

# The Rotation Rate and its Evolution Derived from Improved Mode Fitting and Inversion Methodology

S.G. Korzennik<sup>(1)</sup>, A. Eff-Darwich<sup>(2)</sup>

(1) Harvard-Smithsonian Center for Astrophysics  
Cambridge, MA, USA

(2) Instituto de Astrofísica de Canarias  
La Laguna, Tenerife, Spain

July 2010

## The Data: Some 13 Years of It

---

---

- GONG *pipe-line* (Anderson *et al*, 1990) 1995.06.29 -- 2009.09.07
  - ★ 147 GONG *months* (36 days),
  - ★ fitted in *overlapping* 108-day long segments,
  - ★ *individually & independently* fitted,
  - ★ no leakage matrix information, symmetric profile.
- MDI *pipe-line* (Schou, 1992) 1996.05.01 -- 2009.12.07
  - ★ 67 MDI non-overlapping epochs (72 days),
  - ★ polynomial expansion in  $m$ ,
  - ★ incl. leakage matrix information, symmetric profile.
- MDI Larson/Schou improvements
  - ★ spatial decomposition: plate scale and image distortion
  - ★ leakage matrix: distortion by differential rotation
  - ★ symmetric *and* asymmetric profile (62 & 57 epochs)

# YAOPBM

- SGK fit (Korzennik 2005, 2008)

- ★ Method

- \* simultaneous fit of indiv. modes w/ sanity rejection,
- \* incl. leakage matrix,
- \* asymmetric profile,
- \* *optimal* multi-tapered spectral estimator,
- \* use time-series of varying lengths.

- ★ *pipe-line* SHC time-series

1996.05.01 -- 2002.01.17

- \* 2088-day long time-series
- \* 728-, 364-, 182-day long, overlapping, time-series

- ★ *improved* SHC time-series

1996.05.01 -- 2009.07.16

- \* incl. distortion by diff. rotation
- \*  $64 \times 72$ ,  $32 \times 72$ ,  $16 \times 72$ -day long, overlapping, time-series

*Yet An Other Peak Bagging Method*

## The Problems: Foreword

- Inverse Theory

$$y_i = \int K_i x(p) dp$$

- ★ Inverse problems are singular,
- ★ require regularization to lift singularity (smoothness),
- ★ produce an *estimate* of the solution  $\hat{x} = x \otimes R$
- ★  $R$  resolution kernels – depend on the input set

- Solar Rotation

$$\delta\nu_{n,\ell,m} = \iint K_{n,\ell,m}(r, \theta) \Omega(r, \theta) dr d\theta$$

- ★ input set is defined by  $\{n, \ell, m\}$  or  $\{n, \ell, a_i\}$
- ★ temporal changes in the input set affect  $R$ , hence  $\hat{x}$

