## Results for HH5 exercise; case of HD 180642

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Number of frequencies modeled : 51 peaks, corresponding to:

- one radial mode
- $31=1$ modes
- $41=2$ modes
- $31=3$ modes

Main properties of the signal:

- The radial mode has an amplitude a lot higher than the others (in power: several orders of magnitude, as shown on figure 1).
- No peak is frequency-resolved (no measurable width). Some modes lie in the same frequency bin when the spectrum is not oversampled (resolution $=0.08 \mu \mathrm{hz}$ ).
- Phases and amplitudes are distributed randomly within a multiplet, the signal to photon noise ratio being high.
- Some multiplets overlap.


## Results of the analysis:

1) Detection.

47 from the 51 peaks were detected. As far as the 4 non-detected peaks are concerned, 2 of them are very close to the radial mode ( 3 or 4 times less than the frequency resolution). The 2 other miss come from the overlap of multiplets, inducing modes very close together; in 2 cases, one mode was also hidden by its significantly more powerful neighbour.

From the 47 detections, 45 modes were identified with a measurable amplitude and phase. The 2 others were detected only through a perturbation of the side-lobes profile of a previously identified mode: both in the energy profile and in the phase profile (both oversampled), the signature of a faint mode can be relatively well localized.
2) Comparaison of the values recovered by the hound with the values set by the hare (Figures 2 to 7 ).

Frequency: the analysis was stopped when a $0.005 \mu \mathrm{~Hz}$ formal precision was reached, which corresponds to less than a tenth of a frequency resolution bin. To sum up, $100 \%$ of the peaks derived are closer than half of a bin to their input value, more than $80 \%$ of the peaks being even closer than $1 / 8^{\text {th }}$ of a bin. (see figure 2; note that the " y " range of the plot corresponds to a frequency resolution bin). In most cases, modes with the highest errors have a low amplitude, as shown by figure 3. In few cases, the localization of the eigenfrequency is biased by the interference with a side lobe of a neighbouring mode.

Amplitude: in most cases, the error obtained for the amplitude of the modes is less than $20 \%$ and decreases with the amplitude (figures 4 and 5).

Phase: the formal precision of the analysis is about $\pm 5^{\circ}$, the difference between the value recovered and the input being sometimes significantly higher than this precision (figure 6). The phase determination is very sensitive to the frequency determination, as confirmed by figure 7 which shows the strong correlation between the error in the eigenfrequency and the error in the phase.

## Main conclusions

The high signal to noise ratio allowed us to identify most of the eigenfrequencies with a very small error. The complete identification of the modes, ie associating a degree and an azimuthal order to every peak, was possible except for one multiplet ( $1=2 ; 103.84 \mu \mathrm{~Hz}$ ), for which one component could not be recovered. Without any other information on the structure of the star, we cannot discrimate between the two cases: either the $\mathrm{m}=-2$ or the $\mathrm{m}=+2$ is missing. The phase analysis was very powerful to detect modes with low amplitudes, but as far as precise value of the phase is concerned, caution is required as its derivation is very sensitive to the frequency bias.

An iterative process could have been tried to better characterize the modes, and to detect the 4 missing modes, but as the hound knows the result, we decided not to perform any deeper analysis.


Figure 1: spectral density of the light curve


Figure 2: error in the peak frequency recovery process.


Figure 3: error in the peak frequency versus input amplitude (radial mode withdrawn for clarity).


Figure 4: error in the peak amplitude recovery process


Figure 5: error in the peak amplitude versus the input (radial mode withdrawn for clarity).


