Amplitudes of solar p modes:

a refined modeling of the eddy time-correlation function

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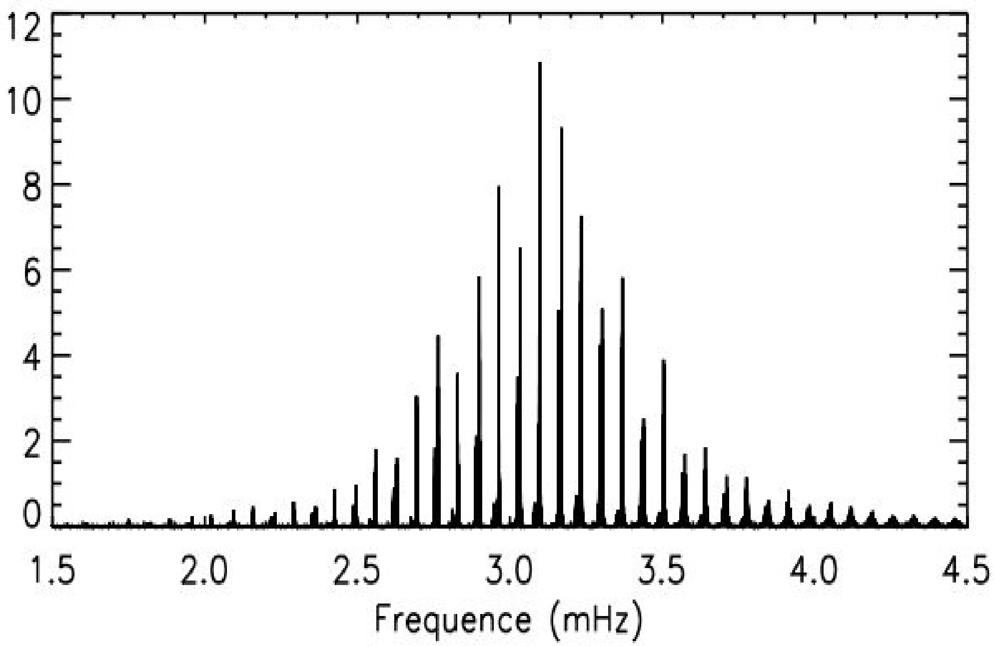
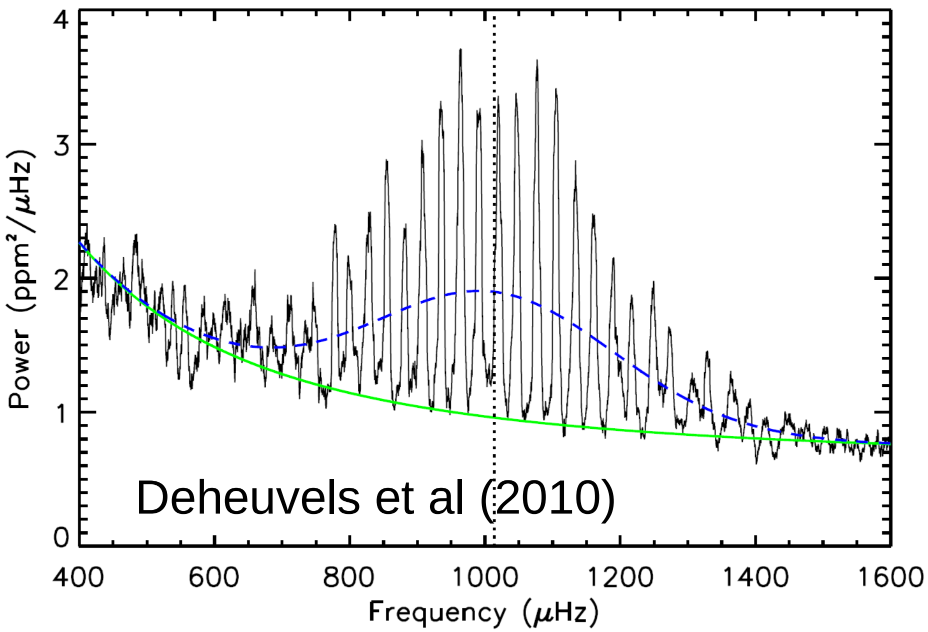
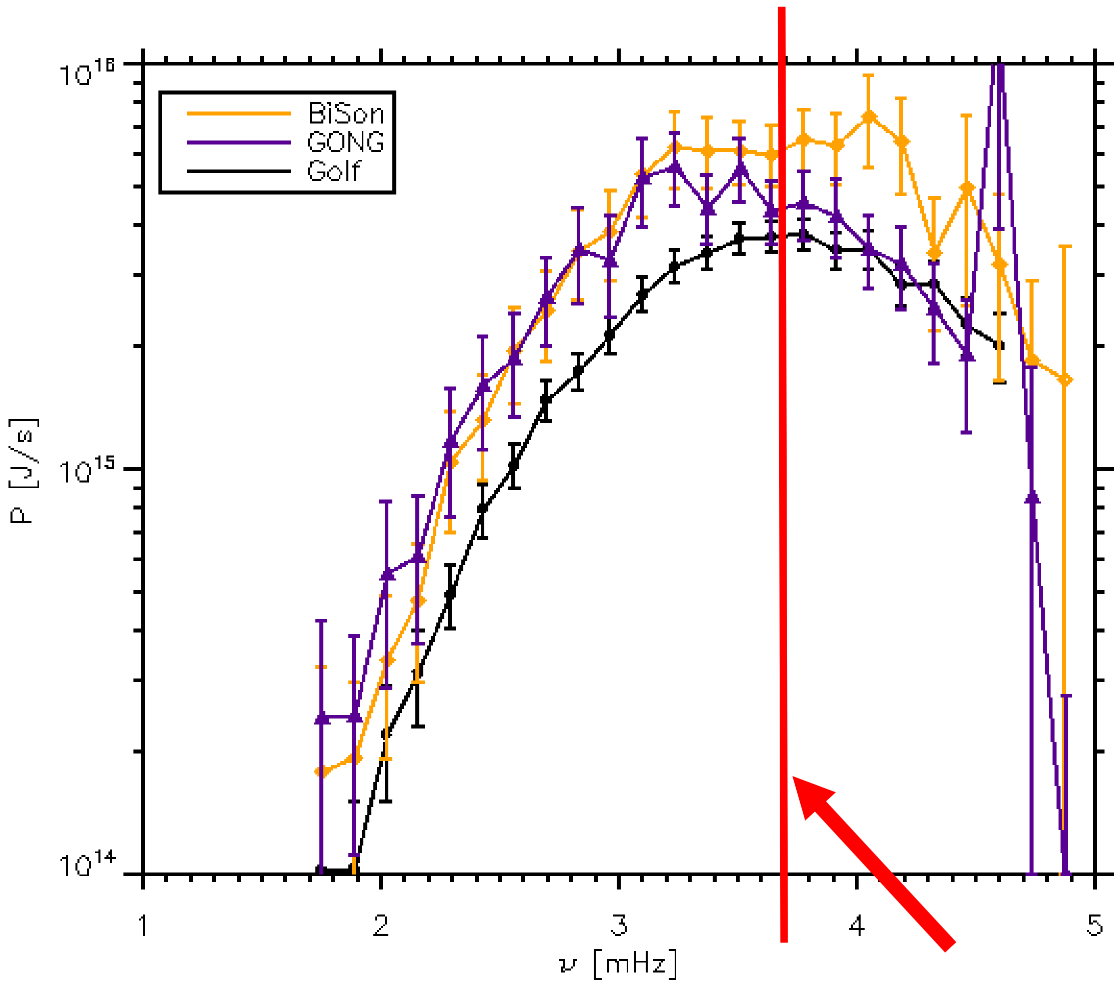
<u>The Sun</u>



<u>HD 49385</u> (CoRoT)



Solar modes: energy supply rate



~ 5 minutes

(Baudin et al 2005)

$$P = 2\ \pi^2\ \ M\ \ \Gamma\left(\Gamma\ \ H\right)$$

H: mode height

Γ: mode line-width

M: mode mass (~ mode inertia)

At high frequency:

P decreases because of a less efficient driving

At low frequency :

P increase with freq. because mode inertia decreases.

Discrepancies between the different instruments are within 1-2 sigma.