⇒ chose to invert a constant input set

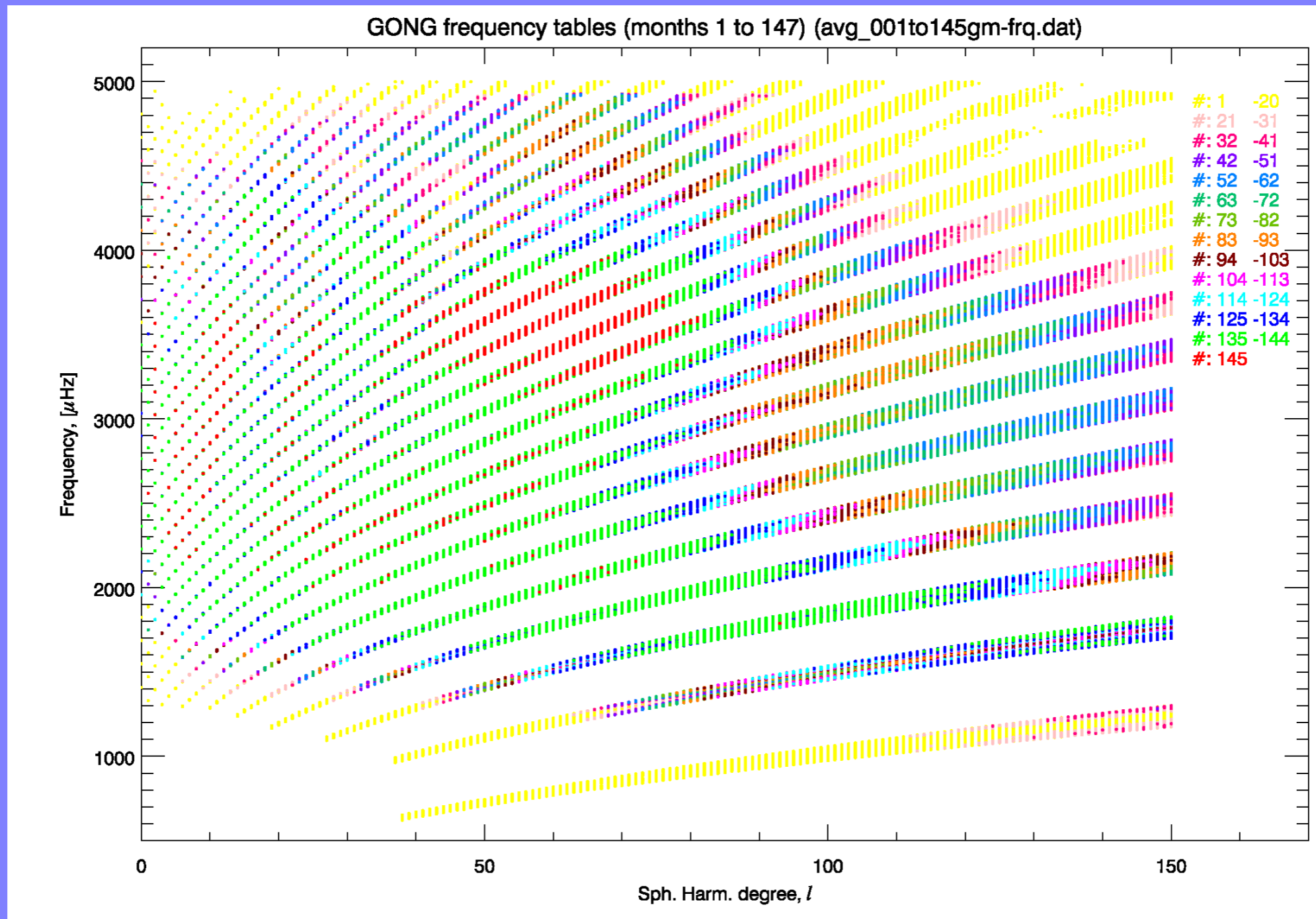
# GONG & MDI



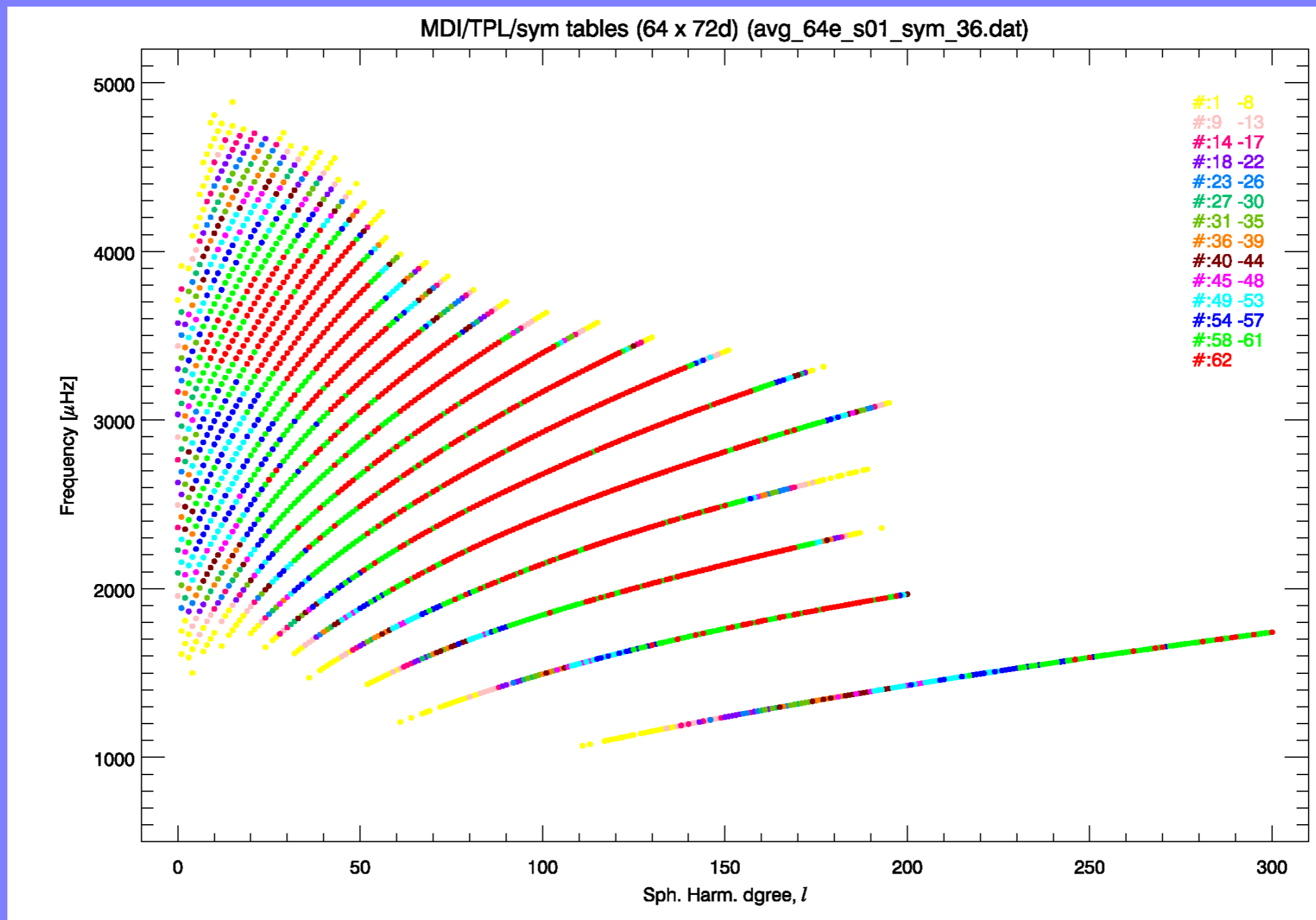
*qui aime bien, châtie bien*

- Frequencies or polynomial expansion?
  - ★ GONG *pipe-line*
    - \* Would like to use  $\nu$ , not  $a_i$
    - \*  $a_i$  computed from  $\nu_{n,\ell,m}$
  - ★ MDI *pipe-line*
    - \* Produces only  $a_i$ , independent of mode visibility
- Mode attrition when reducing to a unique input set
- Fill factor not constant with time
- Error bars estimate