Figure 6: error in the phase recovery process


Figure 7: error in phase versus error in frequency

Recovered values

| Structure | Frequency (microHz) | Amplitude (fraction unité) | Phase ( ${ }^{\circ}$ ) |
| :---: | :---: | :---: | :---: |
| Triplet (or quintuplet ?) <br> Degree $1=1$ (or 2 ?) | $\begin{aligned} & 61.285 / 62.02 \\ & 62.74 \\ & (63.5 \text { ???) / } 64.20 \end{aligned}$ | $\begin{aligned} & 1.8 \mathrm{e}-4 / 1.5 \mathrm{e}-4 \\ & 4.2 \mathrm{e}-4 \\ & ? / ? \end{aligned}$ | $\begin{aligned} & -70 /-10 \\ & 0 \\ & ? / ? \end{aligned}$ |
| Main Pulsation (radial) | 63.5075 | 0.0355 | 170 |
| $\begin{aligned} & \text { Quintuplet (or septuplet ?) } \\ & 1=2 \text { (or } 3 \text { ?) } \end{aligned}$ | $\begin{aligned} & 62.74 \text { ? / ( } 63.5 \text { ??? }) / 64.30 \\ & 65.08 \\ & 65.86 / 66.63 / 67.38 \end{aligned}$ | $\begin{aligned} & ? / ? / 4.7^{\mathrm{e}}-4 \\ & 1.3^{\mathrm{e}}-4 \\ & 2.4 \mathrm{e}-4 / 2 \mathrm{e}-4 / 7 \mathrm{e}-5 \end{aligned}$ | $\begin{aligned} & ? / \text { / } / 50 \\ & 180 \\ & 50 /-60 / 65 \end{aligned}$ |
| Triplet $1=1$ | 67.85 / 68.65 / 69.45 | 3.6e-4 / 4e-4 / 2e-4 | $80 / 180 / 150$ |
| Quintuplet $1=2$ | $\begin{aligned} & 78.91 / 79.59 \\ & 80.27 \\ & 80.96 / 81.64 \end{aligned}$ | $\begin{aligned} & 1 \mathrm{e}-4 / 3 \mathrm{e}-5 \\ & 4.4 \mathrm{e}-4 \\ & 3.4 \mathrm{e}-4 / 1.5 \mathrm{e}-4 \end{aligned}$ | $\begin{aligned} & 150 /-100 \\ & -10 \\ & -70 / 140 \end{aligned}$ |
| Septuplet $1=3$ | $\begin{aligned} & 84.3 / 85.07 / 85.82 \\ & 86.59 \\ & 87.30 / 88.04 / 88.76 \end{aligned}$ | $\begin{aligned} & 3.5 \mathrm{e}-5 / 3.4 \mathrm{e}-4 / 6 \mathrm{e}-4 \\ & 3 \mathrm{e}-5 \\ & 5.7 \mathrm{e}-4 / 3.5 \mathrm{e}-4 / 3 \mathrm{e}-5 \end{aligned}$ | $\begin{aligned} & 0 ? /-65 / 180 \\ & -145 \\ & -120 /-40 /-10 \end{aligned}$ |
| $\begin{aligned} & \text { Triplet } \\ & 1=1 \\ & \hline \end{aligned}$ | 89.83 / 90.65 / 91.47 | $2 \mathrm{e}-4 / 1 \mathrm{e}-3 / 1.7 \mathrm{e}-4$ | $-20 /-20 /-20$ |
| Quintuplet $1=2$ | $\begin{aligned} & 96.15 / 96.91 \\ & 97.71 \\ & 98.48 / 99.27 \end{aligned}$ | $\begin{aligned} & 5 \mathrm{e}-5 / 2.6 \mathrm{e}-4 \\ & 1 \mathrm{e}-4 \\ & 4.5 \mathrm{e}-4 / 8 \mathrm{e}-5 \end{aligned}$ | $\begin{aligned} & -20 / 160 \\ & 180 \\ & 50 / 70 \end{aligned}$ |
| Septuplet $1=3$ | $\begin{aligned} & 102.17 / 102.92 / 103.68 \\ & 104.43 \\ & 105.19 / 105.95 / 106.70 \end{aligned}$ | $\begin{aligned} & 1 \mathrm{e}-4 /<1 \mathrm{e}-4 / 6 \mathrm{e}-4 \\ & 1.2 \mathrm{e}-4 \\ & 3.5 \mathrm{e}-4 / 2 \mathrm{e}-4 / ? \end{aligned}$ | $\begin{aligned} & 0 / 180 / 40 \\ & -110 \\ & 180 /-65 / 70 ? \end{aligned}$ |
| Quadruplet (al=2 mode with a non excited component?) | $\begin{aligned} & 103.01 / 103.82 \\ & 104.67 / 105.49 \end{aligned}$ | $\begin{aligned} & 1.1 \mathrm{e}-4 / 3 \mathrm{e}-4 \\ & 1.6 \mathrm{e}-4 / 1.7 \mathrm{e}-4 \end{aligned}$ | $\begin{aligned} & -20 / 180 \\ & 30 / 30 \end{aligned}$ |
| $\begin{aligned} & \text { Triplet } \\ & 1=1 \end{aligned}$ | 107.28 / 108.09 / 108.90 | 1.6e-4 / 7.6e-4 / 3.5e-4 | $-20 / 90 / 130$ |

