The structure of the solar core: an observer's point of view

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ABSTRACT

Since the beginning of helioseismology, most of the internal and dynamics structure of the Sun has been revealed or so we thought. The last island where our powerful tools start to fail is the solar core, where nuclear reactions take place. With the advent of SOHO and GONG, we have now a quality of helioseismic data without precedence that should enable us to understand better the physics of the deepest solar regions. This goal can be partially achieved by measuring low-degree rotational splitting of p modes, and by detecting the elusive g modes.

In a first part, I will review the fitting techniques that are being used for inferring the rotational splittings of low-degree p modes. I will particularly focus on Fourier spectra fitting developed by Schou (1992) and refined by Appourchaux et al (1998a). I will show how one can visualize from the data, the leakage matrix and how one can clean the data from the mode leakages. I will give examples of systematic errors introduced by the leakage matrix and by modes of aliasing degrees. I will also compare the Fourier spectra fitting technique to others techniques that use power spectra. I will give some recent results from SOHO and GONG.

In a second part, I will report on the progress of the Phoebus group for detecting g modes. The Phoebus group is composed of team members of BiSON, VIRGO and SOI/MDI. I will summarize some of the techniques we used for finding g modes, and how one can use those for finding low-order low-frequency p modes. I will, most probably, not report on g-mode detection but most likely stress that the future ahead of us is brighter than ever.

Key words: rotation - p modes - g modes - Sun

1. Introduction

In this review paper, I was specifically asked to give the status of the low-degree splittings and g-mode detection. In order to bring together these two disparate 'observational' subjects, I decided to use the *solar core* as a guideline, mainly because the knowledge of its structure and dynamics is derived from the measurement of the frequencies and splittings of solar oscillations modes.

The determination of the splittings of the low-degree p modes has always been difficult. It seems that the splittings determined by various authors decreased since the 80's (Fossat, 1995; Toutain and Fröhlich, 1992 and references therein). It is likely that the source of the decrease lies more in the reduction of systematic errors than in a Sun slowing down. Until the mid 90's, only full-disk integrated instruments were able to measure reliably the low-degree splittings (Elsworth et al., 1995; Loudagh et al., 1993; Toutain and Fröhlich, 1992). Appourchaux et al.(1995) were the first ones to measure low-degree rotational splittings using an imaging instrument such as the Luminosity Oscillations Imager (LOI). They could image all the azimuthal order for $l > 1$. Unfortunately, the atmospheric noise was too large to allow the detection of the $l = 1$ splitting, and the poor signal-to-noise ratio in intensity provided rather large error bars for the other degrees. The real breakthrough came from Tomczyk et al.(1995) who gave at the time the best low-degree splittings ever measured.

The detection of g-mode is still the main quest in our field. There has been several claims of g-mode detection. The most famous one was by Delache and Scherrer (1983) which has been never confirmed since then. The detection of the 160-minute oscillation was also a big debate in the 80's (Scherrer et al., 1979). This oscillation has not been observed by the helioseismology instrument aboard SOHO (Scherrer; Pallé et al., these proceedings). Recently, Thompson et al.(1995) claimed to have detected g modes in the particle data of the Ulysses spacecraft. There are two difficulties with their finding: how can the solar oscillations propagate into the solar wind, and how can we trust the statistical analysis. The latter has been recently answered by Hoogeveen & Riley (1998). They have shown that the statistical analysis of Thompson et al.(1995) was not correct, and was bound to 'lock' onto artifacts, e.g. noise peaks. Nevertheless, we are not yet giving up on detecting g modes.

Hereafter, I review in the first section the various fitting techniques that have been used for splitting determination, and then focus on 2 commonly-used techniques and their associated systematic errors. I also give the latest results regarding the low-degree splittings determination from SOHO and GONG. In the second section, I present the effort of a new working group whose aim is to detect g modes. I present some of our detection techniques that we used and the results we obtained. I conclude with a prospective view of the future of these 2 topics.

2. On splitting determination

2.1 The techniques

Various techniques have been used for measuring the splittings of low-degree p modes:

- m-averaged spectrum
- auto-correlation
- mean profile
- MLE on power spectra
- MLE on Fourier spectra

The first technique was used by Brown (1985). A spectrum averaged over m, for a given l, is cross-correlated with each m, and the shift measured will provide the a_i coefficients. This technique was used by Tomczyk (1988) and by Korzennik (1990). The main disadvantage is that the splitting used to produce the m-averaged spectrum tends to bias the measured splitting; for convergence some iterations is required.

Alternative techniques have been used for the GOLF and IRIS data (Lazrek et al., 1997; Lazrek et al., 1996). They

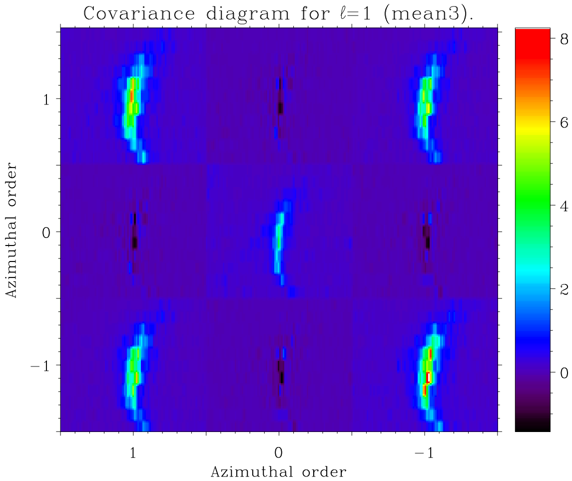

Figure 1: Covariance diagram of the LOI for $l = 1$ *before* cleaning. This diagram is made of 9 echelle diagrams of the cross-spectra of the 3 signals. For each echelle diagram the window size is 10 μHz. Note the strong correlation between the signals for $m = -1$ and $m = +1$.