Forced oscillators

(Goldreich & Keely (77)'s approach)

We solve an ***inhomogeneous*** wave equation :

$$L(\delta r) + \omega_0^2 \delta r - D(\delta r) = S$$

L : linear operator

S : source terms

damping

$$\boxed{\delta r = A(t) \vec{\xi}(\vec{r}, t)}$$

A(t) : instantaneous amplitude

$\vec{f}_t$ Reynolds Tensor

\+

$\frac{\partial}{\partial t} \vec{\nabla} h_t$ entropy source term

General solution

$$A \propto \int \int dt' d^3x \ \vec{S} \cdot \vec{\xi} \ e^{i \omega_0 t'}$$

ω_0 : eigenfrequency

Source terms

eigenfunction

- Stochastic process : ➪ observational constraints provides the mean square: <**A**^{2}>
- The source function (S) can be modeled **statistically**

$$\langle A^2\rangle \propto \int dm \int d^3r \, d\tau \; \vec{\xi} \cdot \langle \vec{S}\vec{S}\rangle \cdot \vec{\xi} \; e^{i\omega_0 \tau}$$

τ : Time correlation length

r : space correlation length

two-point correlation products (space and **time**)

Theoretical expression

For radial modes and Reynolds stress only (Samadi & Goupil 2001):

$$P(\omega) \propto \eta \langle A^2\rangle \propto \int dr \;\; 4\pi \;\; r^2 \left(\frac{d\xi_r}{dr}\right)^2 \; S_R(\omega_{0,} r)$$

$S_R(\omega_{0,} r)$ the source function

Theoretical expression

The source function : $$S_R(\omega_{0,} r) = \int dk\, d\omega\, \frac{E^2(k)}{k^2} \chi_k(\omega_0+\omega)\chi_k(\omega)$$

- kinetic energy spectrum : $E(k)$
- mode compressibility : $d\xi_r/dr$

➪ controls the excitation **strength**

- **frequency spectrum** : $\chi_k(\omega)$

➪ controls the excitation **efficiency**

$\chi_k(\omega)$ is the Fourier transform of the Eulerian velocity time-correlation: $\langle u_k(t+\tau)u_k(t)\rangle$

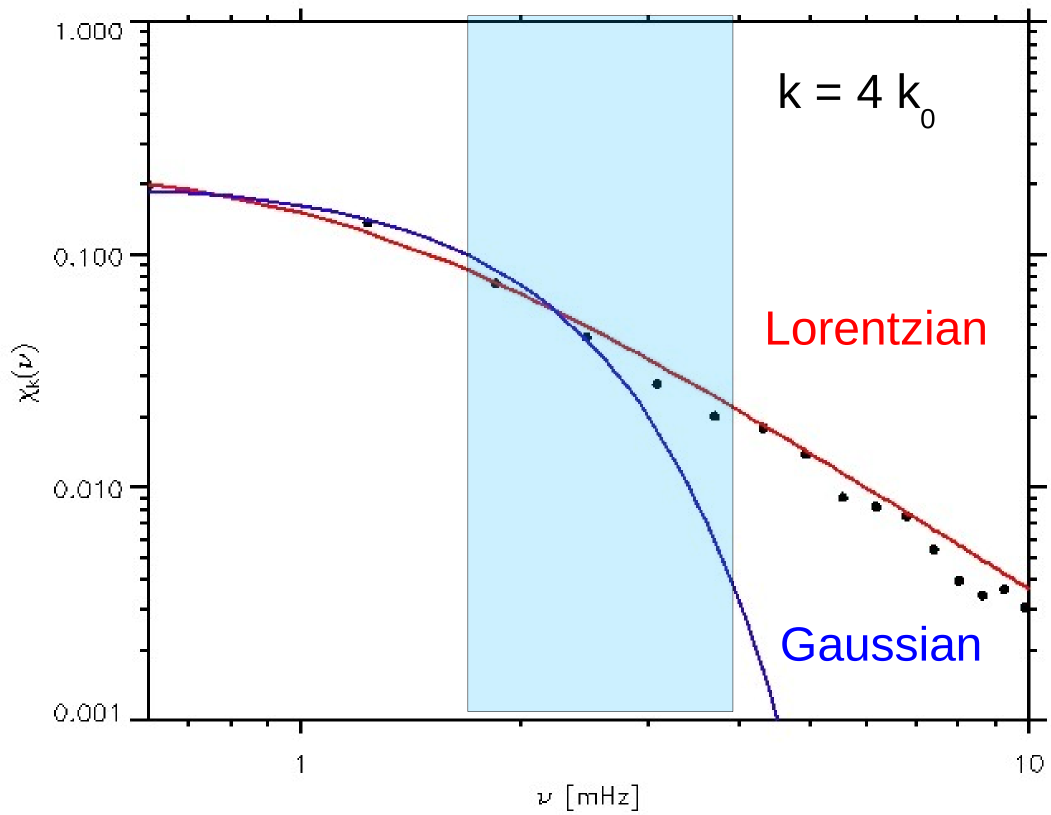
Goldreich & Keelly (1977), Bamlforth (1992), GMK (1994), Houdek (1999), Chaplin et al (2005) ➪ **Gaussian** $\chi_k(\omega)$

Gaussian $\chi_k(\omega)$?

- $\chi_k(\omega)$ computed with Stein & Nordlund (1998)'s code.
- Gaussian function underestimates $\chi_k(\omega)$ above ~1.5 mHz

- Lorentzian frequency spectrum at large scale : predicted by Gough (1977)'s time-dependent Mixing-Length theory

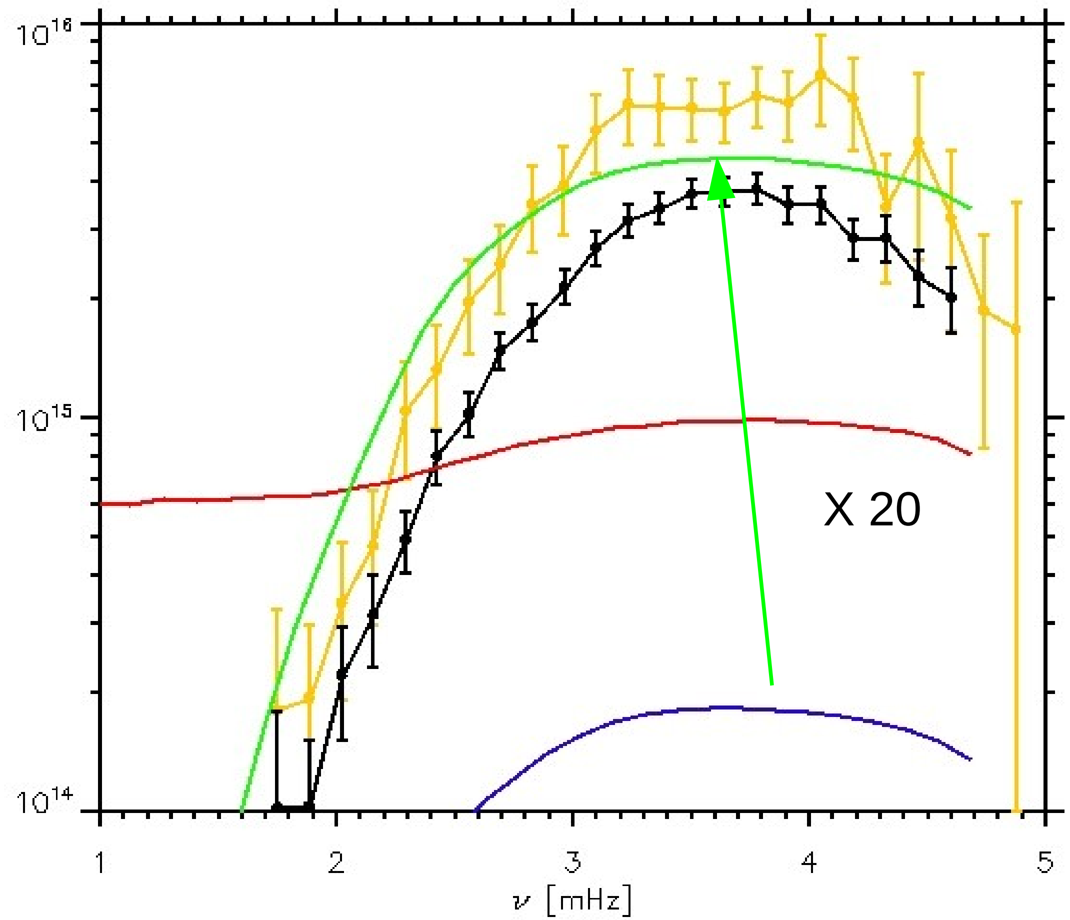
Frequency spectrum $\chi_k(\omega)$



Samadi et al (2003)

