding (Gabriel et al. 2002) or try to tune for the frequency by using bin shifting (Chaplin et al. 2002). This latter technique involves creating many different time series of similar but different length, but without zero padding (Chaplin et al. 2002). In either case, the statistics of the observation is indeed affected. For oversampling, the analytical calculation is somewhat difficult. It has been replaced by Monte-Carlo simulation showing that there are typically 3 independent frequency bins when the time series is oversampled by a factor 5 (Gabriel et al. 2002). The bin-shifting method produces also 3 independent power spectra from the many spectra generated (Chaplin et al. 2002). This factor 3 is certainly not a coincidence and we await theoretical calculations to confirm the numerical observations. An other way of reducing, the frequency mismatch would be to use multitapers (Rafael García, private communication). Given, the possible phase change produced by say 5 tappers, it might be quite likely that the signal would be less missed than it is with Fourier spectra, even at the expense of broadening the peak.

Time-frequency spectrum or wavelet analysis has been used by Gabriel et al. (1998) and by Finsterle & Fröhlich (2001) for looking for the lifetime of candidate g modes. Its statistical properties can be derived from those of the Fourier spectrum.

Last but not least, a new technique for spectrum estimation has been presented during the conference (Grec and Renaud, these proceedings). This is a technique that uses randomly generated window function for mode detection. The generated power spectrum has a well known statistics. This promising technique has been applied to the GOLF time series but should be tried to other time series such as those of GONG.

2.2. Mode masking

Mode masking is required by instruments making images of the Sun. There are several types of masks:

- spherical harmonics masks
- g-mode masks
- optimal masks

Spherical harmonics masks are widely used in helioseismology. They correspond to the displacement or temperature perturbations (velocity or intensity observations). These masks have been used for detecting g modes as in Appourchaux et al. (2000); in which their visibilities are given.

Unfortunately, these masks are rather well adapted for most p modes but not for g modes. For velocity observations, the horizontal displacement must be taken into account. For intensity observations, the oscillations perturbs the surface of the Sun and the light it emits (Berthomieu & Provost 1990; Toutain et al. 1999). For either observations, the additional contribution is a non-spherical harmonics function (Unno et al. 1989; Appourchaux & Andersen 1990). In either case, specific g-mode masks can be devised (for intensity observations see Appourchaux & Andersen (1990)).

Masks that are optimal for specific purpose has also been developed. For instance, Appourchaux & Andersen (1990) developed masks that minimize both the leakage from other degree and from other m. Similar masks have been derived by Appourchaux et al. (1998b) and by Toutain & Kosovichev (2000). Recently, Wachter et al. (2002) has produced masks optimizing (i.e. minimizing) the noise contribution from the supergranulation noise varying across the solar disks, producing lower detection limits than those of Appourchaux et al. (2000). There are other interesting masking techniques that could make use of the properties of supergranulation. The solar rotation causes the supergranulation to enter and leave the observing window. A way to reduce the resulting noise would be to have a window following the rotation in a region close to the centre of the solar disk where the supergranulation noise is weaker than at the limb.

2.3. Statistical testing

Statistical testing is related to hypothesis testing. This branch of mathematics require proper expertise that the present author may lack. In short, we have basically tried two type of hypothesis:

- H0 hypothesis: what is the probability of detecting pure noise?
- H1 hypothesis: what is the probability of detecting a signal?

This is a simplification of what is performed, and I would like to apologize to the experts for doing that. The use of hypothesis is related to a decision process (Papoulis 1991); should we accept or reject the hypothesis? It should not be used for implying that say g modes are being detected: i.e. reject H0.

The H0 hypothesis was used by Appourchaux et al. (2000) for providing upper limit to g-mode amplitudes. The statistical method is based on the knowledge of the statistical distribution of the power spectra of full-disk integrated instruments; namely this is a χ^2 distribution with 2 degrees of freedom (d.o.f). Appourchaux et al. (2000) provide a simple formula for the relative level $\sigma_{\rm det}$ for which a peak due to noise has a 10% probability to appear in a 70-μHz bandwidth. This relative level will depend upon the observing time (T) because the number of frequency bins in the bandwidth increases with time. And we have:

