

G-MODE DETECTION: WHERE DO WE STAND?

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ABSTRACT

We review the recent developments in determining the upper limits to g-mode amplitudes obtained by SOHO instruments, GONG and BiSON. We address how this limit can be improved by way of new helioseismic instruments and/or new collaborations, hopefully providing in the not too distant future unambiguous g-mode detection.

Key words: intensity - p modes - SOHO - Sun.

1. INTRODUCTION

Since the beginning of helioseismology, much of the internal structure and dynamics of the Sun has been revealed. The last island where our powerful tools start to fail is the solar core, where nuclear reactions take place. The detection of g modes is still the main quest in our field. There have been several claims of g-mode detection (e.g. Delache & Scherrer, 1983; Thomson et al., 1995). None of these detections has ever been confirmed. Since the conception of the SOHO mission, one of the goals of this mission was to detect g modes. In 1997, following the lack of g-mode detection by SOHO experimenters, the Phoebus group was formed, with the aim of detecting g modes. The Phoebus group already reported on its activity at the Boston meeting (Appourchaux, 1998; Appourchaux et al., 1998; Fröhlich et al., 1998).

In the first section, we report on the latest results of the Phoebus group related to the upper limit to the amplitude of g modes. In the second section, we develop new ideas for lowering the limit that we set.

2. UPPER LIMIT TO G-MODE AMPLITUDES

Several techniques for detecting g modes were described by Appourchaux (1998). Two of these techniques have been used on SOI/MDI, GONG, BiSON and VIRGO data for detecting g modes: a statistical method and the collapsogramme ($m - \nu$ averaged spectra). These two techniques are described in detail by Appourchaux et al. (2000). Here we have just extracted the main points of this latter paper.

The statistical method is based on the knowledge of the statistical distribution of the power spectra of full-disk integrated instruments; namely this is a χ^2 distribution with 2 degrees of freedom. Appourchaux et al. (2000) provide a simple formula for the relative level σ_{det} for which a peak due to noise has a 10% probability to appear in a $70\text{-}\mu\text{Hz}$ bandwidth. This relative level will depend upon the observing time (T) because the number of frequency bins in the bandwidth increases with time. And we have:

$$\sigma_{\text{det}} = 10 + \log(T_y) \quad (1)$$

where T_y is the observing time in years. This limit is sometimes incorrectly called the $10\text{-}\sigma$ limit; as one can notice this limit will increase with time. A similar calculation to that leading to Eq. (1) can be carried out using spectra obtained by making an $m - \nu$ averaged spectra: the so-called *collapsogramme* (Appourchaux et al., 2000). The advantage of the collapsogramme is that it enhances mode multiplets while reducing at the same time artifacts due to instrumental effects (Appourchaux et al., 2000). Any technique aiming at detecting g modes should be able to detect also low-frequency p modes. Since the low-frequency p modes have long lifetimes, they can mathematically mimic g modes. Fröhlich et al. (1998) showed that these two techniques could help to detect these

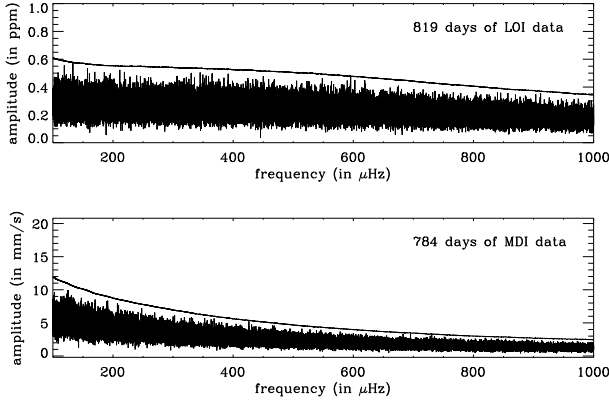


Figure 1. Collapsogramme for $l = 2$ for the LOI (top), MDI (bottom) resolved data corrected for the spatial filter function. The 10%-probability level is indicated; the σ estimate is derived from the spectra smoothed on $62 \mu\text{Hz}$.

low-frequency p modes.

We have used these two techniques on BiSON, GONG, SOI/MDI and VIRGO data. In order to compare the different instruments, we took into account the different temporal filters, duty cycles and geometrical visibilities. Figure 1 shows a collapsogramme with its associated 10%-probability level. From this level, we deduced an upper limit to the amplitude of g modes of 10 mm s^{-1} and 0.5 ppm at $200 \mu\text{Hz}$ (Appourchaux et al., 2000). These levels are to be compared with the theoretical predictions of Andersen (1996) and Kumar et al. (1996), which are no greater than 0.5 mm s^{-1} . It is quite clear that other strategies need to be explored in order to have a successful g-mode detection.