## Input values

| Quintuplet: |  |  |  |
| :---: | :---: | :---: | :---: |
| $2-2$ | 61.288 | 182.7 | 153.4 |
| $2-1$ | 62.014 | 145.4 | 103.5 |
| 2 | 62.740 | 459.8 | 97.0 |
| 2 | 63.467 | 132.6 | -105.7 |
| 22 | 64.194 | 106 | 75.4 |

Main radial mode:
$0 \quad 0 \quad 63.507 \quad 35779.8 \quad-81.9$


Triplet:

| 1 | -1 | 67.855 | 357.5 | 6.4 |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0 | 68.649 | 386.4 | -93.2 |
| 1 | 1 | 69.444 | 228.3 | -58.3 |

Quintuplet:

| 2 | -2 | 78.905 | 100.0 | -39.9 |
| ---: | ---: | ---: | ---: | ---: |
| 2 | -1 | 79.589 | 29.4 | -169.0 |
| 2 | 0 | 80.274 | 444.6 | 95.6 |
| 2 | 1 | 80.958 | 342.0 | 164.3 |
| 2 | 2 | 81.642 | 141.3 | -63.8 |
| Septuplet: |  |  |  |  |
| 3 | -3 | 84.329 | 46.7 | 104.1 |
| 3 | -2 | 85.072 | 344.3 | 152.7 |
| 3 | -1 | 85.814 | 627.7 | -79.8 |
| 3 | 0 | 86.556 | 25.8 | -55.5 |
| 3 | 1 | 87.299 | 544.3 | -151.2 |
| 3 | 2 | 88.041 | 335.5 | 122.7 |
| 3 | 3 | 88.784 | 33.2 | 27.3 |

Triplet:

| 1 | -1 | 89.836 | 201.1 | 91.1 |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0 | 90.654 | 921.9 | 117.2 |
| 1 | 1 | 91.471 | 174.0 | 127.8 |

Quintuplet:

| 2 | -2 | 96.129 | 48.3 | 151.5 |
| ---: | ---: | ---: | ---: | ---: |
| 2 | -1 | 96.912 | 269.4 | -67.6 |
| 2 | 0 | 97.696 | 104.4 | -66.8 |
| 2 | 1 | 98.479 | 452.0 | 36.7 |
| 2 | 2 | 99.262 | 75.8 | 36.3 |

Septuplet:

| 3 | -3 | 102.156 | 51.5 | 128.9 |
| ---: | ---: | ---: | ---: | ---: |
| 3 | -2 | 102.914 | 88.7 | -77.2 |
| 3 | -1 | 103.672 | 648.4 | 55.9 |
| 3 | 0 | 104.430 | 128.3 | -162.0 |
| 3 | 1 | 105.187 | 348.6 | -79.5 |
| 3 | 2 | 105.945 | 234.1 | 156.9 |
| 3 | 3 | 106.703 | 21.4 | -68.2 |

Quintuplet:
2 -2 102.189
2 -1 103.015
20103.842

21 104.669
$\begin{array}{lllll}2 & 2 & 105.495 & 165.9 & 43.5\end{array}$
Triplet:
$\begin{array}{lllll}1 & -1 & 107.278 & 142.2 & 126.5\end{array}$
$1 \quad 0 \quad 108.089 \quad 747.3-2.8$
$1 \begin{array}{llll}1 & 108.900 & 336.0 & -42.8\end{array}$