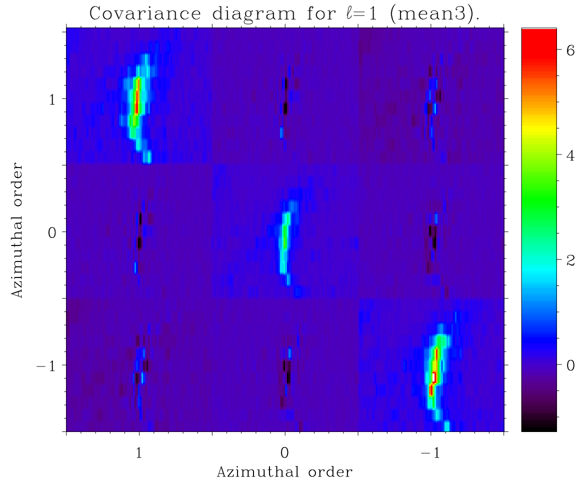

Figure 2: Covariance diagram of the LOI for $l = 1$ *after* cleaning. This diagram is made of 9 echelle diagrams of the cross-spectra of the 3 signals. For each echelle diagram the window size is 10 μHz. Note the *absence* of correlation between the signals for $m = -1$ and $m = +1$.

include making auto-correlation of a single (l, n) to derive the splitting from the shape of the correlation; or constructing for a given l, a mean profile computed over selected n for which the linewidth is similar.

An approach to a better statistical description of p-mode spectrum was introduced by Duvall and Harvey (1986). They suggested to use the *a priori* knowledge of the statistics for applying Maximum Likelihood Estimation (MLE). In the case of power spectra, the statistics is assumed to be a χ^2 with 2 degree of freedom (Duvall and Harvey, 1986). Anderson et al.(1990) showed that MLE would lead to less biased p-mode parameters (See also Toutain and Appourchaux, 1994). This technique has been since then applied to full-disk integrated instrument such as IPHIR (Toutain & Fröhlich, 1992), IRIS (Gelly et al., 1997) and BiSON (Chaplin et al., 1996). Very often, the MLE is applied for a pair of degree ($l = 0 - 2$ or $l = 1 - 3$) and for a given n. A few authors have attempted to fit the spectra as a whole (Toutain & Kosovichev, 1994; Lazrek, 1998, private communication).

For measuring splittings, imaging instruments such as GONG are intrinsically superior to full-disk instruments, mainly because they can detect the $2l+1$ components of an (l, n) multiplet. Even with imaging instruments, the probability assumed for the MLE has to be as close as possible to the real statistics. For the GONG data, Hill et al.(1996) assumed that the statistics of an (l, n, m) spectra was χ^2 with 2 d.o.f, that the $2l+1$ spectra were independent from each other, and that there were no mode leakage. For the LOI, Appourchaux et al.(1995) did the same as GONG but took into account the mode leakage. It is somewhat inconsistent to assume that the spectra are independent of each other while the mode leakage is taken into account! Schou (1992) was the first one to include the mode leakage in a correct statistics: a multi-normal distribution. This technique has been applied to the Fourier Tachometer data (Schou, 1992), the LOWL instrument (Tomczyk et al., 1995), the SOI/MDI instrument (Kosovichev et al., 1997), the GONG data (Rabello-Soares and Appourchaux, these proceedings) and the LOI data (Appourchaux & the VIRGO team, these proceedings). Schou's technique is called *Fourier spectra* fitting, while the former is called *power spectra* fitting. In the next section, I will focus on these 2 techniques which are the most commonly used in our field.

2.2 Fourier spectra versus Power spectra

2.2.1 The differences.

The fitting of Fourier spectra is often perceived as complicated and unnecessary. For full-disk integrated instruments without gaps, power spectra fitting is *not* an approximation (Appourchaux et al., 1998a). For imaging instruments, one could argue that fitting power spectra is an approximation which is good enough in most cases. But as we will see in the next section the systematic errors associated with each technique argue in favor of fitting Fourier spectra.

The mathematical background of fitting Fourier spectra was laid out by Schou (1992) and by Appourchaux et al.(1998a). A simple way of understanding the difference between fitting power spectra and fitting Fourier spectra is shown in Fig. 1 (On the CD-ROM supplement, see file `mmm/appour_1/figure1.ps`). This figure helps you to visualize the correlations between the 3 signals generated by the LOI/SOHO instrument for the $l = 1$ modes (Appourchaux et al., 1997). To grasp the significance of such a figure, we should go back to *echelle diagrams* that are sometimes used for visualizing full-disk integrated power spectra (Appourchaux et al., 1998a). These diagrams are used to visualize the *power* spectra, which, when they are smoothed, give an estimate of the *variance* of the spectra. For imaging instruments, getting an estimate of the covariance between the signals generated for different (l, m) and (l', m') is of paramount importance for understanding what is fitted, i.e. we should visualize the covariance matrix. An estimate of this matrix can be obtained by computing cross-spectra of the (l, m) and (l', m') signals as shown by Appourchaux et al.(1998c). Thus, Fig. 1 shows an estimate of the covariance matrix for $l = 1$ (On the CD-ROM supplement, see file `mmm/appour_1/figure1.ps`). This covariance matrix is related to the leakage matrix of $l = 1$ (Appourchaux et al., 1998a). For instance, one can clearly see the correlations between the $m = 1$ and $m = -1$ signals due to the LOI mode leakage of about 45% between these two signals. When one fits Fourier spectra all the information contained in the covariance diagram is used, while when one fits power spectra *only the diagonal part of the diagram* is used; leaving out some information located off the diagonal.

Here I would like to add a word of clarification. The fitting of Fourier spectra is often referred to as *taking into account the phase information.* This wording is inaccurate. In my opinion, the inclusion of a phase information would mean that the real parts and the imaginary parts of the $2l+1$ signals of a mode are statistically dependent upon each other. As a matter of fact, when one takes into account in the data reduction the orientation of the Sun onto the detector (i.e. P=0.), the real parts and the imaginary parts are statistically independent of each other! If the leakage matrix would be complex, one would have to take into account the correlations between the real and imaginary parts (Appourchaux et al., 1998a), e.g. you would take *the phase information* into account. Fortunately, we can always remap our data such that the leakage matrix is real, and we take into account only a *sign information.*

Nevertheless, the use of a *phase information* was indeed used by Appourchaux et al.(1996) for deriving the phase difference between the SOI/MDI, GOLF velocity instruments and the LOI instruments. But this phase information was not produced by the presence of mode leakage but by the Sun itself.