3. ON LOWERING THE LIMIT: THE PRESENT AND THE FUTURE

There are other possibilities for detecting the g modes. We know that we have still a long way to go (at least a factor 20). Hereafter we summarize the possible orientation that our search will take:

- Other observables: limb measurement
- New (or newer) data analysis techniques
- Longer and additional time series

Hereafter, we describe each direction in more detail.

3.1. Limb measurement

The limb provides two types of observable: a physical displacement of the surface of the Sun, and a relative intensity fluctuation.

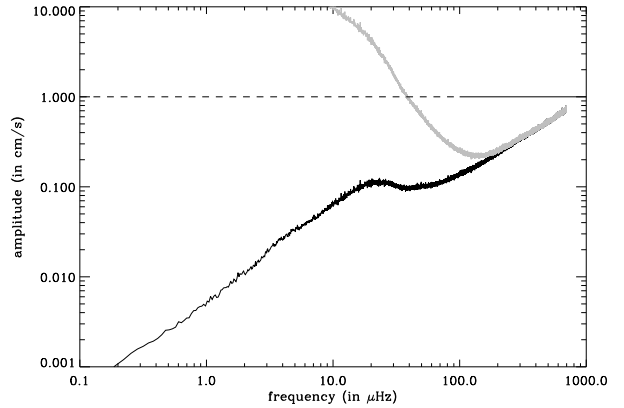


Figure 2. 10% probability level of the equivalent velocity amplitude derived from limb displacement measured by SOI/MDI as a function frequency: (black) for radial velocity (grey) for total velocity. The continuous line indicates the upper limit to g-mode amplitude as given by Appourchaux et al. (2000); the dashed line below $100 \mu\text{Hz}$ indicates the extrapolation of this limit. As the g modes become more and more horizontal the equivalent total velocity increases more and more towards lower frequencies (grey curve).

Both types of measurement have been used by Kuhn et al. (1997) for detecting solar modes. His work has been the basis of a SMEX proposal to NASA called SPHERIS which has unfortunately not been approved (for more on SPHERIS look at www.ifa.hawaii.edu/users/kuhn/spheris). The Phoebus group using SOI/MDI data has been able to reproduce Kuhn's finding related to the limb displacement. For instance, Fig. 2 gives the velocity equivalent to displacement measured on the limb of the Sun; a similar plot can be found in the SPHERIS proposal. Unfortunately, if we were to detect the g modes with the limb displacement only, we would not be able to go to frequencies lower than $100 \mu\text{Hz}$ because the g modes becomes more and more horizontal, e.g., there is no limb displacement detectable (See Fig. 2).

Using intensity fluctuations, Appourchaux & Toutain (1998) showed that p modes could be detected in the guiding signals of the VIRGO/LOI instrument (See Fig. 3). Thanks to their large amplitude at the limb, the modes could be detected even with noise levels as large as $10 \text{ ppm}^2/\mu\text{Hz}$. The full-disc p-mode amplitudes are typically of the order of $3 \text{ ppm}^2/\mu\text{Hz}$ at $3000 \mu\text{Hz}$ which are to be compared with their amplitudes at the limb ranging from $15 \text{ ppm}^2/\mu\text{Hz}$ to $50 \text{ ppm}^2/\mu\text{Hz}$. This amplification at the limb, predicted by Toutain et al. (1999), has also been observed by Toner et al. (1999) with SOI/MDI data. This is the amplification which will be used for detecting g modes with the PICARD instrument (Damé et al., 1999). PICARD is an approved CNES mission to be launched in 2002. An amplification factor of 3 to 5 in amplitude might not be enough to detect the g modes, but

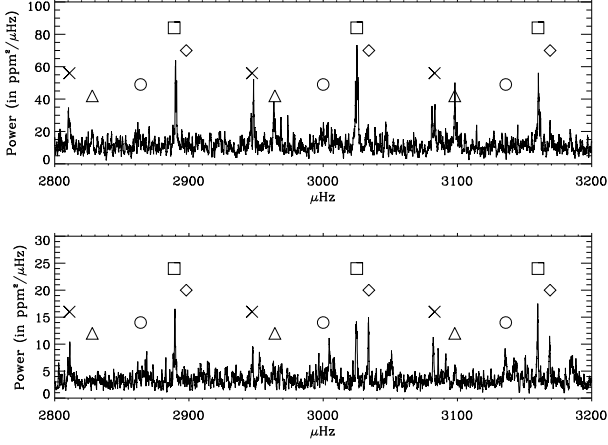


Figure 3. Power spectrum for the East-West (top) and North-South (bottom) guiding pixels; \diamond $l = 0$, \triangle $l = 1$, \square $l = 2$, \times $l = 3$, \circ $l = 4$. The frequency resolution is 54 nHz. The spectra are smoothed to 10 points to enhance the p modes. The maximum p mode amplitude is 8 ppm. The $l > 0$ modes in the East-West pixels have a power about five times higher than those in the North-South pixels. This difference cannot be attributed to a scaling error because the $l = 0$ modes have the same power above the noise levels.

we hope that by combining the PICARD data with other observables (intensity, velocity) we may be able to detect the g modes.