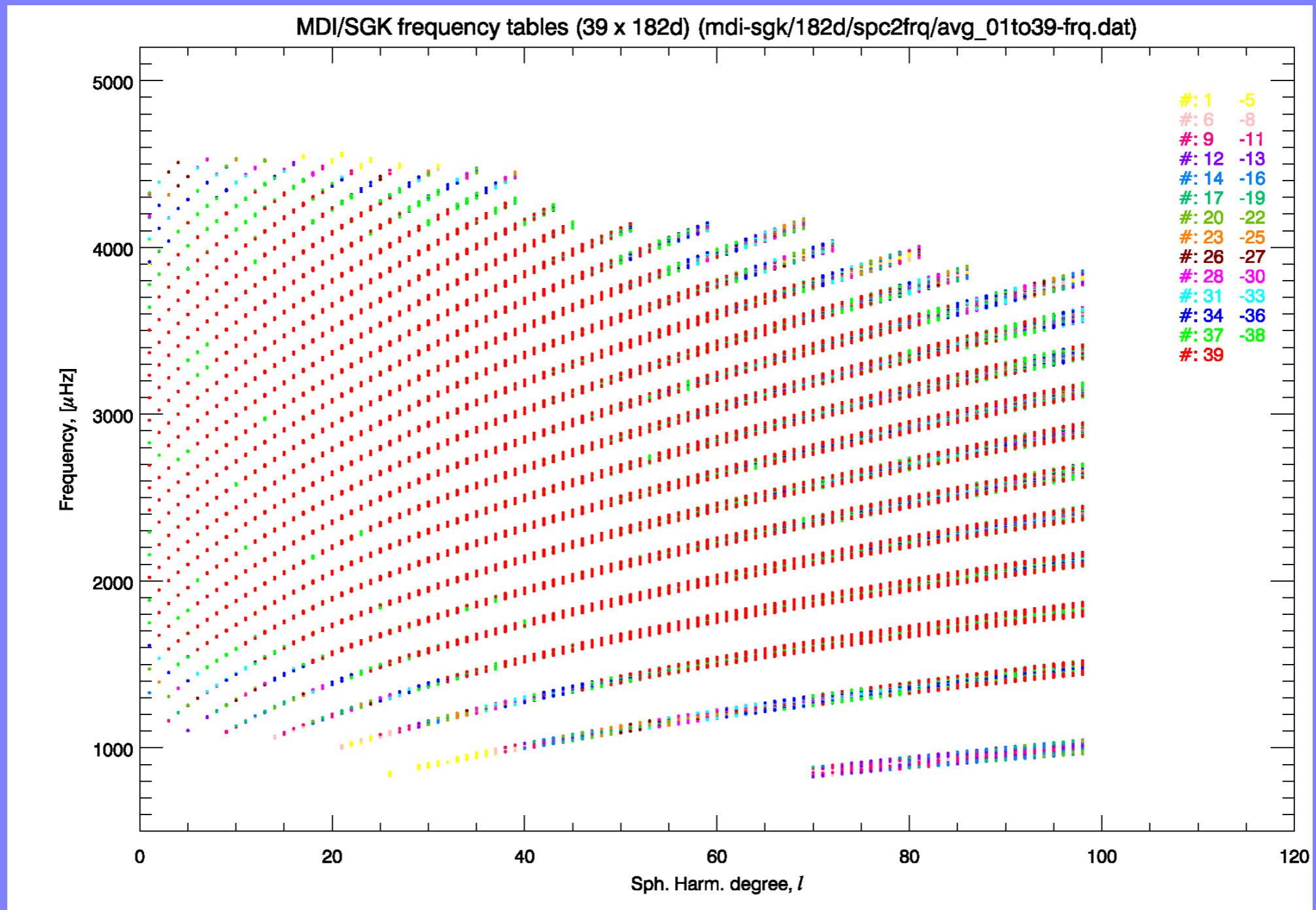
# Attrition – GONG Tables



# Attrition – MDI Tables



# SGK Fitting (*YAOPBM/MDI*) – Attrition





## More Problems

- Leakage matrix

- ★ closest leaks –  $\Delta\nu_{\delta m=2, \delta \ell=0}$  – are *rarely* resolved

- \*  $\Delta\nu \gg \Gamma$ ,  $\Delta\nu \simeq 2 \times \frac{\Omega}{2\pi} \simeq 0.8 \mu\text{Hz}$

- ★ plate scale, distortions,  $B_0$

- \* new MDI Sph. Harm. Coefs

- accounts for plate scale and image distortion

- \* distortion by differential rotation (Woodard 1989)

1 – 6% effect

- \*  $B_0 = B_0(t)$

3 – 15% effect

- \* other geometric variations negligible

⇒ very long time-series indicate remaining mismatch for f-mode

- Independent leakage computation

# Leakage Matrix – Refinements I

---

---

Leakage Matrix – Distortion by Differential Rotation



# Leakage Matrix – Refinements II

---

---

Leakage Matrix – Bo effect



# Leakage Matrix – f & p-modes

---

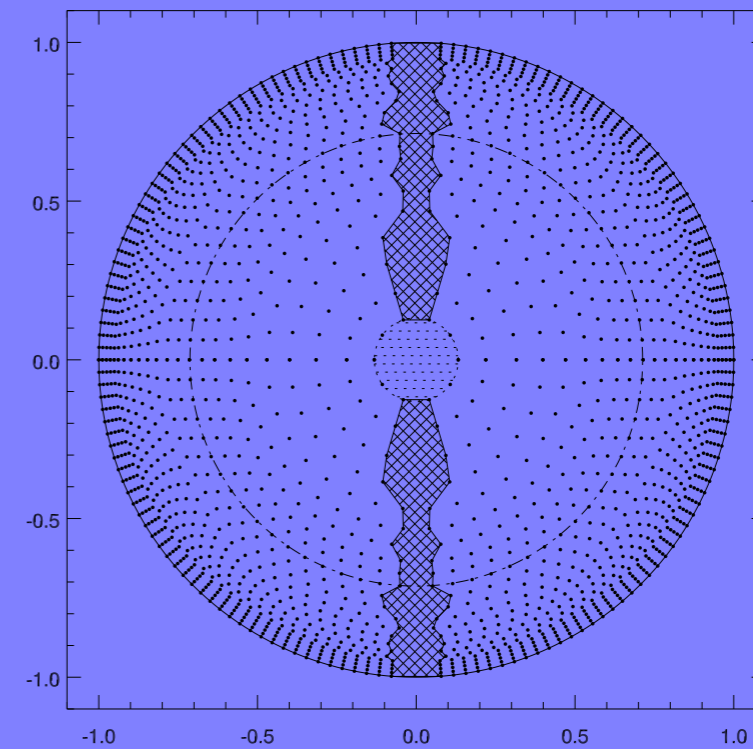
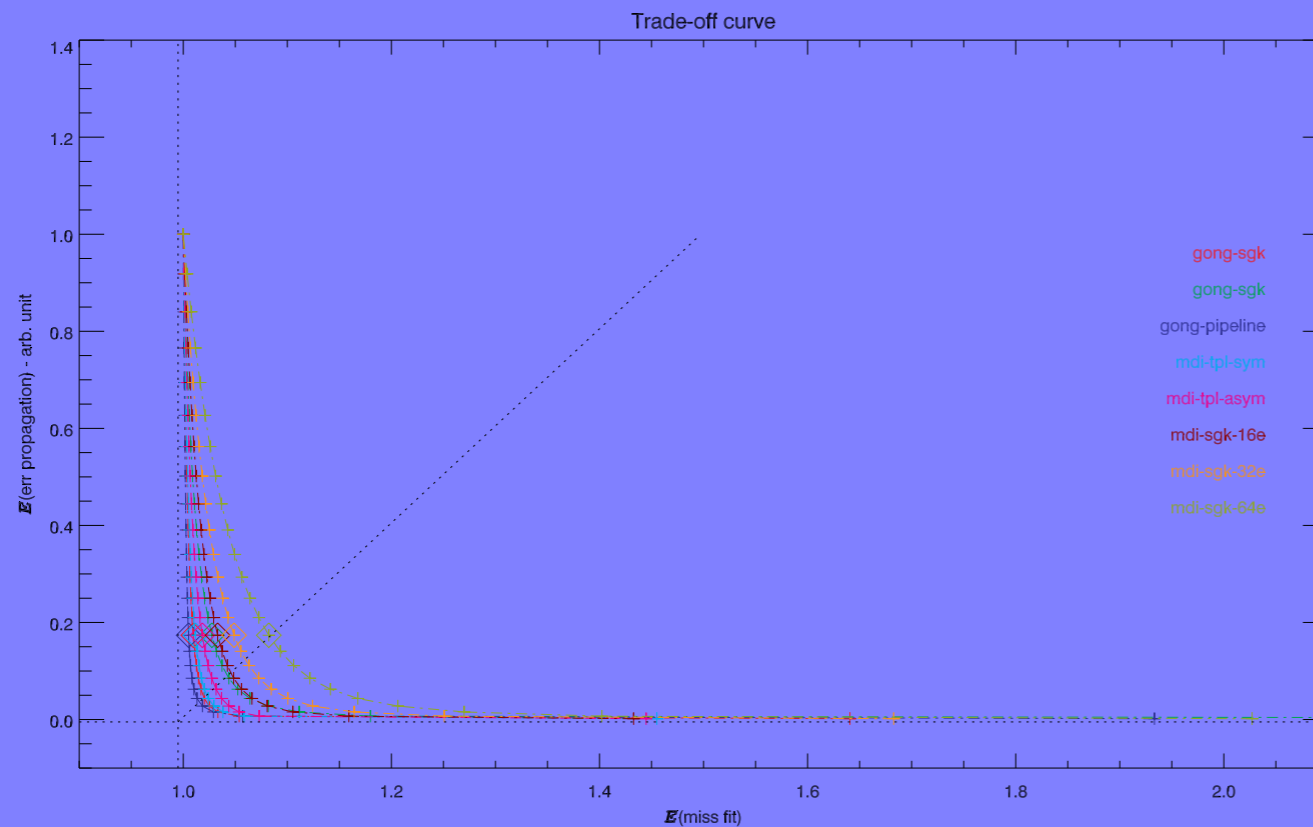
---

f, p<sub>1</sub>, p<sub>2</sub>, & p<sub>3</sub> modes,  $\ell = 20, (5), 200, 64 \times 72d$