2.2.2 Cleaning m and l leaks.

It has been pointed out by Appourchaux et al.(1998a) that fitting Fourier spectra was equivalent to apply the inverse of the leakage matrix to the data. As an example, Fig. 2 shows how the covariance diagram are modified after applying the inverse of the leakage matrix to the data: all artificial correlations between the $m=-1$ and $m=+1$ signals are removed (On the CD-ROM supplement, see file `mmm/appour_1/figure2.ps`). This very simple property has been used not only to clean data from m leaks, but also to clean the data from l leaks (Appourchaux et al., 1998c; Rabello-Soares and Appourchaux, these proceedings). As we will see later, data cleaning provides less biased estimate of the splitting especially when spurious modes interfere with the target mode. Last but not least, data cleaning for intermediate- or high-degree modes may provide a way to produce data for which the $2l+1$ spectra of an (l,n) mode multiplet are very nearly independent from each other; and have each a χ^2 with 2 d.o.f. statistics. This is an approximation as the covariance matrix of the noise will *not* be diagonal (Appourchaux et al., 1998a).

2.3 Systematic errors

The source of systematic errors is a very important matter in any scientific field. The inferences that we make about the Sun could be biased by these errors. Then not only the measurements will be biased, but so will be our view of the physics ongoing inside the Sun. We should not forget that the detection (or non-detection) of a polar jet by SOI/MDI (or GONG) (Howe et al., these proceedings) could be induced by the presence of such systematic errors.

Schou (1992) was one of the few to study systematic errors of p-mode parameters. He focused on effects such as image distortion, temporal aliases and spatial aliases. Hereafter, I studied in some details additional source of systematic errors coming from:

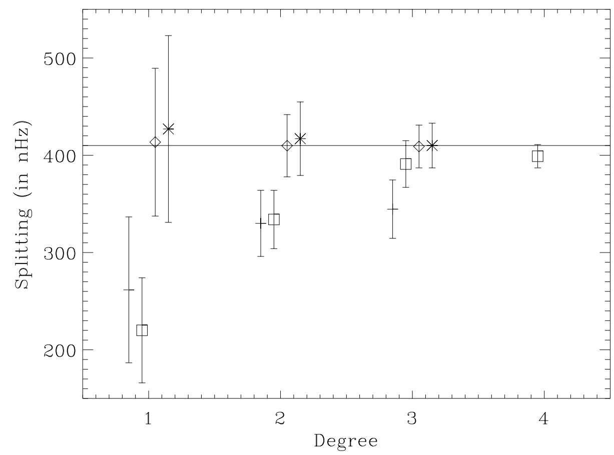

Figure 3: Comparison of splitting measured on LOI and GONG simulated data. Plus: LOI power spectra fitted *without* taking into account the mode leakage (as in Hill et al., 1996) and using a common linewidth; Star: LOI power spectra fitted taking into account the mode leakage and using a common linewidth, as in Appourchaux et al., 1995; Diamond: LOI Fourier spectra fitted taking into account the mode leakage and using a common linewidth, as in Schou (1992). The target a_1 splitting is 410 nHz. The linewidth of the mode is 0.92 μHz. The signal-to-noise ratio in the power spectra is about 30. The simulated a_i coefficients are the following: a_1=410 nHz, a_2=30 nHz, a_3=10 nHz, a_4=-50 nHz and a_5=40 nHz, of course when applicable. Square: GONG power spectra fitted *without* taking into account the mode leakage and a linewidth dependent upon $|m|$, as in Hill et al.(1996). For the GONG simulation the signal-to-noise ratio is 10 times higher, and the a_i coefficients for $i>1$ are zero.

- the fitting techniques
- the leakage matrix
- the presence of l leaks

Each contribution is studied hereafter.

2.3.1 Fitting techniques.

Using LOI simulated data, Appourchaux et al.(1998a) have shown that the p-mode parameters are returned without significant bias. Figure 3 shows the result of simulation aimed at comparing 3 types of fitting techniques:

- MLE of $(2l+1)$ independent power spectra without mode leakage:
  - -linewidth dependent upon $|m|$
  - -common linewidth for the multiplet
- MLE of $(2l+1)$ independent power spectra with mode leakage, and common linewidth for the multiplet
- MLE of $(2l+1)$ dependent Fourier spectra with mode leakage, and common linewidth for the multiplet

The first technique has been used by the GONG project (Hill et al., 1996) for deriving p-mode parameters combined with the first strategy. The second technique was used by Appourchaux et al.(1995) with the ground-based version of the LOI instrument. The last technique is

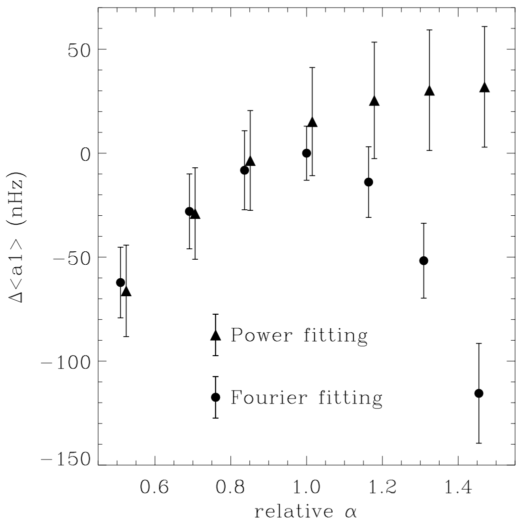

Figure 4: $l = 1$ splitting difference as a function of the relative leakage element α for 2 fitting techniques. α represents the mode leakage between $m = -1$ and $m = +1$ (After Rabello-Soares and Appourchaux, these proceedings).

the one devised by Schou (1992). From Fig. 3, it is clear that neglecting the mode leakage will dramatically pull down the estimate of the rotation of the solar core. The underestimation is very sensitive to the mode leakage and the a_i coefficients introduced: the LOI simulation always give a much lower splitting than for the GONG simulation. On the other hand, even with the inclusion of the mode leakage, fitting power spectra will tend to slightly overestimate the splittings. This effect has also been found by Chang (1996, 1998) and confirmed by Appourchaux et al.(1998b). Numerous publications in these proceedings mention similar effects (Chaplin et al.; Roca-Cortés et al.). This overestimation affects less the imaging instruments than the full-disk integrated instruments, since these latter cannot detect individually each component of the multiplets. Fortunately, at low frequency where the linewidth of the modes is smaller than the mode separation, the bias becomes minute (Chang, 1998; Appourchaux et al., 1998b). Even though, it seems that power spectra fitting including mode leakage would return an unbiased a_1 coefficient, it is not advisable to do so mainly because the other a_i coefficients, the linewidth and the amplitudes will be biased. The technique of fitting Fourier spectra will provide the smallest bias and the minimum variance for any parameters (Appourchaux et al., 1998a; Schou 1992).