(50 mn, 25 km,253x253x163)

Comparison with the observations



Gaussian $\chi_k(\omega)$

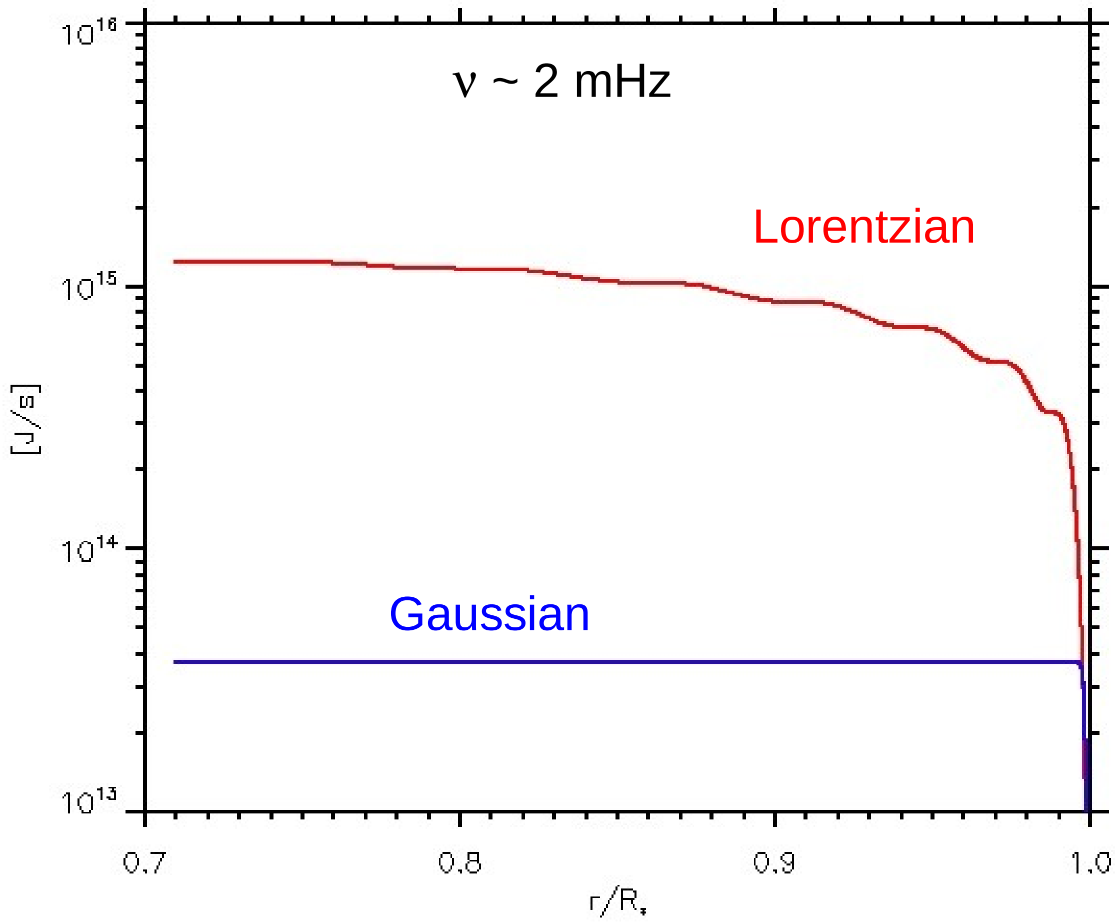
Lorentzian $\chi_k(\omega)$

blue x 20

Calculation as in Chaplin et al (2005) :

Kolmogorov spectrum, no entropy contribution

Extend of the driving region



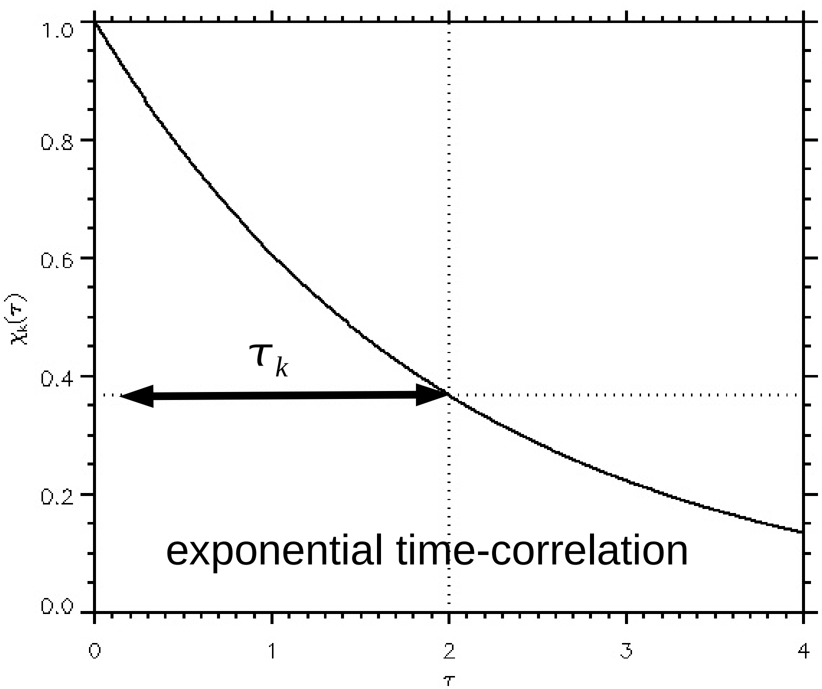
➔ Cumulated mode excitation rates (from the surface down to the bottom of the CZ)

➔ A substantial part of the driving comes from the inner part of the CZ

➔ There is something wrong with the Lorentzian $\chi_k(\omega)$

Lorentzian $\chi_k(\omega) \Leftrightarrow$ exponential time-correlation $\chi_k(\tau)$

$$\chi_k(\omega) = \frac{2}{\pi\,\omega_k}\;\frac{1}{1+(2\,\omega/\omega_k)^2} \;\Leftrightarrow\; \chi_k(\tau) = \exp\left[-\left|\frac{\tau}{\tau_k}\right|\right]$$ with $\tau_k = \frac{2}{\omega_k}$ (integral time-scale)



The **Langevin equation**:

a prototypical stochastic process (see Pope (1994)'s review)

$$\Delta u_k = \underbrace{-u_k(t)\,\frac{\Delta t}{\tau_k}}_{\text{Viscous damping}} + \underbrace{\sqrt{2\,u_0^2\,\frac{\Delta t}{\tau_k}}\;\xi}_{\text{Random acceleration}}$$