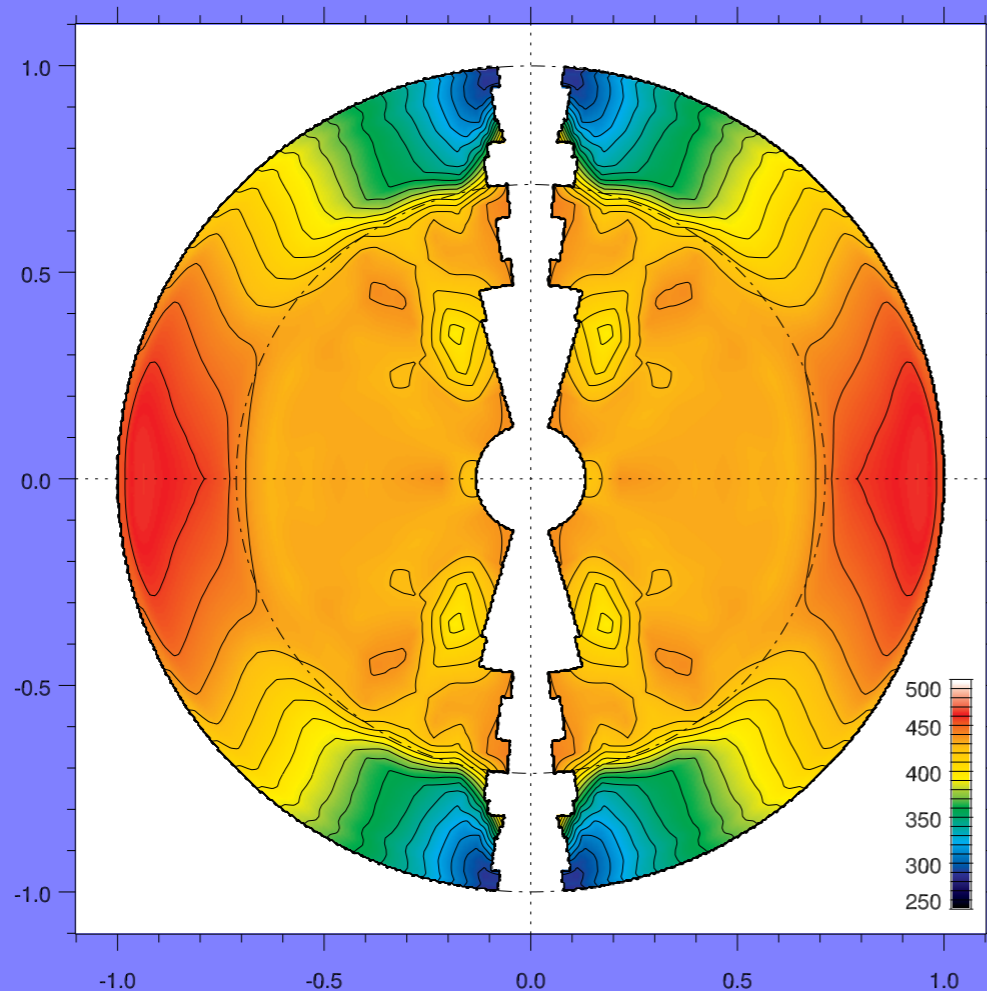
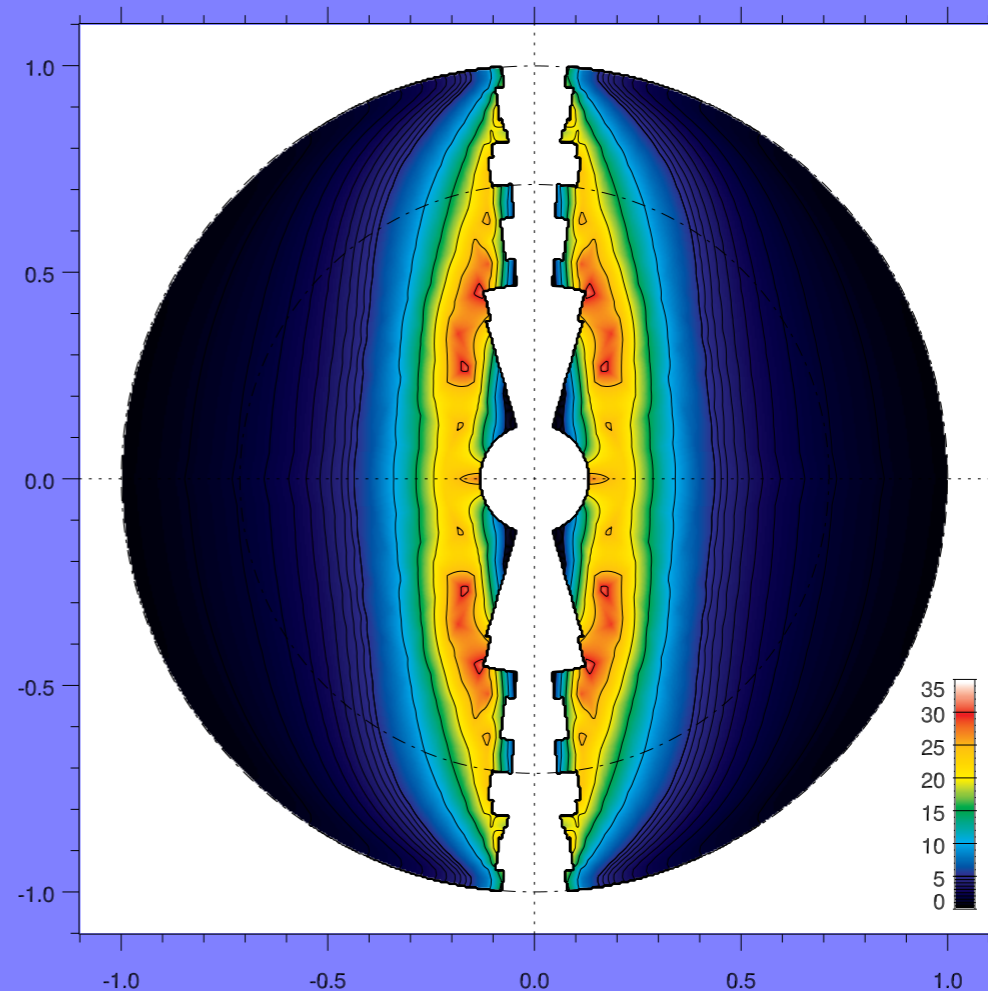
# The Inversion Methodology