$$\sigma_{\rm det} = 10 + \log(T_{\rm y}) \qquad (1)$$

where $T_{\rm y}$ is the observing time in years. A similar calculation leading to Eq. (1) can be carried out using spectra obtained by making an $m - \nu$ averaged spectra: the so-called *collapsogramme* (Appourchaux et al. 2000). The advantage of the collapsogramme is that it enhances mode multiplets while reducing at the same time artifacts due to instrumental effects (Appourchaux et al. 2000). For these 2 cases, the H0 hypothesis can only be used if the statistical properties of the spectrum estimator is known. That is why I emphasized in the corresponding section whether the statistics of the estimator is defined. For instance, the RLSSA and RLSCSA of Varadi is an interesting technique lacking a proper knowledge of its statistical properties. This renders the low-frequency p-mode detection of Bertello et al. (2000) of limited value. I must outline that similar results could be obtained by say raising a power spectrum to some unknown and random power; large peaks due to noise might be enhanced by this procedure but given the random nature of the exponent their significance would be doubtful. That is the reason why for any spectrum estimator a proper derivation of the statistics should be performed either through analytical calculation or Monte-Carlo simulations as in Gabriel et al. (2002) or in Chaplin et al. (2002).

The H1 hypothesis has also been used in Gabriel et al. (2002) for setting a probability of detecting a sine wave given the noise in the GOLF data (Gabriel et al. 2002). In this latter case, the statistics is a non-centered χ^2 with 2 degrees of freedom.

Composite hypothesis testing is also of interest but is currently not being developed in our field yet.

2.4. Patterns

The asymptotic behaviour of g-mode frequencies (or periods) was used for easing detection. This was pioneered by Delache in 1983 leading to a claimed detection of g modes (Delache & Scherrer 1983); leading other observers to try the method on their own data (Memorie della Società Astronomica Italiana, vol 55 and references therein). This approach is only of relevance to high order g modes (or very low frequency below 100 μHz) for which the asymptotic behaviour applies. Unfortunately, the solar noise increases towards lower frequencies, and the mode spacing dramatically decreases; in addition the situation is even complicated by the introduction of rotational splitting (Fröhlich & Andersen 1995). The mode degeneracy being lifted by the internal rotation of the Sun, it also contributes to the overall pattern.

The collapsogramme technique pioneered by Appourchaux et al. (2000) makes use of the pattern created by the rotational splitting for detecting the modes. It can also be used for full-disk instruments producing a single power spectrum: in this case it is called an *overlapogramme*. Chaplin et al. (2002) has devised a statistical technique based on the detection of an ordered multiplet (due to rotational splitting) in power spectrum of full-disk integrated data. It has allowed to lower the detection level depending on the number of peaks searched for. The limit (under the H0 hypothesis) they derived can be translated into the equivalent sigma level in a power spectrum for a one-year observing time: 5.9σ, 4.5σ, 3.8σ for a doublet, triplet and a quadruplet, respectively. A similar approach has been used by García et al. (2001) for the GOLF data. They derived levels for detecting modes of various degree that can be translated for 1-year time series to 5.4σ for an $l = 1$ doublet, 5.9σ for an $l = 2$ doublet and 4.3σ for an $l = 2$ triplet.

An artificial way of reducing the detection limit is to reduce the window over which we want to detect the mode, e.g., by looking in a window centred around theoretical g-mode frequencies. Denison & Walden (1999) provided a simple formula to derive the number of peaks that one can find in a power spectra given a list of given frequencies and a window around these frequencies. Under the H0, hypothesis, it is written as follows:

$$N = N_{\rm l}(1 - (1 - p_{\rm det})^{2N_{\rm w}+1}) \qquad (2)$$

where $N_{\rm l}$ is the number of frequencies guiding the search, $p_{\rm det}$ is the probability level needed for identifying a peak and $N_{\rm w}$ is half the window size in units of bins. When $(2N_{\rm w} + 1)p_{\rm det}$ is much smaller than 1, we can rewrite Eq (2) as:

$$N = N_{\rm l}(2N_{\rm w} + 1)p_{\rm det} \qquad (3)$$

This simple formula is quite useful to realize that the number of identified peaks will increase with the size of the window. This is the drawback of such a method: spurious peaks will be detected in this manner that are likely to be wrongly identified as g modes. Here we should remind the reader, that theoretical p mode frequencies had been in error of a few tens of μHz until it was realized that the error came from our inability to model properly the surface of the Sun (Christensen-Dalsgaard 1990). Therefore care should be exerted when using theoretical frequencies as a guideline for searching for g modes.