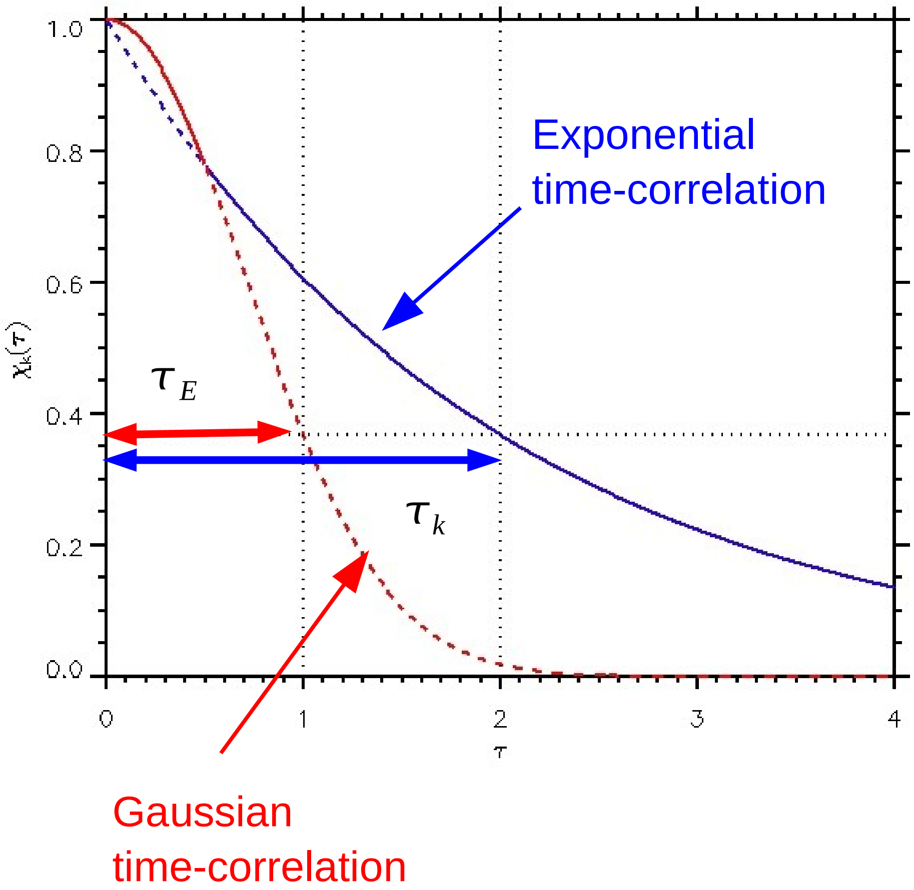
Random variable

$$\langle u_k(t+\tau)\,u_k(t)\rangle = u_0^2\;\exp\left[-\left|\frac{\tau}{\tau_k}\right|\right]$$

➔Langevin stochastic model: results in an exponential time-correlation

➔But derivative not continuous at $\tau = 0$ (the time-correlation is symmetric w.r.t τ=0)

The Eulerian time-scale



Two time-scales

Integral time scale (τ_k):

➔ time-scale of turbulent loss of coherence, or memory time-scale of turbulence

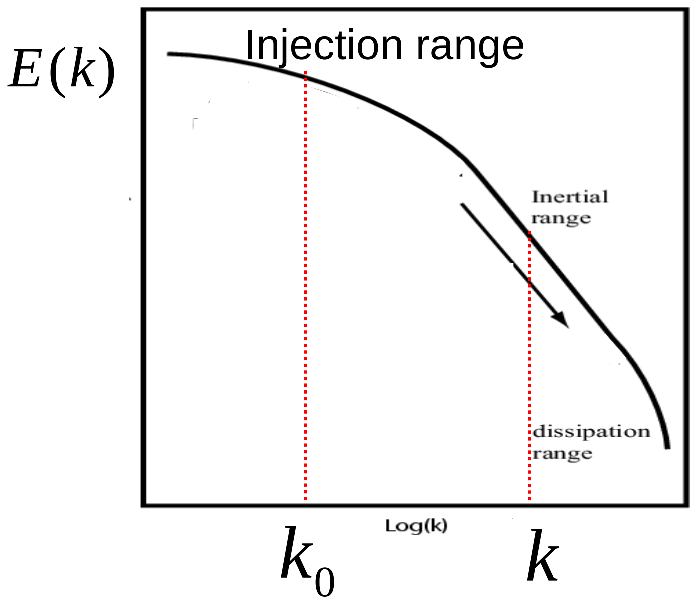
Eulerian time micro-scale (τ_E):

➔ Controls the curvature of $\chi_k(\tau)$ at $\tau = 0$

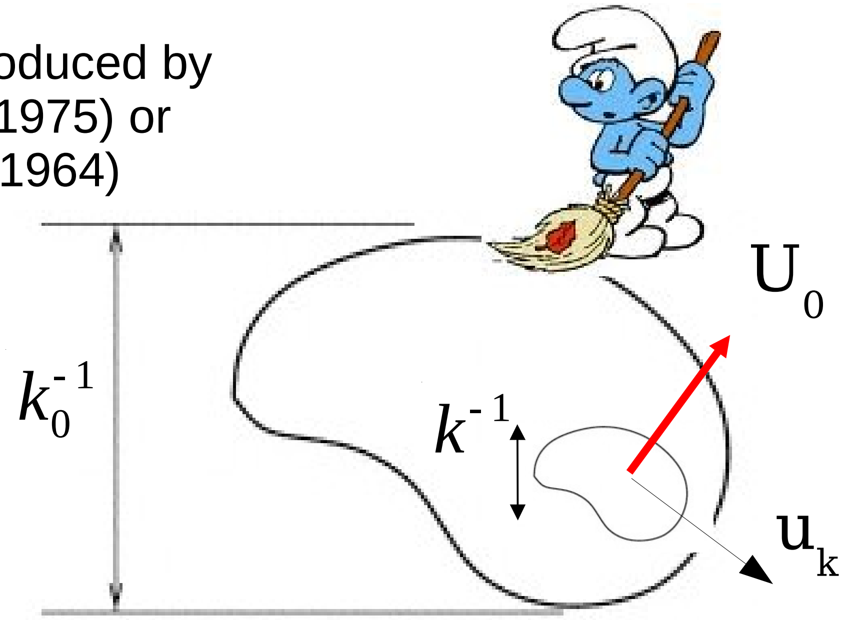
➔ Time-equivalent of the Taylor micro-scale (largest scale at which viscosity affects the dynamics)

The sweeping hypothesis

<u>Sweeping</u>: *to clean a floor by using a brush to collect the dirt into one place from which it can be removed*



➔ First introduced by Tennekes (1975) or Kraichnan (1964)



- At the scale k, an observer will see the sub-inertial eddies advected by energy bearing eddies of velocity U_0
- Equivalent of the Taylor's hypothesis (frozen turbulence approximation)
- Equivalent to a Doppler shift

Short-time analysis

Short-time analysis (see e.g. Kaneda 1993) : $\chi_k(\tau) = \chi_k(\tau=0)\left(1 - \frac{|\tau|}{\tau_k} + \frac{1}{2}\left(\frac{\tau}{\tau_E}\right)^2\right)$

first order term

second order term

The sweeping assumption gives: $\boxed{\tau_E = (k\ U_0)^{-1}}$

(Kraichnan 1964, Tennekes 1975, Kaneda 1993, Kaneda et al 1999, Rubinstein & Zhou 2002)

$\boxed{\text{We always have: } \tau_E < \tau_k}$

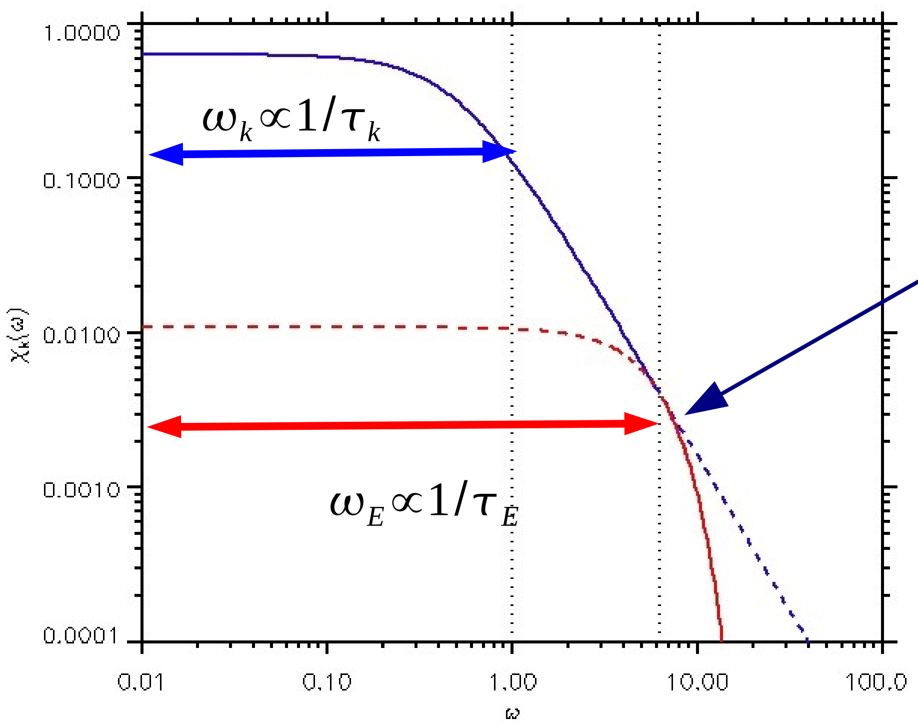
- For $\tau > \tau_E$ (*low frequency*): the first order term dominates