3.2. New (or newer) data analysis techniques

Numerous techniques for extracting weak signals from noise have been developed either by the Phoebus group or by other colleagues in the field. Here is a non-exhaustive list:

- Multitaper spectra
- Random Lag Singular (Cross-) Spectrum Analysis (RLSSA and RLSCSA)
- Multivariate analysis
- Use of theoretical g-mode frequencies
- Reduction of noise

The multitaper spectra technique is a technique used in geophysics for estimating the mean spectrum from a single time series (Percival & Walden, 1993). It makes use of different tapers that make the resulting time series statistically independent. The spectra are later optimally combined to produce a spectrum smoother than the original one. This technique is the starting point for the claim of g-mode detection made by Thomson et al. (1995). In order to assess

the power of this technique, the Phoebus group initiated a hare-and-hound exercise with real SOI/MDI data in which fake g modes were embedded. The results are so far negative, and only an *a posteriori* detection has been possible (Thomson, private communication). The signal-to-noise ratio of the fake modes was about 1 which shows the limit of the method.

The RLSSA technique has been developed by Varadi et al. (1999). It is derived from techniques used in geophysics. It seems to be a powerful technique as shown by Varadi et al. (1999) but it seems that its main drawback lies in the considerable amplification of noise which can be mistaken for signals (Varadi et al., 1999). A modification to the RLSSA technique (RLSCSA) has been also developed by Varadi et al. (2000); it uses data from different instruments to detect the solar modes. Using the latter technique, Bertello et al. (2000) claimed to have found low-degree low-frequency p modes, at frequencies as low as 535 μ Hz.

Assuming that the modes are predominantly coherent over the observing time, we have used the Multivariate Spectral Regression Analysis (MSRA) to search for coherent structures in the signal (Koopmans, 1974; Appourchaux et al., 2000). The basic assumption is that low-order, low-degree p modes and g modes will have lifetimes that are significantly longer than the observing time. The VIRGO SPM blue channel and the MDI full-disc signal were used. From each of the time strings, three strings consisting of consecutive thirds of the data were generated (about 1200 days in total). In order to maintain the phase between the different strings, we set the unwanted values in the original time string to zero. With the MSRA algorithms the coherent signal of these three parts of the full time string was calculated. This was carried out for the MDI and SPM data independently. Thereafter the correlated part of the SPM and MDI signals was calculated. Figures 4 and 5 show the results in two regions where low-degree low-order p modes are expected. For these regions there are seemingly interesting signals close to the expected frequencies. However, if we look at the frequency interval from 500-1000 μ Hz, we find as many significant peaks far away from theoretical frequencies as close to them.

An artificial way of reducing the detection limit is to reduce the window over which we want to detect the mode, e.g., by looking in a window centred around theoretical g-mode frequencies. Denison & Walden (1999) provided a simple formula to derive the number of peaks that one can find in a power spectrum given a list of given frequencies and a window around these frequencies. It is written as follows:

$$N = N_1(1 - (1 - p_{\text{det}})^{2N_w + 1}) \quad (2)$$

where N_1 is the number of frequencies guiding the search, p_{det} is the probability level needed for identifying a peak and N_w is half the window size in units of bins. When $(2N_w + 1)p_{\text{det}}$ is much smaller than 1,

we can rewrite Eq (2) as:

$$N = N_1(2N_w + 1)p_{\text{det}} \quad (3)$$

This simple formula is quite useful to realize that the number of identified peaks will increase with the size of the window. This is the drawback of such a method: spurious peaks will be detected in this manner that are likely to be wrongly identified as g modes. Here we should remind the reader, that theoretical p mode frequencies had been in error of a few tens of μHz until it was realized that the error came from our inability to model properly the surface of the Sun (Christensen-Dalsgaard, 1990). Caution and scepticism is the name of the game.

There are other interesting techniques which we will not discuss here such as the one developed by García et al. (1999). It makes use of a higher sampling time for creating 2 (or more) independent time series. There are other interesting schemes, such as the one making use of the properties of supergranulation. The solar rotation causes the supergranulation to enter and leave the observing window. A way to reduce the resulting noise would be to have a window following the rotation in a region close to the centre of the solar disk where the supergranulation noise is weaker than at the limb.