2.5. Data combination

The use of different data sets related to different observables and/or wavelengths could very well be the solution to the g-mode detection. There is no doubt that the combination of more than 2 signals could considerably lower our detection limit. Observables such as radial velocity, intensity fluctuation, limb displacement and/or brightening are polluted by different source of noise such as supergranulation and active regions that produce different signatures. We can list the possible combinations in 2 categories:

- use of one instrument
- use of more than one instrument

Autocorrelation falls into the first category. The collapsogramme technique used by Appourchaux et al. (2000) belongs to the first category. An interesting technique developed by García et al. (1999) use a longer sampling time (twice as long) to create two independent time series of the GOLF data.

The Multivariate Spectral Regression Analysis (MSRA) belongs to the second category (Koopmans 1974; Appourchaux et al. 2000). It assumes that the modes are predominantly coherent over the observing time. The basic assumption is that low-order, low-degree p modes and g modes will have lifetimes that are significantly longer than the observing time. The MSRA has been used on VIRGO by Finsterle & Fröhlich (2001) allowing to claim detection of low-frequency low-degree p modes.

The RLSCSA described above is also a technique making use of different data sets for mode detection.

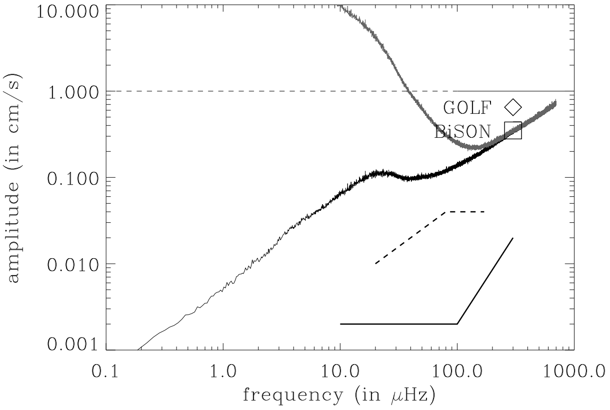

Figure 1. Current upper limit for GOLF (diamond), BiSON (square) and MDI (black and grey curve) derived under the 10-% probability limit as in Appourchaux et al. (2000). The GOLF limit is obtained by Gabriel et al. (2002) using nearly 6 years of data. This latter limit is close to that derived by Wachter et al. (2002) for sectoral modes and for 2 years of MDI data. The BiSON limit is derived for a quadruplet from Chaplin et al. (2002) using 9 years of data. The MDI limit is derived from Appourchaux et al. (2001); Kuhn et al. (1997) for 1 year of data; the black curve corresponds to the radial displacement and the grey curve to the total displacement (radial and horizontal. The black thick continuous and dashed curves are the simplified version of the theoretical g-mode amplitude for l=1 predicted by Kumar et al. (1996) and Gough (1985), respectively.

3. ON LOWERING THE LIMIT: THE PRESENT

At the time of writing, the canonical figure of 10 mm/s given by the Phoebus group in Appourchaux et al. (2000) has already been lowered by time and by newer analysis technique. Figure 1 shows the current upper limit obtained by various instrument. The limit is now closer to 3.5 mm/s for BiSON due to the use of a nine-year time series and of the multiplet ordered technique described above. For GOLF the limit is about 6.5 mm/s due to a data set about 6 years long; similar to actual limit of the Phoebus group.

As mentioned earlier by Appourchaux (1998) the limit does not decrease like $T^{-\frac{1}{2}}$ but like $T^{-\frac{1}{2}}\log(T)$. That is a slower decrease that for instance shows that the limit of 10 mm/s would be 4.6 mm/s for a 100-year time series instead of the 1 mm/s commonly thought. This is due to the probability limit which needs to be kept constant (Appourchaux 1998). If we were to do otherwise it would mean that we would lower our probability level and accept more peaks that would likely be due to noise. Therefore, the decrease is bound to be not as fasr as expected. So it is quite clear that a large amount of cleverness will be required to detect the g modes. The hope for detecting the g modes in our lifetime remains quite weak. That is why we need to explore other ways and means of lowering the limit.

4. ON LOWERING THE LIMIT: THE FUTURE

There are other possibilities for detecting the g modes. We know that we have still a long way to go (at least a factor 20). Hereafter I summarize the possible orientation that our search will take:

- New observable: Limb measurement
- New instrumentation for measuring solar radial velocities
- New techniques: general-relativity effects

Hereafter, we describe each direction in more detail.