2.3.2 Leakage matrix.

The fitting of Fourier spectra makes use of the knowledge of the leakage matrix. Systematic errors can arise from an imperfect model of the matrix. Of course, one could argue that this is the main culprit of the method. On the other hand, a better modeling of the data is unlikely to be worse than a model neglecting the mode leakage.

Schou (1992) and Appourchaux et al.(1998a) showed that the leakage matrix can be theoretically computed. The sub-matrices can also be measured directly from the data (Appourchaux et al., 1998c). Unfortunately, this can become a rather tedious task even for $l \approx 4$. Clearly, a first order approximation using the theoretical leakage matrix should be a good starting point before adjusting it from the data. Appourchaux et al.(1998c) have shown that their computation of the GONG leakage matrix was accurate enough for $l < 10$. They used a simple model of the observation that took into account the remapping of the velocity map onto the (x, ϕ) space. Of course, there are additional uncertainties in their computation of the leakage matrix such as the one induced by: the image distortion, the instrumental point spread function, the scattered light, the spatial sensitivity variation, or the neglecting of the horizontal velocity. Their model has not been tested for higher degree modes where it would be most likely inaccurate.

The influence of an inaccurate knowledge of the leakage sub-matrices has been studied by Appourchaux et al.(1998a) and by Rabello-Soares & Appourchaux, these proceedings. They varied the off-diagonal elements of the a leakage sub-matrix by $\pm 50\%$ of their nominal values. Figure 4 shows for the $l = 1$ modes of the GONG data, the effect of the uncertainty of the α element on the splitting determination (α represents the mode leakage between $m = -1$ and $m = +1$). It is quite remarkable that the bias induced when fitting Fourier spectra is quadratic. Intuitively, it means that the first order effects are taken care of and that the computed leakage matrix is close too the optimal one. For rotation inversion, the knowledge of the quadratic variation matters more than the value of the 'maximum' splitting. On the other hand, when fitting power spectra, taking into account mode leakage, the splitting dependence is monotonic and never reaches an optimum. This shows the advantage of fitting Fourier spectra over fitting power spectra. The quadratic dependence has also been measured on other a_i coefficients (Rabello-Soares & Appourchaux, these proceedings). Since the number of off-diagonal elements increases like l^2, it is very likely that there are substantial systematic errors arising in the splitting determination; the presence of a polar jet in the SOI/MDI data should be taken with great care.

2.3.3 Presence of l leaks.

Since the presence of m leaks produce systematic errors, the l leaks must also be taken care of. Appourchaux et al.(1998a,b) have shown that data could be cleaned from l leaks by applying the inverse of a leakage sub-matrix to the data. For instance, they showed for the GONG data how the $l = 1$ modes could be cleaned from the $l = 6$ and $l = 9$ modes, and vice versa. Not only the $l = 1$ modes benefit from the cleaning but also the $l = 6$ and $l = 9$ modes. Of course one could argue rightly that higher degree modes will be introduced in the new spectra, but this 'new' pollution will be farther away from the target modes than the 'old' pollution. The influence of the l leaks is obvious in the a_1 coefficients resulting from the analysis of the LOI data (Appourchaux and the VIRGO team, these proceedings). Other instruments are sensitive to the l leaks as reported by Tomczyk for the LOWL data (1998, private communication). Rabello-Soares and Appourchaux, these proceedings, have detected the effect of the l leaks on the GONG data; but they have also shown how the cleaning mentioned above can take care of the systematic errors induced by the l leaks. Figure 5 and 6 compare the a_i coefficients they have measured with and without cleaning (On the CD-ROM supplement, see file `mmm/appour_1/figure5.ps` and `mmm/appour_1/figure6.ps`). These two figures show

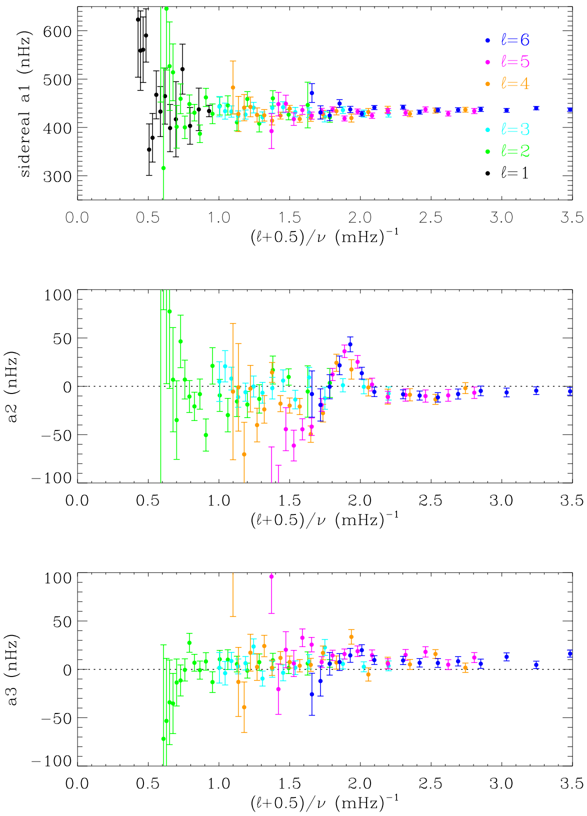

Figure 5: a_i coefficients for the GONG data as measured by Rabello-Soares and Appourchaux, these proceedings. The data are *not* cleaned from l leaks. Note the weird bump in a_2 where the $l = 4 - 7$ and $l = 5 - 8$ modes interact; and the jump in a_1 for $l = 1$ where the $l = 1 - 6$ modes interact.

