DEPENDENCE ON AZIMUTHAL ORDER OF THE AMPLITUDES OF LOW-DEGREE P MODES

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ABSTRACT

The m-dependence of the amplitudes of the multiplets of low-degree p modes contains information about the latitudinal distribution of the power in the excitation of the oscillations. We present estimates of those amplitudes from 4 years of VIRGO/LOI observations. Variation of the excitation with magnetic activity is studied.

Key words: excitation - p modes - Sun.

1. INTRODUCTION

The first results from VIRGO (Fröhlich et al., 1997) indicated that there was a latitudinal variation in the vigour with which solar p modes are excited by the turbulent convection, and suggested that perhaps the excitation is stronger in latitudes of greater magnetic activity. If that were the case, one would expect there to be a temporal variation in the pattern of excitation with the solar cycle. We report in this poster the results of a similar analysis of four years of LOI data in an attempt to confirm or deny this suggestion.

2. AMPLITUDE RATIO FROM LOI

The LOI time series analysed here starts on 27 March 1996 0:00 TAI and ends on 26 March 2000 23:59 TAI. The fitting of the LOI spectra is described by Appourchaux (1998) and by Appourchaux in these proceedings. Owing to the small number of pixels (12), the modes with $l \geq 4$ are strongly contaminated by modes of higher degree (Appourchaux et al., 1998). In order to have more reliable inferences, we decided to use only the amplitude ratios inferred for $l \leq 3$. Figure 1 shows the amplitude ratio for $l = 1$ to $l = 3$; the amplitude ratios here are homogeneous to relative intensity. The ratios have been corrected for the LOI sensitivities as computed in Appourchaux et al. (2000).

3. EXCITATION FUNCTION INVERSION

We adopt the following model equation satisfied by the temporal Fourier transform $a_{lm}(\omega)$ of the instantaneous amplitude of a forced oscillation of a degree l and azimuthal order m:

$$< \left(\omega^2 - \omega_{lm}^2 + 2\mathrm{i}\omega\kappa\right) a_{lm} Y_{lm}^2 > = < \mathcal{F}(\mu) Y_{lm}^2 > \quad (1)$$

where $\mathcal{F}$ is the amplitude of stochastic forcing, considered to be a function of $\mu = \cos\theta$, where θ is co-latitude, and the brackets $< \ldots >$ denote an average over the sphere.

The quantity ω_{lm} is the corresponding adiabatic eigenfrequency and we have represented the natural damping in terms of a damping coefficient κ. Because part of the damping process depends on turbulent interactions, we have in mind that it too depends on μ; we assume that the part with zero average has the same functional form as the variable part of $\mathcal{F}$, and we thus set $\mathcal{F} = \mathcal{F}_0 + \mathcal{F}_1 f(\mu)$, $\kappa = \kappa_0 + \kappa_1 f(\mu)$, and we expand f in a series of even-degree Legendre polynomials $P_n(\mu)$:

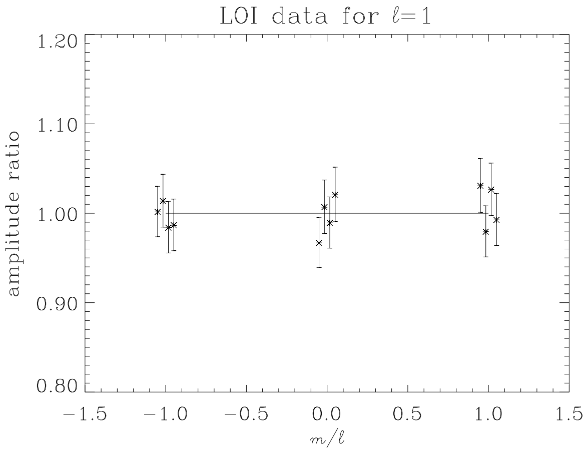
$$f(\mu) = \sum_{k=1}^{k_{\rm n}} f_k P_{2k}(\mu). \quad (2)$$