4.1. Limb measurement

The limb provides two types of observable: a physical displacement of the surface of the Sun, and a relative intensity fluctuation. Both types of measurement have been used by Kuhn et al. (1997) for detecting solar modes using MDI limb data. Unfortunately, if we were to detect the g modes with the limb displacement only, we would not be able to go to frequencies lower than 100 μHz because the g modes becomes more and more horizontal, e.g., there is no limb displacement detectable (See Fig. 1). Using intensity fluctuations, Appourchaux & Toutain (1998) and Toner et al. (1999) showed that p modes could be detected in the guiding signals of VIRGO/LOI instrument and of the MDI limb data, respectively. This observed amplification at the limb, predicted by Toutain et al. (1999) will be used for detecting g modes with the PICARD instrument (Thuillier & Meissonnier 2002). PICARD is an approved CNES mission to be launched in 2006. An amplification factor of 3 to 5 in amplitude might not be enough to detect the g modes, but we hope that by combining the PICARD data with other observables (intensity, velocity) we may be able to detect the g modes. The predicted upper limit for PICARD is 1 mm/s in 2 years (Damé et al. 1999). We may hope that a similar limit be reached by the recently approved Helioseismic and Magnetic Imager (HMI) of NASA's Solar Dynamic Observatory (SDO) (for more information on HMI look at hmi.stanford.edu).

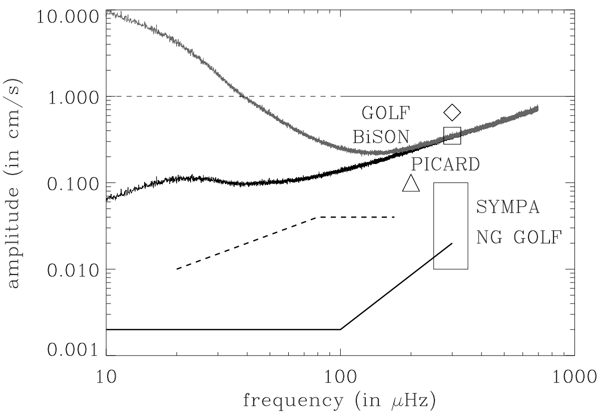

Figure 2. Predicted upper limit for a one-year observing time for various instruments planned or to be flown: SYMPA and NGGOLF (Open box), and PICARD (triangle); and current upper limit for GOLF (diamond), BiSON (square) and MDI (black and grey curve) derived under the 10-% probability limit as in Appourchaux et al. (2000). The GOLF limit is obtained by Gabriel et al. (2002) using nearly 6 years of data. The BiSON limit is derived for a quadruplet from Chaplin et al. (2002) using 9 years of data. The MDI limit is derived from Appourchaux et al. (2001); Kuhn et al. (1997) for 1 year of data; the black curve corresponds to the radial displacement and the grey curve to the total displacement (radial and horizontal. The black thick continuous and dashed curves are the simplified version of the theoretical g-mode amplitude for l=1 predicted by Kumar et al. (1996) and Gough (1985), respectively.

4.2. New instrumentation for measuring solar radial velocities

There are now projects being developed for making better measurements of the solar radial velocities. The instrument called MR15 or NGGOLF proposed to derive solar radial velocities from 15 measurements in the profile of the Sodium line (Turck-Chièze et al. 2001). Although no mention of the capability of the instrument is described, it is hope that the improvement will range between 0.1 and 1 mm/s, with a goal of going down to 0.1 mm/s (Turck-Chièze et al. 2001) provided that the solar noise can be beaten. An instrument of a very novel design is being developed by the Université de Nice (Jacob 2002, and their web site www-astro.unice.fr). It is based on a Mach-Zender interferometer making use of the whole spectrum for deriving radial velocities for planets, stars or the Sun. It has great potential for space use given its compact nature, its lack of moving parts and the heritage of MDI[1] regarding solid Michelson interferometers. The performance of the instrument

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[1] Michelson Doppler Imager aboard SOHO