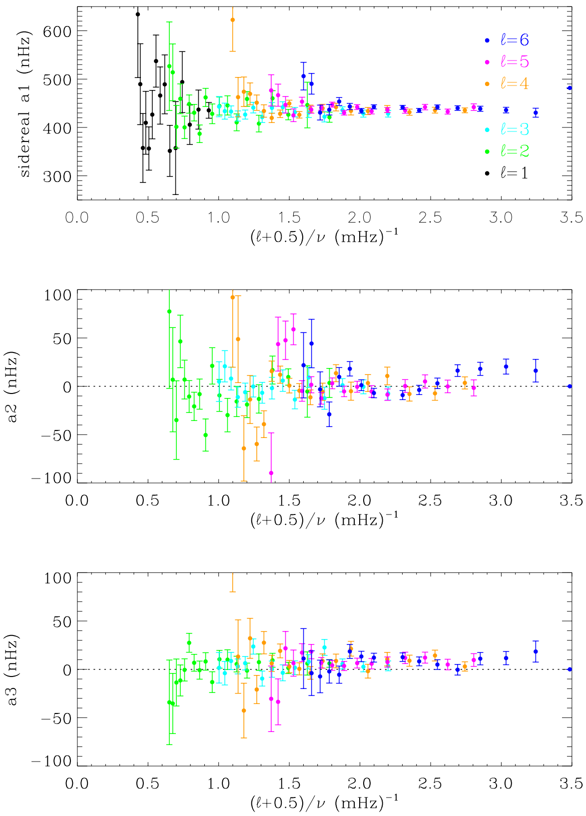

Figure 6: a_i coefficients for the GONG data as measured by Rabello-Soares and Appourchaux, these proceedings. The data are cleaned from l leaks. The bump in a_2 disappeared, and so did the jump in a_1.

how effective data cleaning is.

Schou (1992) reported on a similar 'bump' or 'jump' that he could not be easily explained. In retrospect, I think it is likely that the temporal aliases combined with the spatial aliases were producing the bump, as indeed already suggested by Schou (1992). He discarded this explanation because the 'bump' did not disappear when treating data with a degraded window function for which the first sidelobes were 70% instead of 45%. A better, but at the time impossible, test would have been to use data without gaps degraded in the same manner. Although Schou's bump was stable in time, it has never been reproduced since then with space-based data.

2.4 Splitting comparison

Now since I believe most of the systematic errors on the splittings are understood, we can start to compare the splittings measured by different instruments as in Fig. 7.

All the imaging instruments provide an $l = 1$ splitting around 432 nHz while the GOLF instrument gives a value higher by about 30 nHz. This bias might be due to the fitting of the power spectra which might tend to give slightly higher splitting as shown by Chang (1998) and Appourchaux et al.(1998b). For $l = 1$, the BiSON data gives a lower splitting than that of GOLF because it is determined for low-frequency modes for which the bias is negligible.

For $l = 2$, the data are very nicely clustering and agree very well with each other. For the $l = 2$ splitting, the bias due to power spectra fitting is negligible for the full-disk instruments.

For $l = 3$, the LOI data seems to be slightly higher than the other instruments. The reason for this bias is not yet known, but it could come from the difference in the observables: intensity vs velocity. In addition, the full-disk integrated instruments do not measure a_1 but a weighted mean depending on the ratio ρ between the $m = 1$ and the $m = 3$ components; from Gizon et al.(1997), I have deduced that $\mathcal{S}_3 = a_1 + a_3 \frac{9\rho - 3}{9\rho + 1}$, the last term is about 4-5 nHz.

For $l = 4-5$, Appourchaux, these proceedings, has shown that the large systematic errors are due to the fitting techniques: the fitting of Fourier spectra cannot be used because the LOI leakage matrix is not invertible for $l > 5$.

From Fig. 7, I can safely deduce that the solar core rotates at the same speed as the radiative zone. The only remaining discrepancy may lie in the determination of the rotation deep in the solar core, say below 0.15 $R_\odot$. In this region, the rotation can be better determined by high-frequency p modes that penetrate deeper into the core. Unfortunately, their linewidths are so large that the $l = 0-2$ and $l = 1-3$ modes are blended. Therefore, the splitting determination at high frequency will suffer from systematic errors that even the cleaning technique by the leakage matrix would not suppress. This is where the detection of g modes would lift all the uncertainties remaining on the structure and dynamics of the solar core.

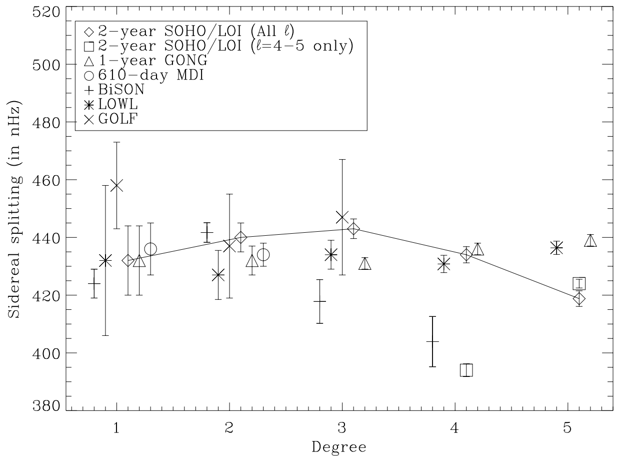

Figure 7: Comparison of sidereal splittings measured by different instruments. Diamond: 2-year LOI data fitted using Fourier spectra (Appourchaux, these proceedings); Square: 2-year LOI data, l=4-5 fitted using power spectra (Appourchaux, these proceedings); Triangle: 1-year GONG data (Rabello-Soares and Appourchaux, these proceedings); Circle: 610-day of SOI/MDI data (Toutain et al., these proceedings); Plus: BiSON data (Chaplin et al., 1996); Star: 2-year LOWL data (Tomczyk, 1998, private communication); Cross: 480-days of GOLF data (Roca-Cortés et al., these proceedings)

3. g-mode detection

3.1 The Phoebus group

This working group is a new entity which was set up in March 1997 by SOHO helioseismologists. The aim of this group is very simple:

To discover the g modes for a better understanding of the solar core structure

This is a very simple minded approach, and no explicit rules have been implemented beyond this approach. Hopefully the scientific ethic of each member of the group should be sufficient for avoiding selfish behaviour! The group was named after the greek god of light (also known as Apollo), and after Gaston Phébus, count of Foix, who wrote a book about hunting (Phébus, XVth century; available at www.bnf.fr/enluminures/manuscrits/man10.htm).