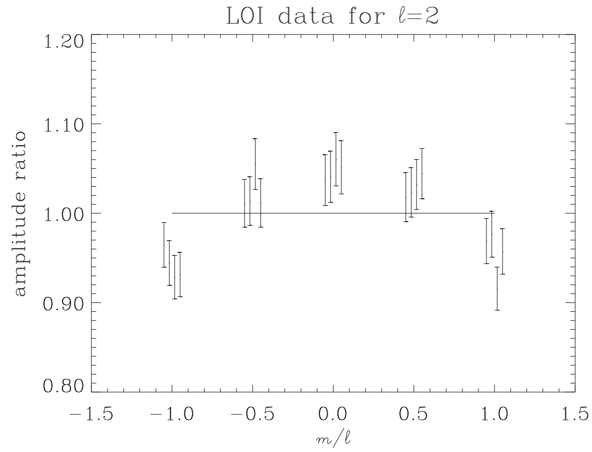
The power in the spectrum from the mode is A_{lm}^2, where

$$A_{lm} \approx \frac{1}{2\omega_{lm}} \left(\frac{\pi}{\kappa_0 + \kappa_1 \hat{f}_{lm}}\right)^{\frac{1}{2}} \left(\mathcal{F}_0 + \mathcal{F}_1 \sum_{k=1}^{k_{\rm n}} Q_{klm} f_k\right) \quad (3)$$

provided $\kappa_0 \ll \omega_{lm}$, where

$$Q_{klm} = \frac{1}{2}(2l+1)\frac{(l-|m|)!}{(l+|m|)!}\int_{-1}^{+1} P_{2k}(\mu)\left[P_l^m(\mu)\right]^2 d\mu. \quad (4)$$





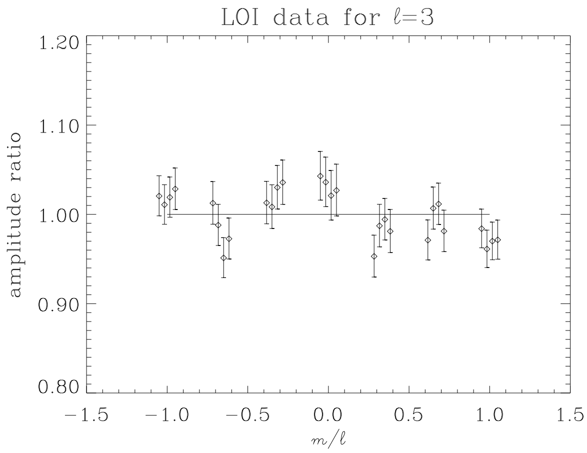


Figure 1. Amplitude ratio as a function of m for $l = 1, 2, 3$ for various years. The ratios are derived from the mode amplitude which is computed using an optimal average over the frequency range 2500-3500 μHz. The leftmost mark is the year 1996, and the rightmost mark is the year 2000. The ratio are corrected from the LOI sensitivities. The ratio have been normalized such that the total power be unity.

If $\kappa_0 \ll \kappa_1$ and $\mathcal{F}_1 \ll \mathcal{F}_0$, expression (3) can be linearized to yield:

$$A_{lm} = \left(\frac{\pi}{\kappa_0}\right)^{\frac{1}{2}} \frac{\mathcal{F}_0}{2\omega_{lm}}(1 + \beta \sum_{k=1}^{k_n} Q_{klm} f_k), \qquad (5)$$

in which $\beta = \mathcal{F}_1/\mathcal{F}_0 - \kappa_1/2\kappa_0$ represents an excess of relative excitation to relative damping.

Equations (5) were solved for $\phi_k = \beta f_k$ in terms of the observed relative amplitudes $\alpha_{lm} = (2l+1)A_{lm}/\sum_m A_{lm}$ of the multiplet of degree l, which are averaged over mode order, by regularized least-squares minimization. We use a penalty function $\mathcal{P}$ to induce flatness:

$$\mathcal{P} = \beta^2 \int_{-1}^{+1} (1-\mu^2) \left(\frac{\mathrm{d}f}{\mathrm{d}\mu}\right)^2 \mathrm{d}\mu = 4 \sum_{k=1}^{k_n} \frac{k(2k+1)}{4k+1} \phi_k \qquad (6)$$

with a tradeoff parameter λ. Thus, we minimize with respect to ϕ_k:

$$\frac{\sum\limits_{l,m} \sigma_{lm}^{-2} \left(\sum\limits_{k=1}^{k_n} Q_{klm}\phi_k - \alpha_{lm} + 1\right)^2}{\sum\limits_{l,m} \sigma_{lm}^{-2} \left(\alpha_{lm} - 1\right)^2} + \lambda \mathcal{P}. \qquad (7)$$