  $\Rightarrow \chi_k(\tau)$ is a **Exponential** $\Rightarrow$ **Lorenztian** $\chi_k(\omega)$
- For $\tau < \tau_E$ (*high frequency*): the second order term dominates

  $\Rightarrow \chi_k(\tau)$ is a **Gaussian** $\Rightarrow$ **Gaussian** $\chi_k(\omega)$

Frequency spectrum

- For $\omega < \omega_E \Rightarrow$ **Lorenztian** $\chi_k(\omega)$
- For $\omega > \omega_E \Rightarrow$ **Gaussian** $\chi_k(\omega)$



Here: $\omega_E = 2\ \omega_k$

Rapid decrease above ω_E

The sweeping frequency (ω_E) acts as a cutoff frequency

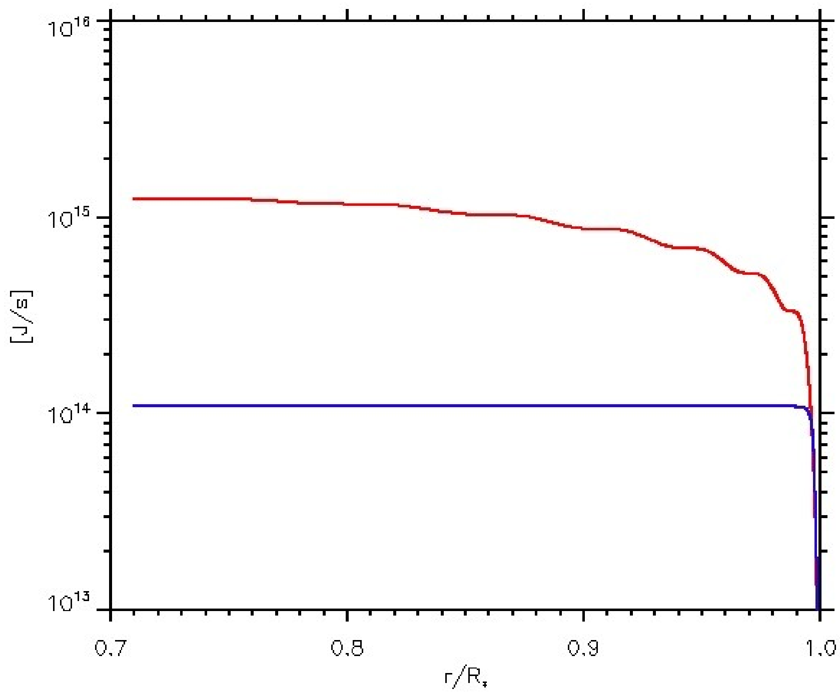
$\Rightarrow$ We impose: $\chi_k(\omega) = 0$ for $\omega > \omega_E$

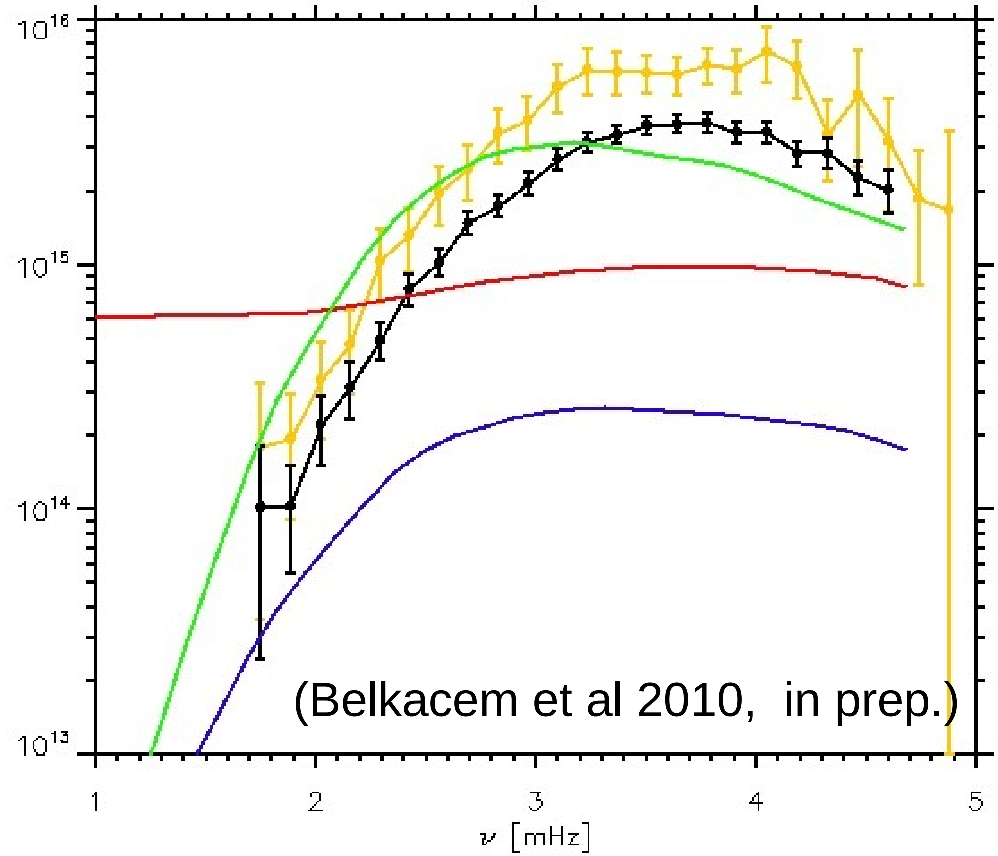
Application to the solar p modes

➔ Low frequency modes are **no longer over-estimated**

➔ But: **systematic under-estimation**

➔ **Better agreement** when we derive E(k) and $\chi_k(\omega)$ from a 3D hydro. simulation (Samadi et al 2003, Belkacem et al 2003, Samadi 2009)





Pure Lorentzian $\chi_k(\omega)$

Lorentzian + Sweeping $\chi_k(\omega)$

Full 3D hydro calculation

Conclusion

- **Resolve the conflict** between Gaussian $\chi_k(\omega)$ and Lorentzian $\chi_k(\omega)$
- **Seismic validation** of the sweeping hypothesis in a stellar context
- **Still important discrepancies** at high frequency between the observations and theoretical calculations based on 3D data

To go further

- Determination of the **Eulerian time micro-scale in the injection region**
- Inclusion of a **realistic non adiabatic treatment** of the eigenfunctions
- More realistic modelling of the **entropy source term** (avoiding the passive scalar assumption)

Sweeping approximation

At small scales, the eddy dynamic is governed by the advection by the largest scales (see Kraichnan 1964):

$$\frac{\partial}{\partial t}\vec{u}(k,t)+\vec{U}_0.\nabla\vec{u}(k,t)=\vec{0}$$

General solution:

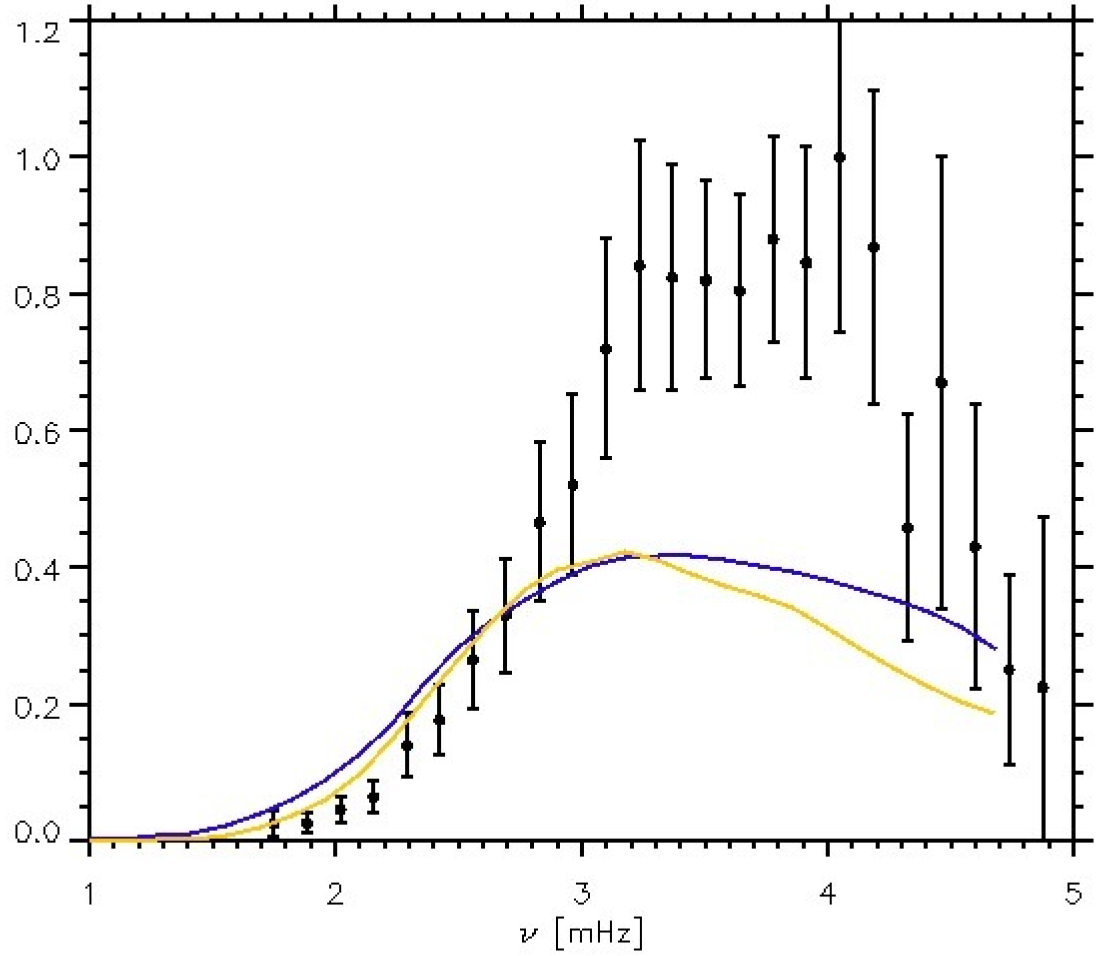
$$\vec{u}(k,t)=e^{i\ \vec{k}.\vec{U}_0\ t}\ \vec{u}(k,0)$$

Normalised eddy time-correlation:

$$R(\tau)=\frac{\langle u(k,t+\tau)u(k,t)\rangle}{\langle u(k,t)^2\rangle}=\langle e^{i\ \vec{k}.\vec{U}_0\ \tau}\rangle=\exp\left[-\frac{1}{2}\ U_0^2\ k^2\ \tau^2\right]$$

We then derive the Eulerian time-scale:

$$R(\tau)=\exp\left[-\frac{1}{2}\ U_0^2\ k^2\ \tau^2\right]=\exp\left[-\frac{1}{2}\ \left(\frac{\tau}{\tau_E}\right)^2\right]\quad\Rightarrow\quad\tau_E=\left(k\ \ U_0\right)^{-1}$$

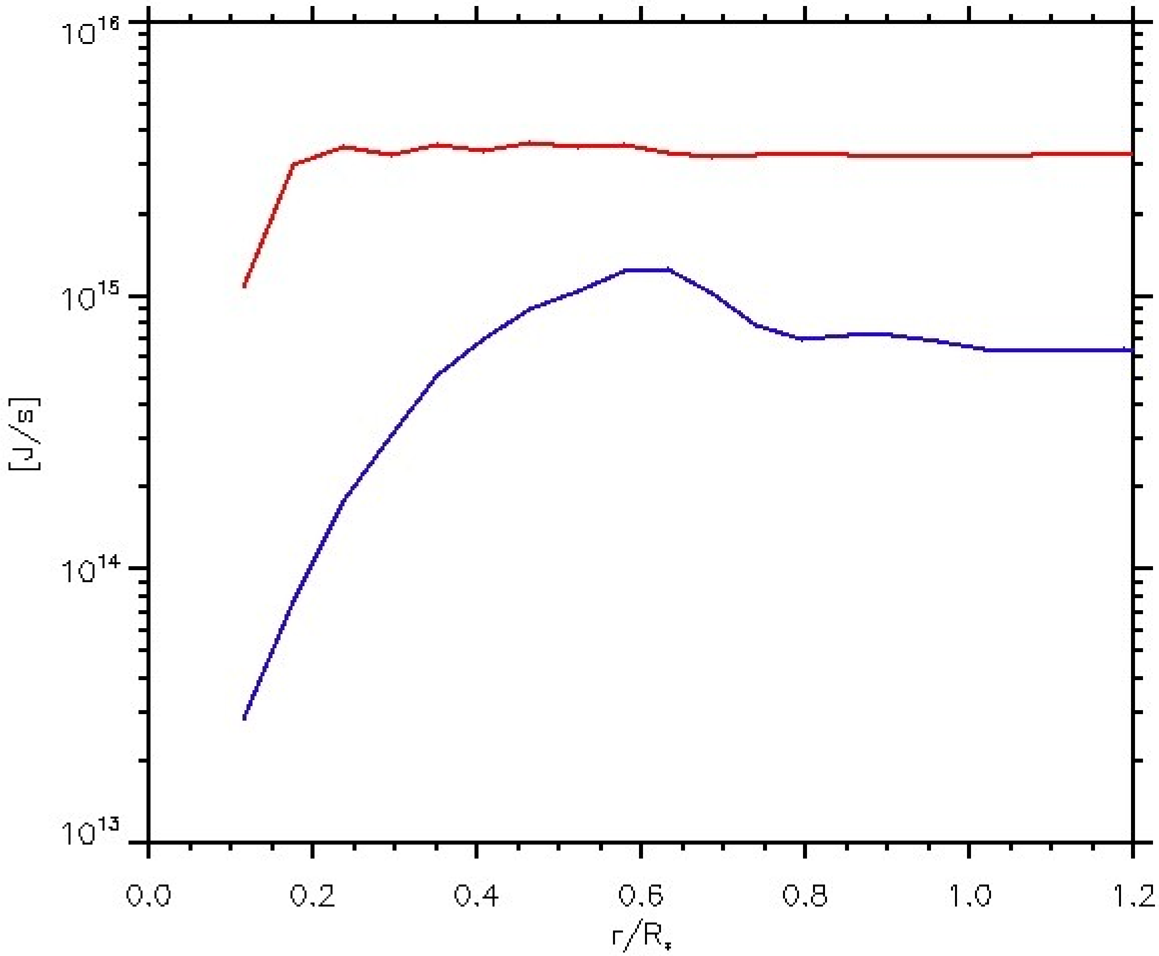


1.2
1.0
0.8
0.6
0.4
0.2
0.0
1
2
3
4
5
ν [mHz]

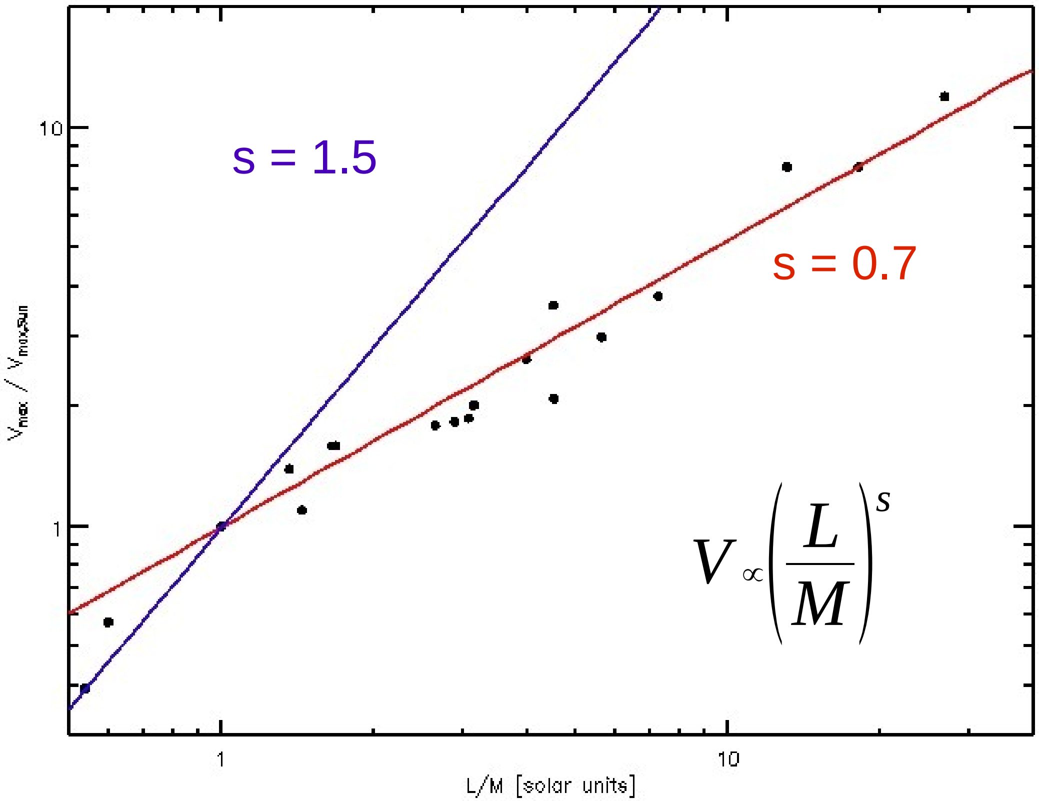
Pure Lorentzian $\chi_k(\omega)$

Lorentzian + Sweeping $\chi_k(\omega)$

HD 49385 (CoRoT)