- Modified RLS (Eff-Darwich & Korzennik, 2007)
  - ★ Iterative approach
  - ★ *optimal* model grid, based on input set
  - ★ *non-uniform* model grid



- $\Omega(r, \theta)$ ,  $64 \times 72d$ , change with trade-off

# Mean Rotation Rate & Precision – MDI $64 \times 72$ days (12.6 yr)

MDI/SGK  $64 \times 72$  d  $\Omega(\mathbf{r}, \mathbf{x})$ , set#10MDI/SGK  $64 \times 72$  d  $\sigma_{\Omega}(\mathbf{r}, \mathbf{x})$ , set#10

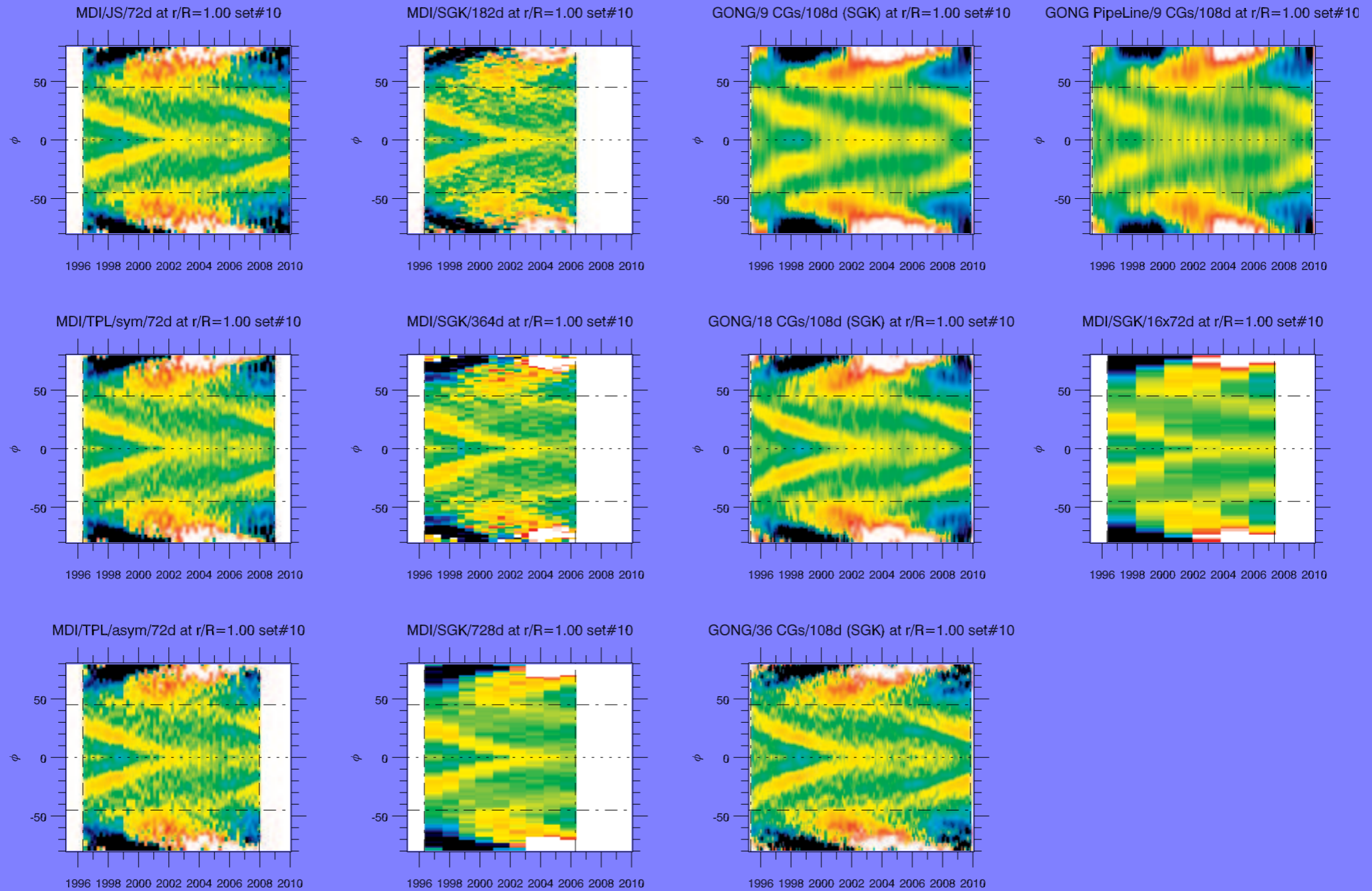
## Rotation Rate – Changes with time, $6 \times 16 \times 72d$