Two techniques that we described (RLSCSA and MSRA) make extensive use of different signals for extracting weak signals. There is no doubt that the combination of more than 2 signals could considerably lower our detection limit. Observables such as radial velocity, intensity fluctuation, limb displacement and/or brightening are polluted by different source of noise such as supergranulation and active regions that produce different signatures. This is also an interesting direction to proceed for the future.

3.3. Longer and additional time series

At the very minute that we are writing this paper, new data are being acquired either from the SOHO spacecraft and from the various helioseismic networks around the world. Unfortunately, the improvement in the detection limit does not go down like $1/\sqrt{T}$ but more like $\log(T)/\sqrt{T}$. This is due to the probability limit which needs to be kept constant (Appourchaux, 1998). If we were to do otherwise it would mean that we would lower our probability level and accept more peaks that would likely be due to noise. Therefore, the decrease is bound to be lower than expected.

As we mention, other observables may be needed to detect the g modes. Unfortunately, other space-based data exist in the field that have not been included yet, such as GOLF data for instance.

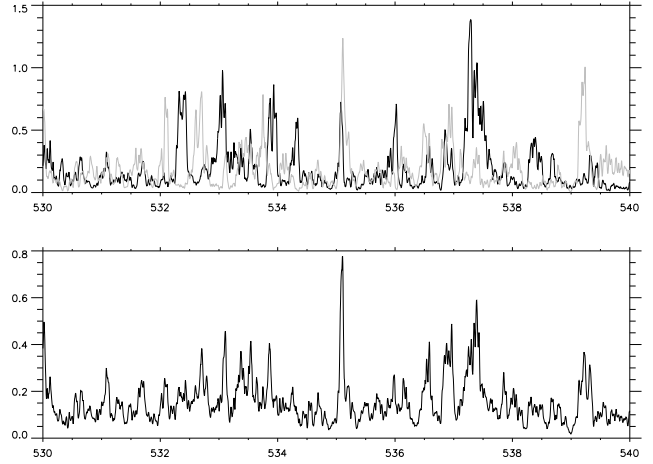


Figure 4. The upper panel shows as a function of frequency (in μHz) as a function of frequency (in μHz) the coherent part of the three consecutive thirds of the SPM (black) and MDI (grey) signals, respectively. The lower panel shows the correlation of the coherent parts of the SPM and MDI signals shown in the upper panel. There is a theoretical $l = 0, n = 3$ at $535.5 \mu\text{Hz}$.

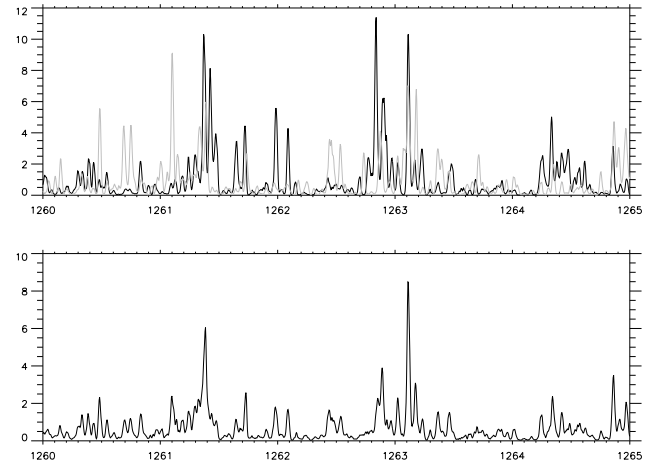


Figure 5. The upper panel shows as a function of frequency (in μHz) the coherent part of the three consecutive thirds of the SPM (black) and MDI (grey) signals, respectively. The lower panel shows the correlation of the coherent parts of the SPM and MDI signals shown in the upper panel. There is a theoretical $l = 0, n = 8$ at $1263.5 \mu\text{Hz}$.

4. CONCLUSION

The upper limit to g-mode amplitudes has been set at $200 \mu\text{Hz}$ to 10 mm s^{-1} in velocity, and to 0.5 ppm in intensity; this is the so-called 10% probability limit. We have not given up yet on finding g modes. Other observables such as limb measurements will be used to achieve this goal. They will come either

from the current SOI/MDI instrument or from accepted missions such as PICARD. New data analysis techniques are being or need to be developed. These new techniques will rely more and more on a combined search using different data sets from different observables. Last, but not least, longer time series and data still out of reach may help to improve the situation.

The Phoebus group still has faith that the goal will be reached. Unlike the search for the Holy Grail, it is not the road for reaching the goal that matters, but the goal itself. Nevertheless, the road may provide interesting social and scientific meetings that will nicely pave the way to the core of the Sun.

ACKNOWLEDGMENTS

SOHO is a project of international collaboration between ESA and NASA.

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