Amplitude of Solar-like oscillations across the HR diagram (2/4)



- Observations => slope s ~ 0.7 – 0.8
- Kjeldsen & Bedding (1995) : s = 1.0
- Houdek et al (1999) : s = 1.5

- With s=1.5 oscillation amplitudes are severely over-estimated for hot stars

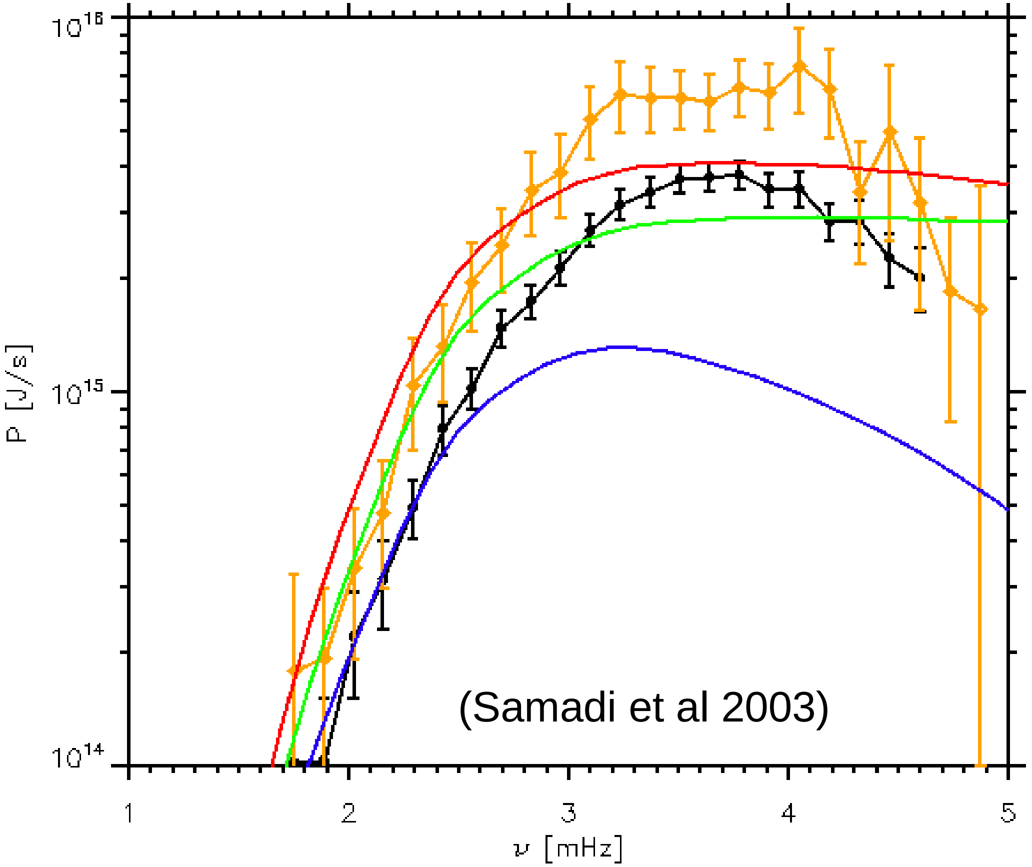
$$L \propto R^2 \quad T_{eff}^4$$

$$g \propto M / R^2$$

$$\frac{L}{M} \propto \frac{T_{eff}^4}{g}$$

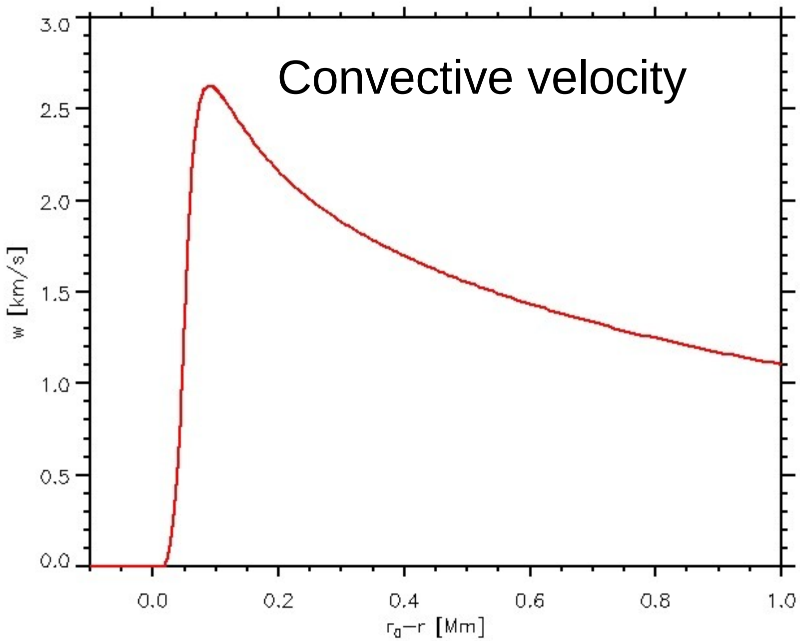
Application to the solar p modes

How far are we from the observations ?



Turbulent spectrum from a solar 3D simulation, Gaussian $\chi_k(\omega)$.

Relevant location



u

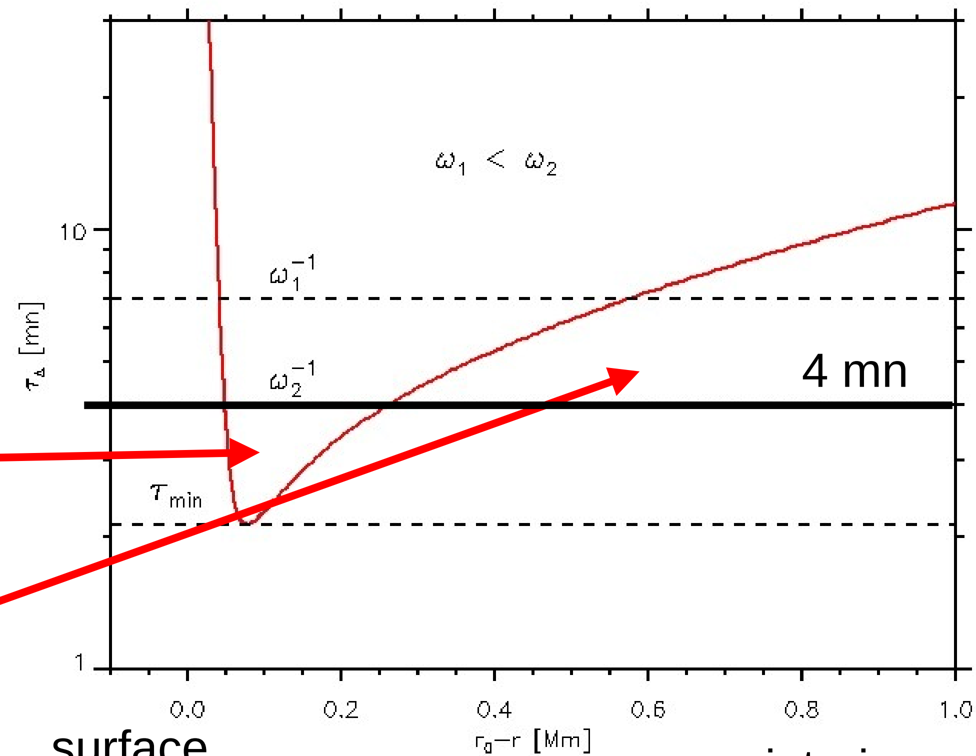
surface

interior

Upper part of the CZ :