$\Omega(r, \theta)$ ,  $6 \times 16 \times 72d$ , change with time



- $\Omega(r, \theta)$ ,  $6 \times 16 \times 72d$ , change with time

# Rotation Rate Changes – $r/R = 1.00$





# Rotation Rate Changes – GONG $r/R = 1.00 - 0.47$

$\Omega(r, \theta)$ , GONG – change with time at various depths

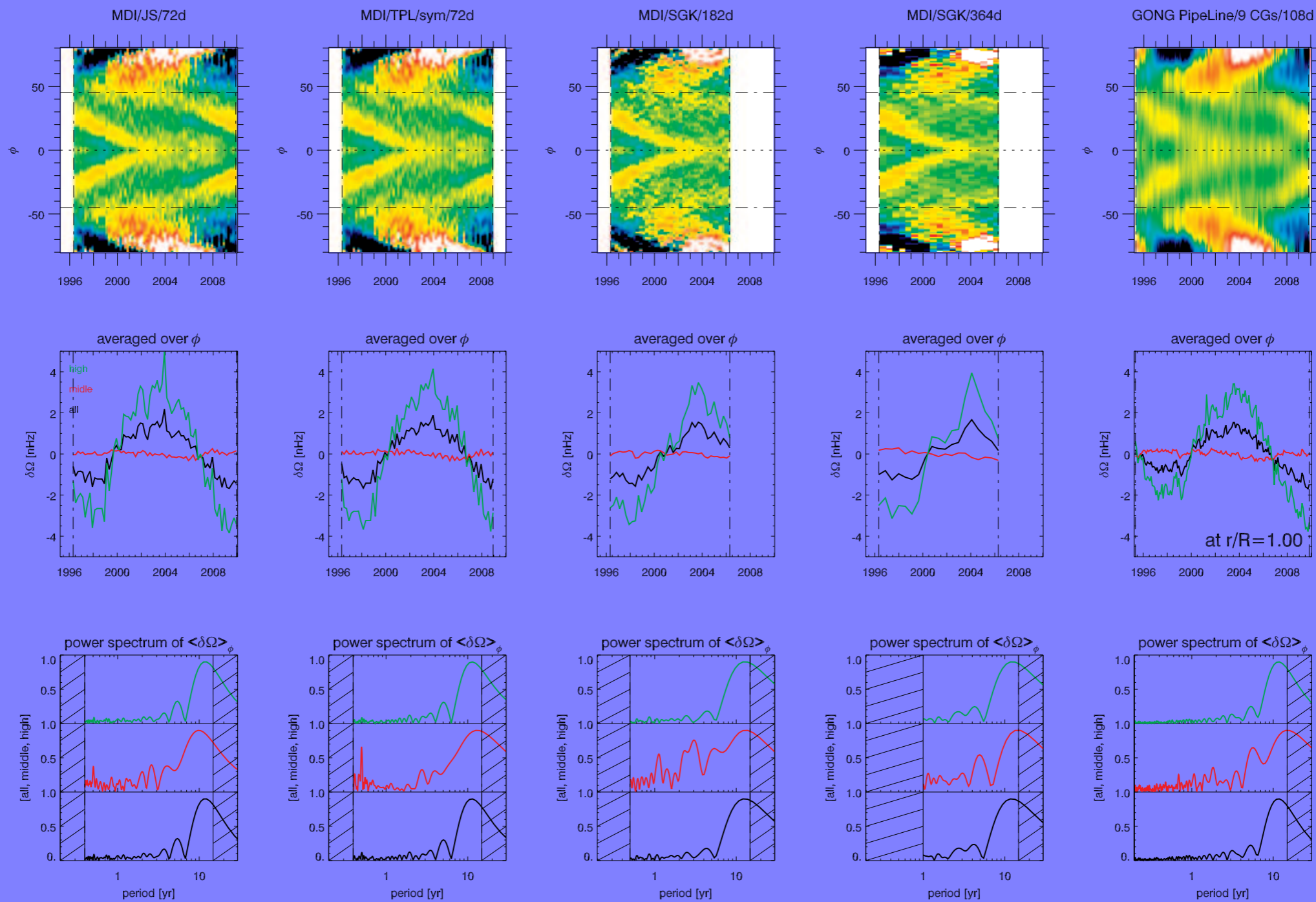


# Rotation Rate Changes – MDI $r/R = 1.00 - 0.47$

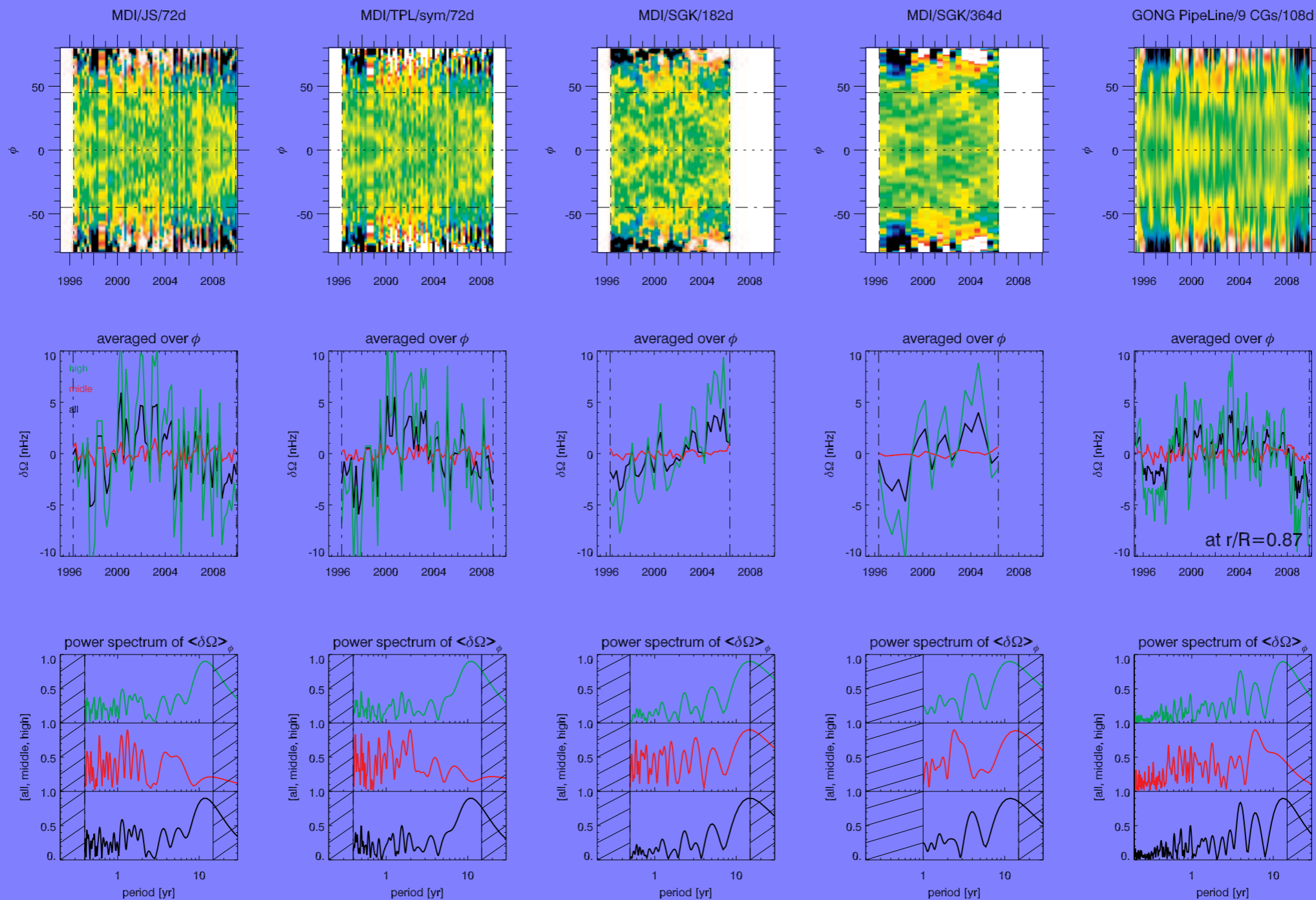
$\Omega(r, \theta)$ , MDI – change with time at various depths