The member of the Phoebus group are the following:

- Bo Andersen (VIRGO/SPM)
- Thierry Appourchaux (VIRGO/LOI, secretary)
- Williams Chaplin (BiSON, as of Jan 1998)
- Yvonne Elsworth (BiSON, as of Jan 1998)
- Wolfgang Finsterle (VIRGO/PMO6)
- Claus Fröhlich (VIRGO/PMO6)
- Douglas Gough (Theoretical support)
- Todd Hoeksema (SOI/MDI)
- George Isaak (BiSON, as of Jan 1998)
- Alexander Kosovichev (SOI/MDI)
- Janine Provost (Theoretical support)
- Phil Scherrer (SOI/MDI)
- Takashi Sekii (Theoretical support)
- Thierry Toutain (VIRGO/SPM, SOI/MDI)

Each member of the group has clear responsibilities regarding data preparation and data analysis. The data from the different instruments are put on a common account that can be accessed by all the members of the group. Teleconferences are held bi-monthly for speeding up data preparation, discussing scientific issues and 'new findings', and improving communication. The Phoebus group had its 1st workshop in November 1997. A 2nd workshop will be held at the end of October 1998. The Phoebus group has a home page linked to the VIRGO home page (virgo.so.estec.esa.nl/html/group.html). The aide-mémoire for g-mode detection written after the 1st workshop can be accessed on the Phoebus home page. Hereafter, I will elaborate on a few techniques that are being used by the Phoebus group for detecting g modes. Another paper by Fröhlich and the Phoebus group, these proceedings, reports on the results obtained with these techniques and on the upper limit for g-mode amplitudes.

3.2 g-mode detection techniques

Several techniques for detecting g modes have been used by several authors. Hereafter, I give a list limited only by my own knowledge:

- Statistical methods
- Patterns:
  - -Splitting (collapsogramme)
  - -Splitting and P_0
  - -Exact fraction technique
- Autocorrelation of spectra
- Data combination
  - -multivariate analysis
  - -cross spectra

The 'statistical methods' and 'collapsogramme' techniques are described in the next sections. The other techniques relying on patterns were used by Fröhlich and Delache (1984) ('Splitting and P_0'), and have been used by Pallé et al., these proceedings ('Exact fraction techniques'). Autocorrelation of spectra were used by Andersen (1997, private communication). The multivariate analysis is described by Finsterle and Fröhlich in these proceedings. The use of cross spectra for reducing noise has also been used by the GOLF team using SOI/MDI data (García, 1997, private communication).

Any technique aiming at detecting g modes should be able to detect also low-frequency p modes. Since the low-frequency p modes have a long lifetime, they can mathematically mimic g modes. This approach has been used by the Phoebus group and resulted in detecting low-frequency p modes (Appourchaux and the Phoebus group, these proceedings). In the next 2 sections, I will concentrate on the 'statistical methods' and 'collapsogramme' techniques used, and on their limitations.

3.2.1 Statistical methods.

The statistical distribution of the power spectra of full-disk integrated instruments is well known: this is a χ^2 with 2 degree of freedom. For imaging instruments, this

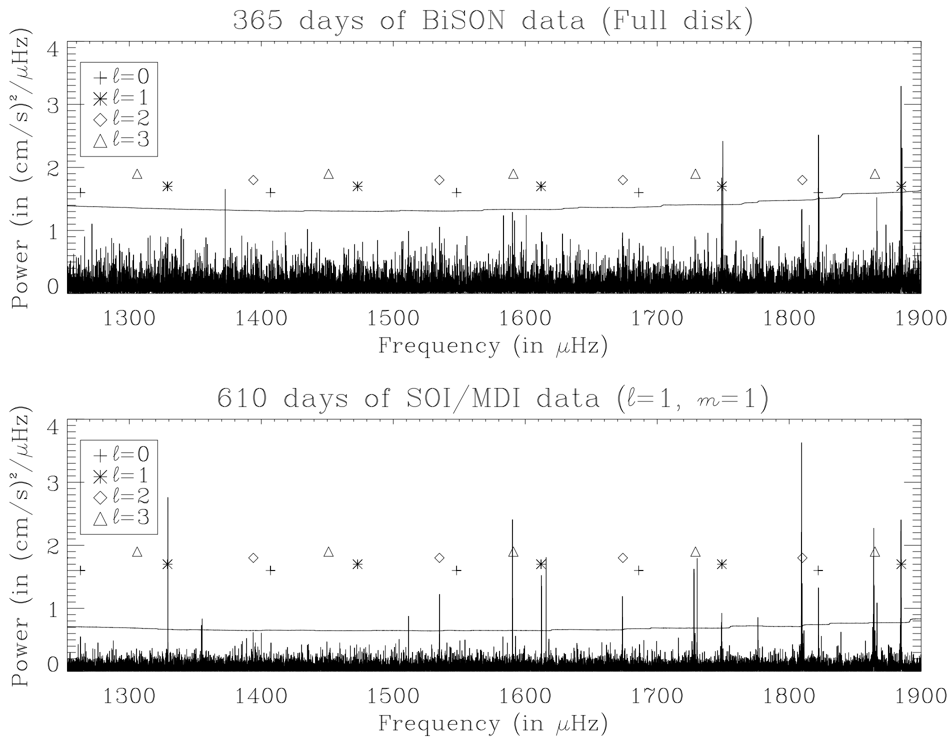

Figure 8: The 0.1 probability level for the Phoebus data. Top: 365 days of BiSON data, the 10-σ level curve is indicated; the σ estimate is derived from a spectra smoothed on 62 μHz. Bottom: 610 days of SOI/MDI data, the 10.5-σ level curve is indicated; the σ estimate is derived from a spectra smoothed on 37 μHz.

is slightly more complicated due to the correlation between the $2l+1$ signals: it is a multinormal distribution (Appourchaux et al., 1998a). For full-disk instruments, we can easily derive the relative power spectra level (σ_{det}) for which a peak due to noise has a probability (p_{det}) to appear in a given frequency bandwidth (Δ_{det}). This level will depend upon the observing time (T) because the number of frequency bins in the bandwidth will increase with time. And we have:

$$\sigma_{det} = \log(T) + \log(\Delta_{det}) - \log(p_{det}) \qquad (1)$$

where T is in units of 10^6 sec, Δ_{det} is in μHz. For a 70-μHz band, a 365-day observing time and a 0.1 probability level, the detection level corresponds to 10 σ. Figure 8 shows the BiSON data and SOI/MDI data with their 0.1 probability level. This is an example where an artifact (e.g. not a mode) is detected in the BiSON data just above 1370 μHz. This 'mode' is not detected in the SOI/MDI data, clearly indicating that the peak detected in the BiSON data should really be taken for what it is: *a peak having a probability of 0.1 to be due to noise.* You can also notice in the SOI/MDI data that peaks at the left-hand side of the $l=2$ modes are getting above the 0.1 probability level: these are $l=4$ modes.