largest u ➪ shortest τ

Eddy turn-over time: $\tau_\Lambda \approx \Lambda / u$



surface

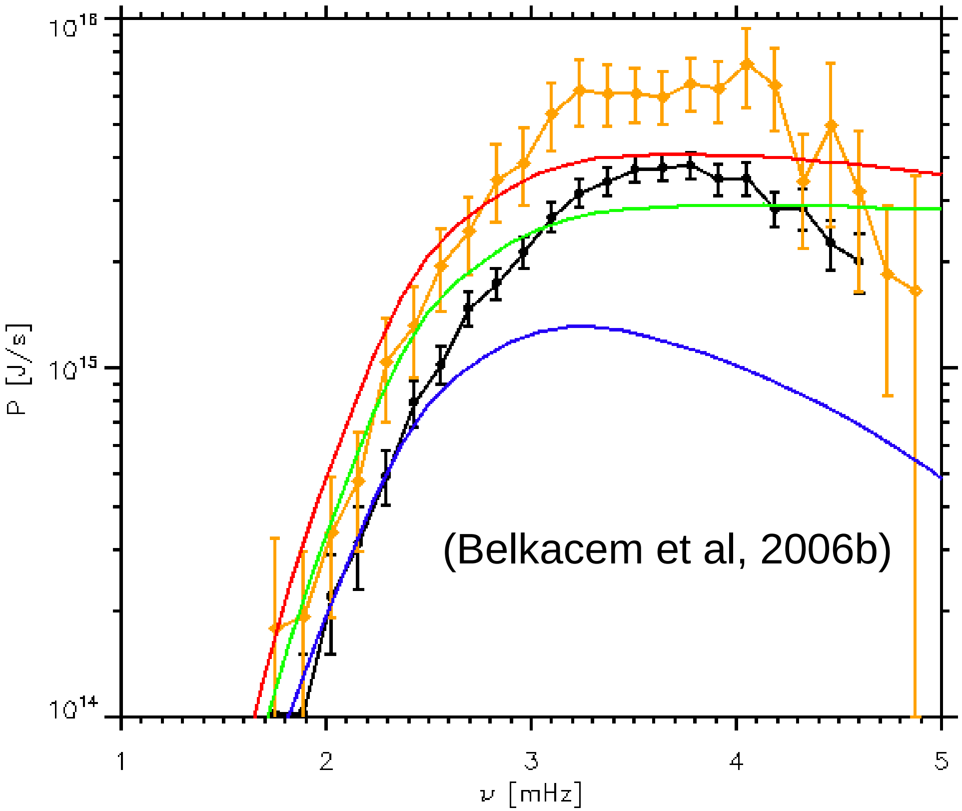
interior

$\tau_\Lambda < P_{osc}$ ➪ **efficient** excitation

$\tau_\Lambda > P_{osc}$ ➪ **inefficient** excitation

More recent results (4/5)

The Sun

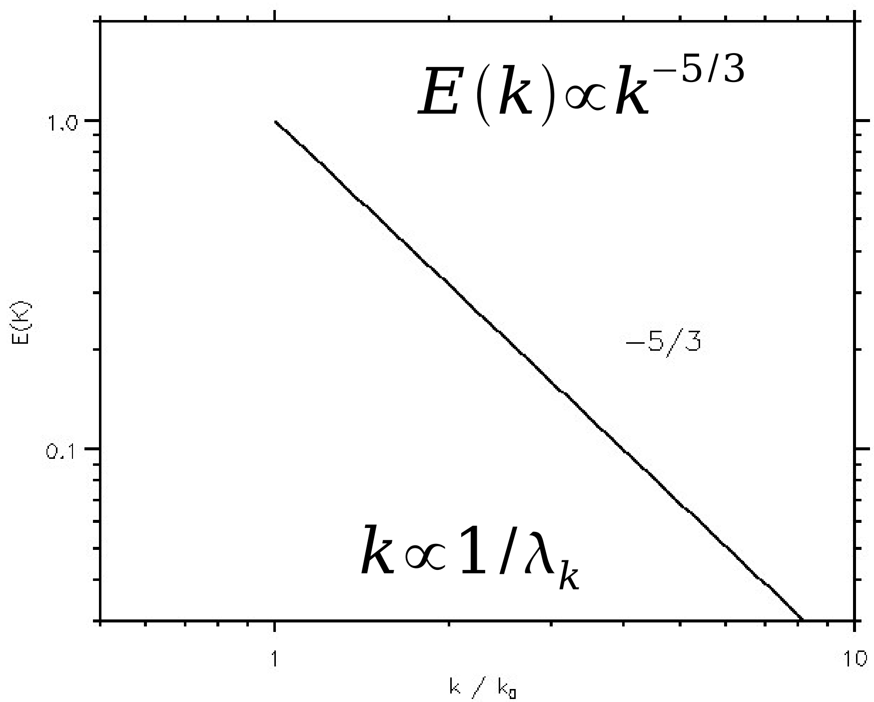


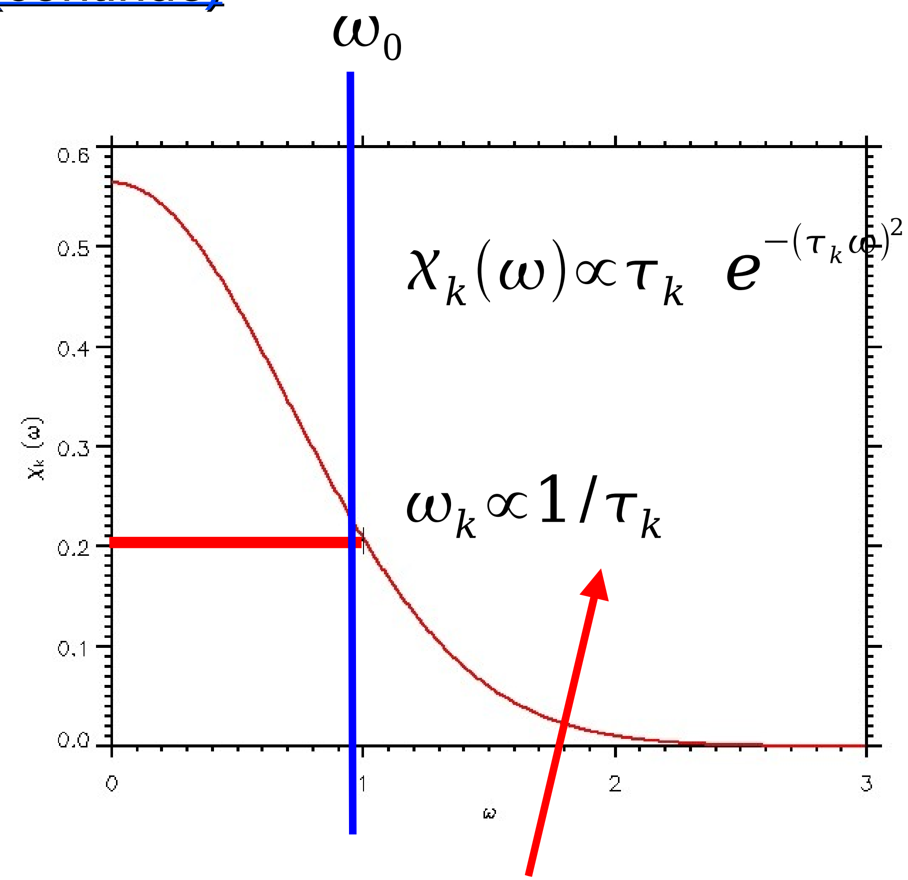
Closure model with plumes (CMP)

QNA

Gaussian $\chi_k(\omega)$.

Relevant quantities (continue)





At fixed mode frequency (ω_0) :

efficient excitation for $\omega_0 < \omega_k \;\Rightarrow\; \tau_k \omega_0 < 1$

Eddy turn-over time : $\tau_k \propto \lambda_k / u_k$

$$E(k) \propto k^{-5/3} \;\Rightarrow \tau_k \approx \tau_\Lambda \; \left(\lambda_k / \Lambda\right)^{2/3}$$

$$\tau_k \omega_0 < 1 \;\Rightarrow\; \lambda_k < \Lambda \left(P_{osc} / \tau_\Lambda\right)$$