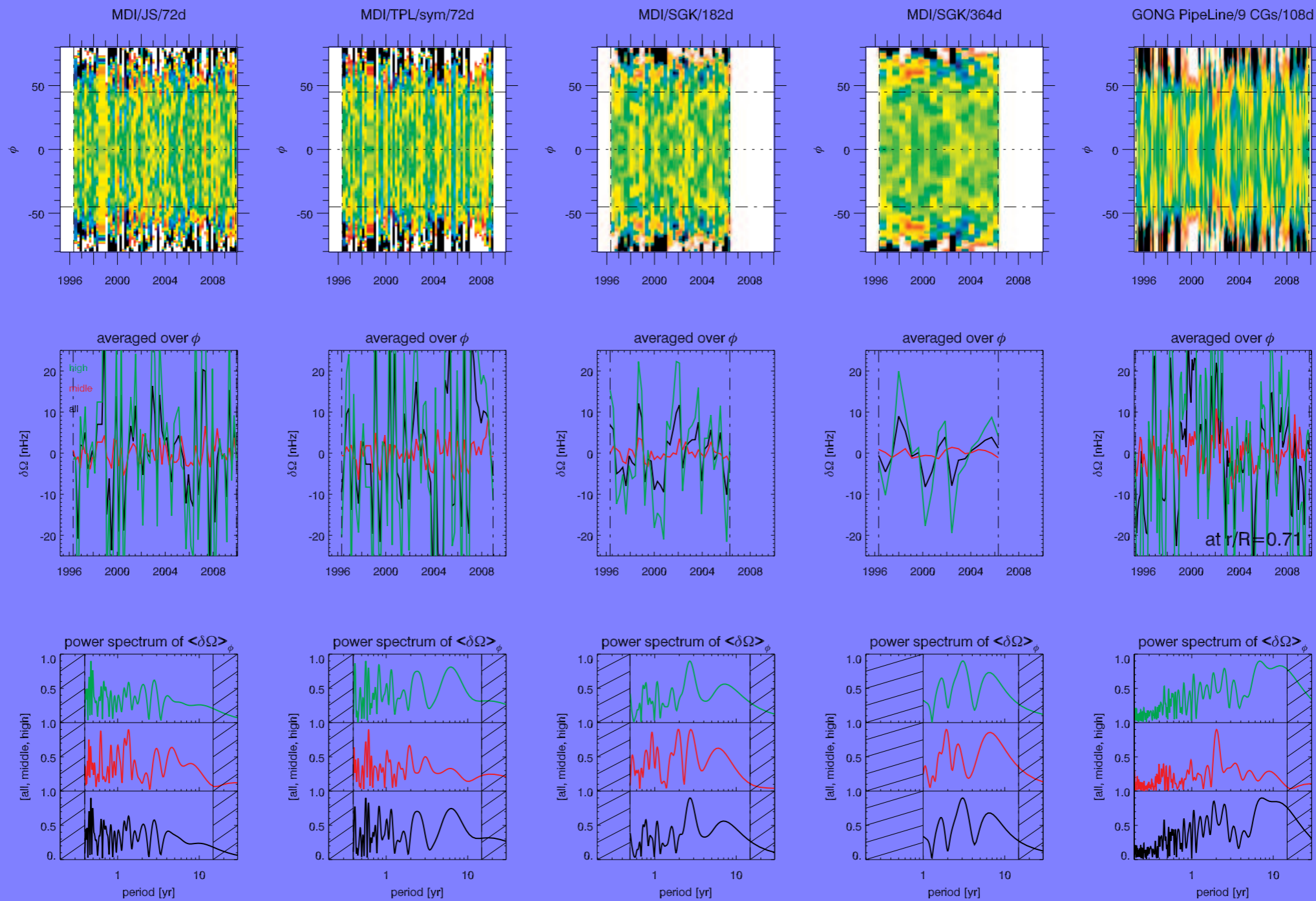
# Rotation Rate Changes – $r/R = 1.00$ (selection)



# Rotation Rate Changes – $r/R = 0.87$ (selection)



# Rotation Rate Changes – $r/R = 0.71$ (selection)



# Conclusions

---

---

- Fitting remains an *issue*
  - ★ attrition
  - ★ f-mode leakage mismatch:
    - \* high  $\ell$  (MTF) or horizontal component ( $\beta = 1$ )
  - ★ plan to complete MDI analysis on shorter epochs
  - ★ expect to apply it on GONG and HMI
- Mean rotation
  - ★ long time-series
  - ★ dip at  $(0.4, 63^\circ) - 1\sigma$ , rising branch of cycle
- Evolution
  - ★ easy at the surface
  - ★ remains challenging down to base of CZ
  - ★ difficult below CZ