The outcome is an estimated relative net variation in the excitation given by:

$$\epsilon(\mu) = \sum_{k=1}^{k_n} \phi_k P_{2k}(\mu) \qquad (8)$$

where ϕ_k are given by:

$$\phi_k = \sum_{k'=1}^{k_n} M_{kk'}^{-1} \sum_{l,m} \sigma_{lm}^{-2} Q_{k'lm}(\alpha_{lm} - 1) \qquad (9)$$

in which M^{-1} is the inverse of the matrix:

$$M_{kk'} = \sum_{l,m} \sigma_{lm}^{-2} Q_{klm} Q_{k'lm} + \frac{4k(2k+1)}{4k+1} \lambda \delta_{kk'} \qquad (10)$$

The standard error in $\epsilon(\mu)$ is estimated, assuming independent data errors, to be:

$$\delta(\mu) = \left[\sum_{j,k} \sum_{j',k'} M_{jj'}^{-1} M_{kk'}^{-1} \times \sum_{l,m} \sigma_{lm}^{-2} Q_{j'lm} Q_{k'lm} P_j(\mu) P_k(\mu)\right]^{\frac{1}{2}}. \qquad (11)$$

Figure 2 shows the resulting inversion following this procedure.

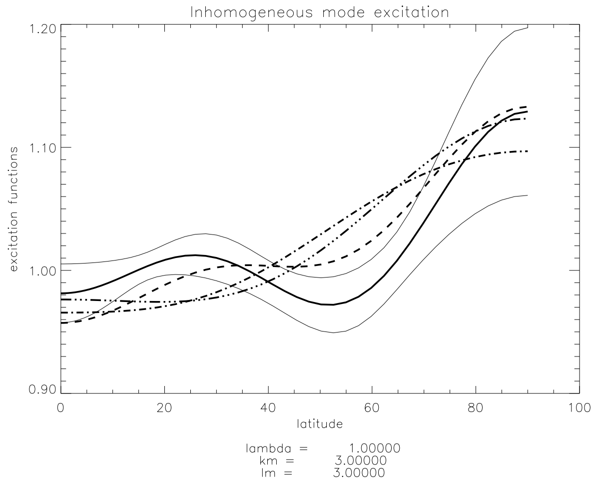


Figure 2. Excitation functions as a function of latitude for various years: 27 March 1996-26 March 1997 (continuous line); 27 March 1997-26 March 1998 (dashed line); 27 March 1998-26 March 1999 (dash-dot line); 27 March 1999-26 March 2000 (dash-three-dot line). For the first time series, the 1-σ error bars are indicated as thinner lines. The error bars for the other years are of similar magnitude.

4. CONCLUSION

The forms of the excitation functions illustrated in the figure are not very sensitive to the value of the regularization parameter λ employed, except that for very large values the functions become flatter. The functions do not exhibit local maxima at the latitudes of active regions, as did the function presented previously by Fröhlich et al. (1997). We suspect that the earlier relatively short data set was too heavily influenced by realization noise. In addition at the time, the data for $l < 4$ were fitted using power spectra and not Fourier spectra, which can influence the amplitude ratios. It is perhaps pertinent to remark that if we include modes of higher degree (fitted using power spectra up to $l = 7$) in our analysis, the inference becomes similar to that reported previously, and is not strongly dependent on the maximum value of l. However, these higher-degree modes cannot be isolated so well from the others in the signal, and therefore are more heavily contaminated by interference.

Besides the fact that the observed distribution is likely a manifestation of instrumental calibration errors, one can still identify possible changes with the increasing solar activity. The weakening of the excitation minimum at ≈ 55 degree, observed during solar minimum, might be due to the increase of activity. The same remark applies to the less significant decrease of the maximum at ≈ 30 degree disappering with increasing solar activity. These trends need obviously more investigation, but may be in line with the observed decrease of the amplitudes with increasing solar cycle, as noted in the present data-set and stated earlier by e.g. Bogdan et al. (1993).

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