From Eq. 1, we can easily deduce that the detection level A_{det} in unit of cm/s or ppm can be written as:

$$A_{det} = \sqrt{s}\sqrt{\frac{\log(T) + \log(\Delta_{det}) - \log(p_{det})}{T}} \qquad (2)$$

where s is the density of the power spectra (in ppm$^2/\mu$Hz or in (cm/s)$^2/\mu$Hz), invariant with time. From Eq. 2, we can deduce that the detection level will not decrease as the inverse of the square root of time $1/\sqrt{T}$, as it is commonly thought, but as $\sqrt{\log(T)}/\sqrt{T}$; which is a slower decrease.

3.2.2 Patterns techniques and collapsogrammes.

All the pattern techniques assume that the g modes are splitted by rotation and/or that their frequencies are derived from an asymptotic formula (Fröhlich and Andersen, 1995; Fröhlich and Delache, 1984). In order for these techniques to be efficient, it is required that the splitted modes be present. Unfortunately, it can happen that, due to beating with the noise, the modes do not appear. As an example, Fig. 9 shows that sometimes not all of the components of a p-mode multiplet are 'excited'. As a matter of fact, the modes are always excited but since the low-frequency p modes have a low amplitude they beat with the noise; sometimes the noise enhances the mode or suppresses it. Figure 10 shows a simulation for 140 days of data of such a beating. This very simple simulation means that the detection of a 'g-mode peak' in one time series may not be confirmed by another independent time series. A proper statistical analysis of many independent time series would be needed before confirming the detection of a g mode.

Keeping the statistical behaviour of the multiplet in mind, I have nevertheless devised a new pattern technique for detecting g modes: *the collapsogramme.* Each m spectrum is shifted from the $m=0$ spectrum by $m\Omega$ (where Ω represents the splitting of the mode), then each spectrum is normalized by an estimate of the variance of the spectrum, and finally the $2l+1$ spectra are added together. The advantage of this technique is that it gets

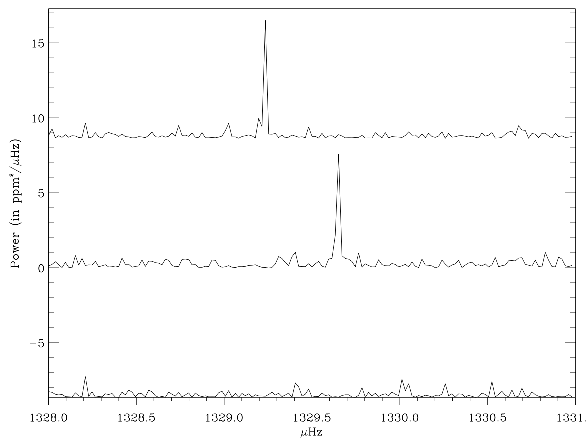

Figure 9: (m, ν) diagram for the $l = 1$, $n = 8$ as observed by SOI/MDI. The frequency resolution is about 19nHz (610 days of data). The power spectra from top to bottom are for $m = +1$, $m = 0$ and $m = -1$, respectively.

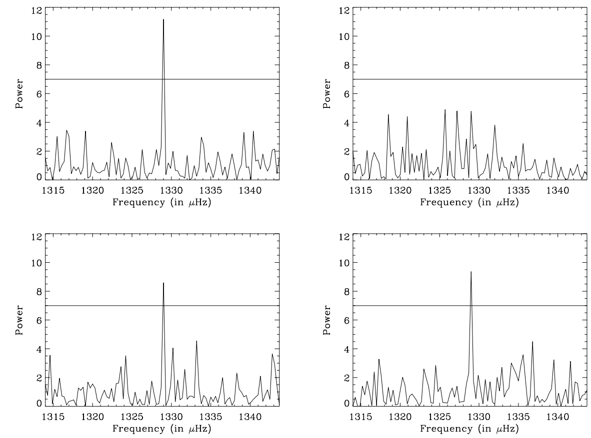

Figure 10: Simulated $l = 1$, $n = 8$ with a signal-to-noise ratio of 6.25 in the power spectrum. The resolution is about 80 nHz. The mean statistical probability that the mode be higher than 7 σ is about 2 chances over 3.