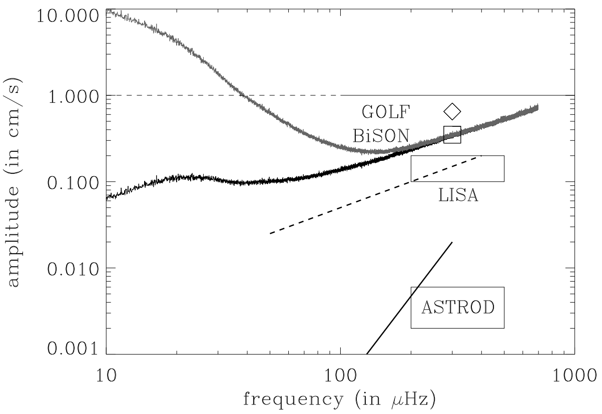

Figure 3. Predicted upper limit for a one-year observing time for various general-relativity space missions planned or to be flown: LISA (Upper open box, after Giamperi et al. (2000)) and ASTROD (Lower open box, after Cruise et al. (2000)); and current upper limit for GOLF (diamond), BiSON (square) and MDI (black and grey curve) derived under the 10-% probability limit as in Appourchaux et al. (2000). The GOLF limit is obtained by Gabriel et al. (2002) using nearly 6 years of data. The BiSON limit is derived for a quadruplet from Chaplin et al. (2002) using 9 years of data. The MDI limit is derived from Appourchaux et al. (2001); Kuhn et al. (1997) for 1 year of data; the black curve corresponds to the radial displacement and the grey curve to the total displacement (radial and horizontal. The black thick continuous and dashed curves are the simplified version of the theoretical g-mode amplitude for l=2 predicted by Kumar et al. (1996) and Gough (1985), respectively.

are expected to be in the same range as of NGGOLF (Fi-X Schmider, private communication; See Fig 2).

4.3. On the use of gravitation

The perturbation of the gravitational fields by the g modes produces tidal perturbations of the Newtonian fields, and generates detectable gravitational waves (Giamperi et al. 2000). The perturbation is non-zero only for modes with $l \geq 2$; for $l = 1$ it is exactly zero (Giamperi et al. 2000). The ESA/NASA LISA[2] mission aims at detecting such gravitational waves. The LISA detection limit for g-mode amplitude and for a one year observing time can be translated to a few mm/s (Culter & Lindblom 1996; Giamperi et al. 2000). This is lower than the traditional helioseismic techniques but not low enough for clear detection. The mission proposed to ESA as F2/F3 mission called ASTROD[3] is believed to provide much lower g-mode detection levels (Cruise et al. 2000). This mission is based on laser ranging of 2 spacecrafts located on the far side of the Sun on the Earth orbit, borrowing some of the technologies being developed for LISA. The one-year detection limit they provided makes the mission extremely appealing to helioseismologists; the limit is below that of the theoretical amplitude predicted by Kumar et al. (1996). Figure 3 summarizes the limits of LISA of ASTROD compared with the theoretical amplitudes for $l = 2$. It must be pointed out that Christensen-Dalsgaard (2002) has independently made a similar analysis leading to conclusions of similar tone; for once it seems that the present paper seems to be slightly more optimistic than that of Christensen-Dalsgaard (2002). Unfortunately, we agreed that the rest of the universe[4] could be the limiting factor for g-mode detection and not the Sun itself.

5. CONCLUSION

There are now numerous techniques that have been developed for g-mode detection, allowing to lower somewhat the upper limit given by Appourchaux et al. (2000). The upper limit to g-mode amplitudes has now been lowered to about 3 mm/s at 200 μHz. Unfortunately, I believe that the techniques currently available will not provide the minimal factor 10 needed for reaching the theoretical amplitudes of Kumar et al. (1996) or of Gough (1985). It is timely to start planning for other space missions that could help us to reach for the goal. This could be achieved with the PICARD mission or even with HMI of SDO that are bound to be launched in 2007. In the more distant future, the LISA mission is going to be launched in 2012 (or so), it will not only provide g-mode detection possibilities but also enabling technologies for a future mission based on spacecraft laser ranging such as ASTROD.

Finally, I truly hope that I will not have to repeat these statements for the next 18 years[5], and that the detection of g modes would have been achieved in this time frame.

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[2] Laser Interferometry Space Antenna

[3] Astrodynamical Space Test of Relativity using Optical Devices

[4] Christensen-Dalsgaard (2002) explicitly mentions the galactic binary stars

[5] Until my retirement...

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