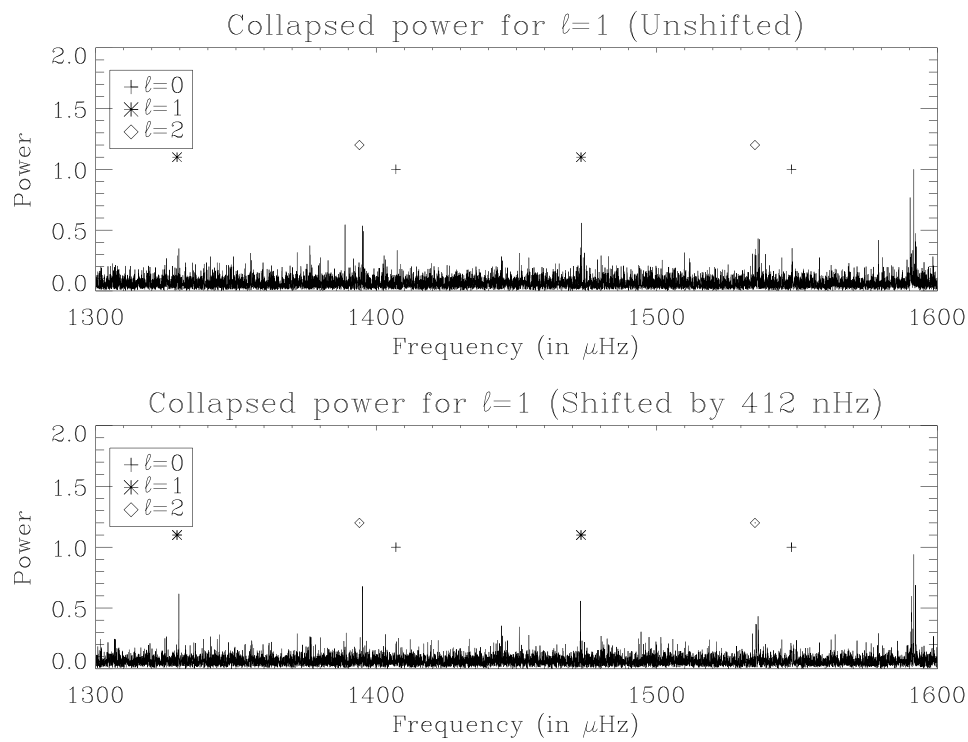

Figure 11: Collapsogramme for 610 days of SOI/MDI data for $l = 1$. Top: unshifted; bottom: shifted by 412 nHz. The $n = 8, l = 1$ mode gets out better in the shifted collapsogramme, as expected.

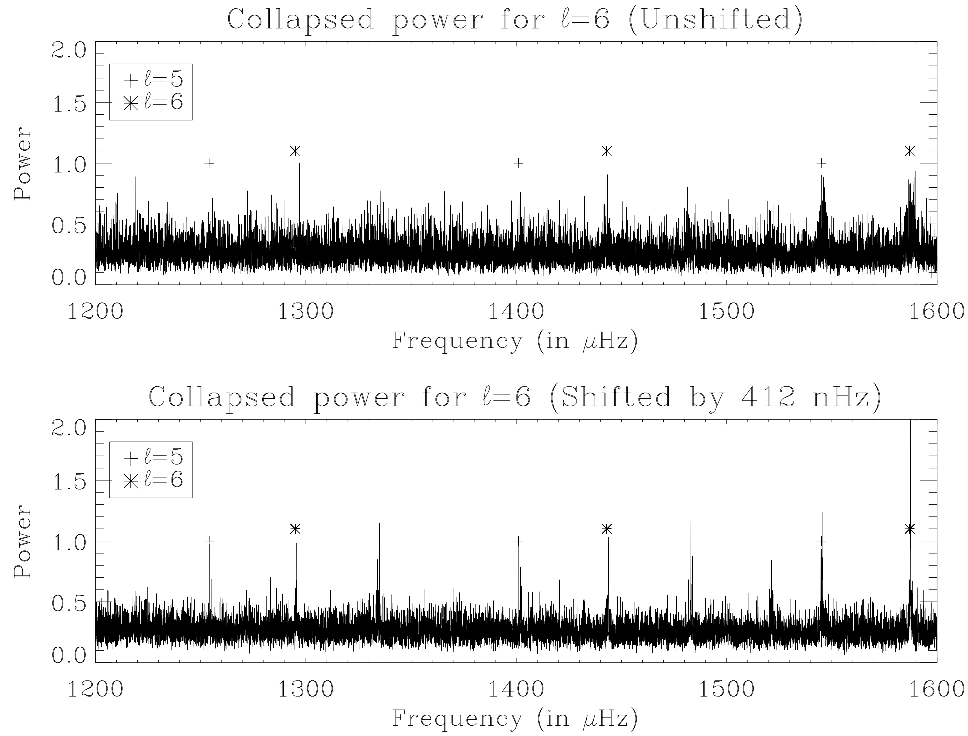

Figure 12: Collapsogramme for 610 days of SOI/MDI data for $l = 6$. Top: unshifted; bottom: shifted by 412 nHz.

rid of the instrumental harmonics (invariant), and produces a spectrum with a well defined statistics (nearly a χ^2 with $4l + 2$ d.o.f.). The disadvantage is that the g-mode splitting varies faster with frequency than for the p modes; the technique should be restricted to frequency band where the splitting varies slowly. Figures 11 and 12 show collapsogrammes in the p-mode frequency range for $l = 1$ and $l = 6$, respectively. Lower-frequency p modes can be detected showing the efficiency of the technique. Results by the Phoebus group using this technique on low frequency p modes can be found in these proceedings (Appourchaux and the Phoebus group).

4. Conclusion and prospects

In this paper, I have shown how one can best determine the splitting of low-degree modes using the fitting of Fourier spectra. I have addressed the systematic errors associated with different fitting techniques, related to the knowledge of the leakage matrix and to the presence of aliasing modes interfering with the target modes. I believe that most of the source of systematic errors on the splitting are now well understood. Some additional work will be needed to assess whether different observables (intensity or velocity) may produce systematic errors. It seems that the agreement between the low-degree splittings given by different authors has never been better. Unfortunately, this agreement tends to make me believe that the solar core rotate rigidly (See also Charbonneau et al., 1998). Increasing the observing time will only improve the precision of this picture as the square root of time.

I have also discussed 2 g-mode detection techniques that a newly formed working group, the Phoebus group, have used. These techniques were tested on low-frequency p modes. The upper limit detection for g modes can be found in these proceedings (Fröhlich and the Phoebus group). In the most optimistic case given the theoretical amplitude of the g modes (1 mm/s after Andersen, 1996, or after Kumar et al., 1996), Fröhlich and the Phoebus group give a 20-year integration time before reaching the 0.1 probability detection level! Such a long time will require a fully functioning SOHO spacecraft and/or ground-based networks. In the mean time, there may be other way to detect the g modes. Toner and Jefferies (1997) attempted to detect the g modes at the solar limb. This is based on a theory predicting an increase towards the limb of the intensity perturbation induced by the modes (Toutain and Gouttebroze, 1993). Jefferies and Toner, these proceedings, have measured such an increase for the p modes. This increase has also been observed in the LOI data (Appourchaux and Toutain, 1998). Obviously the atmosphere may still produce enough seeing noise for preventing the detection of g modes from the ground. A future CNES space mission called PICARD will attempt to detect the g modes at the limb by measuring the Sun's figure (Damé et al., 1998).

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