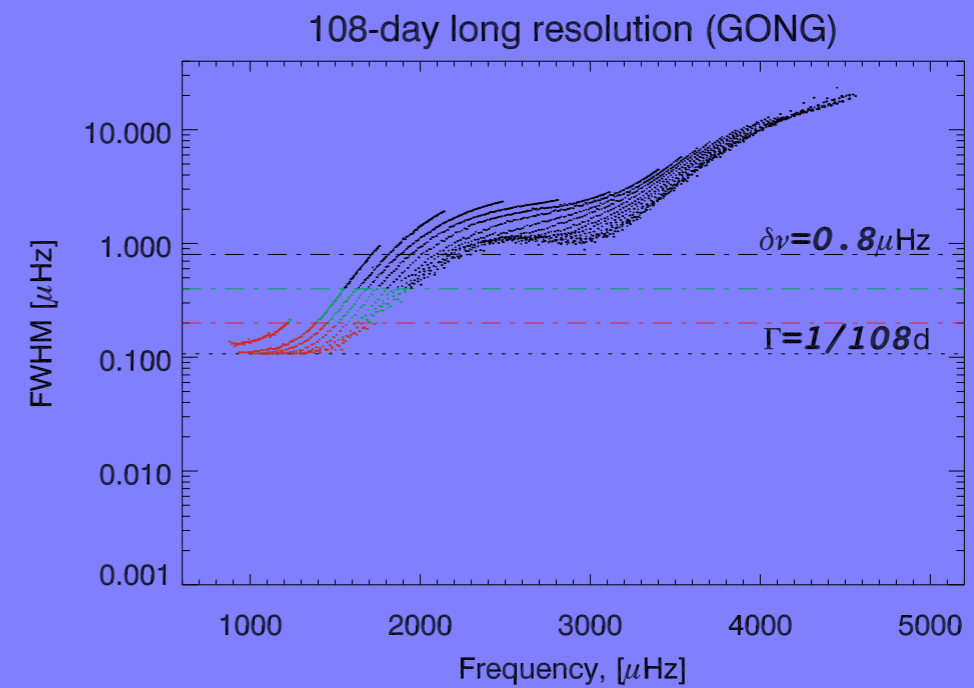
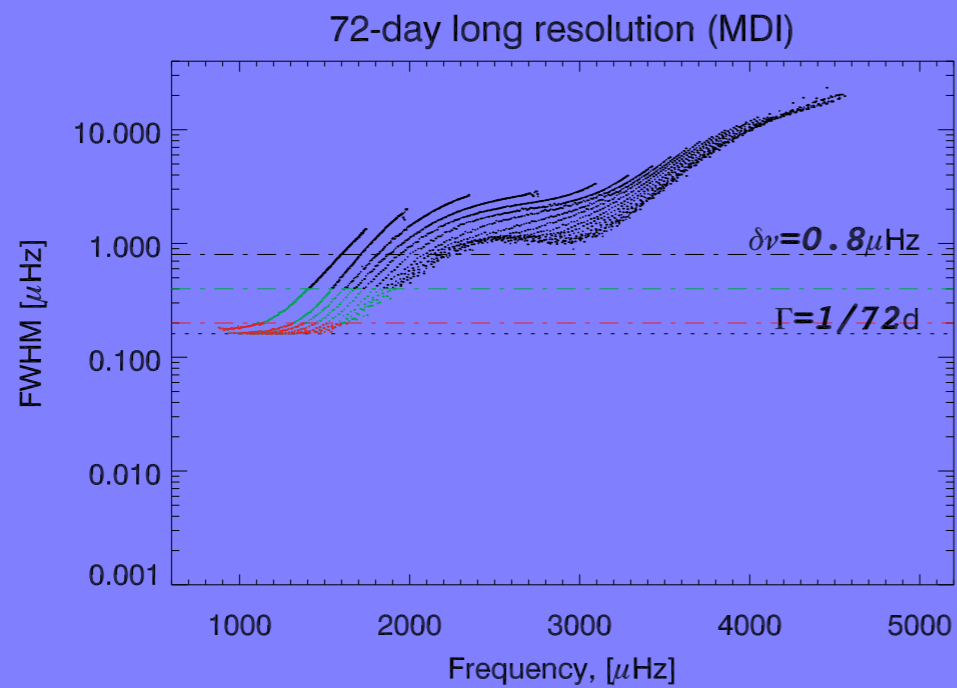
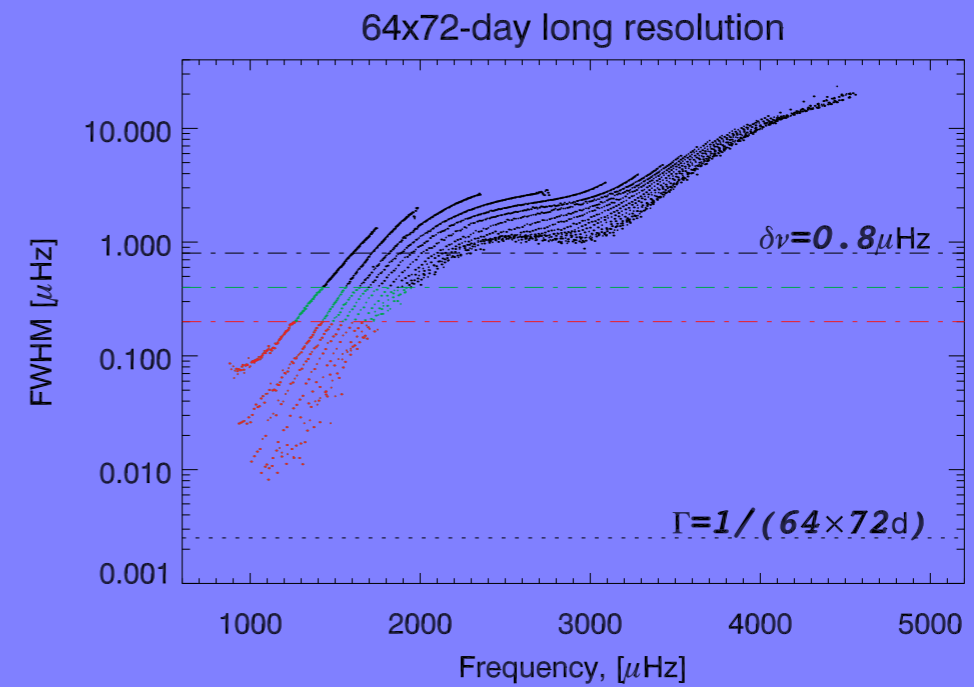
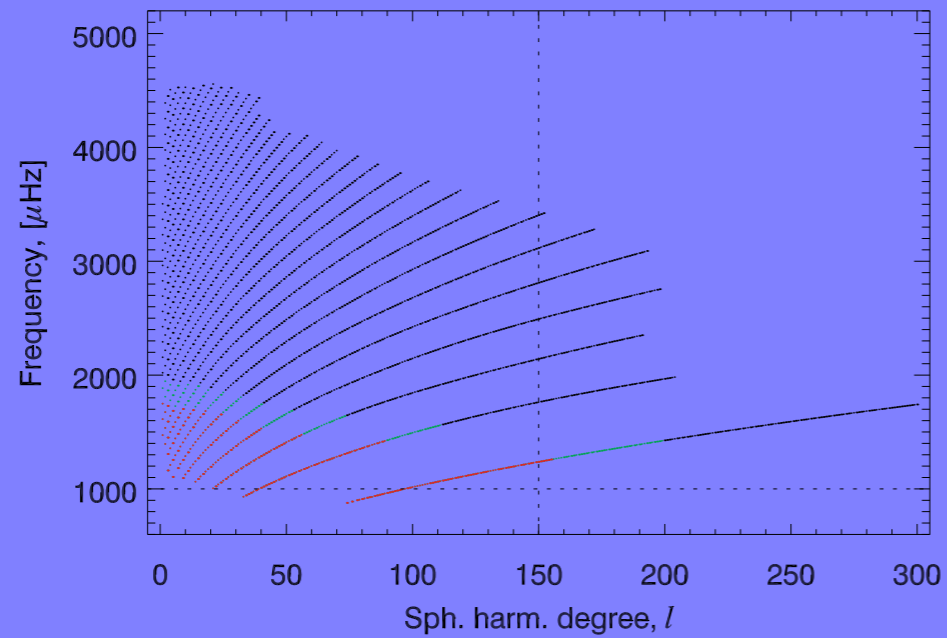
# That's all folks!

ADV – See Also Posters:

- CDF-4 Antia & Basu
- CDF-6 Howe & *al*
- CDF-7 Howe & *al*
- CDF-9 Komm & *al*
- CDF-10 Eff-Darwich & Korzennik
- M-9 Eff-Darwich & Korzennik



# Leakage Matrix – Resolution





# Leakage Matrix – The f-mode

---

---

f-mode –  $64 \times 72d$  &  $2 \times 72d - \ell = 120, (2), 300$



# Leakage Matrix – f-mode Mismatch

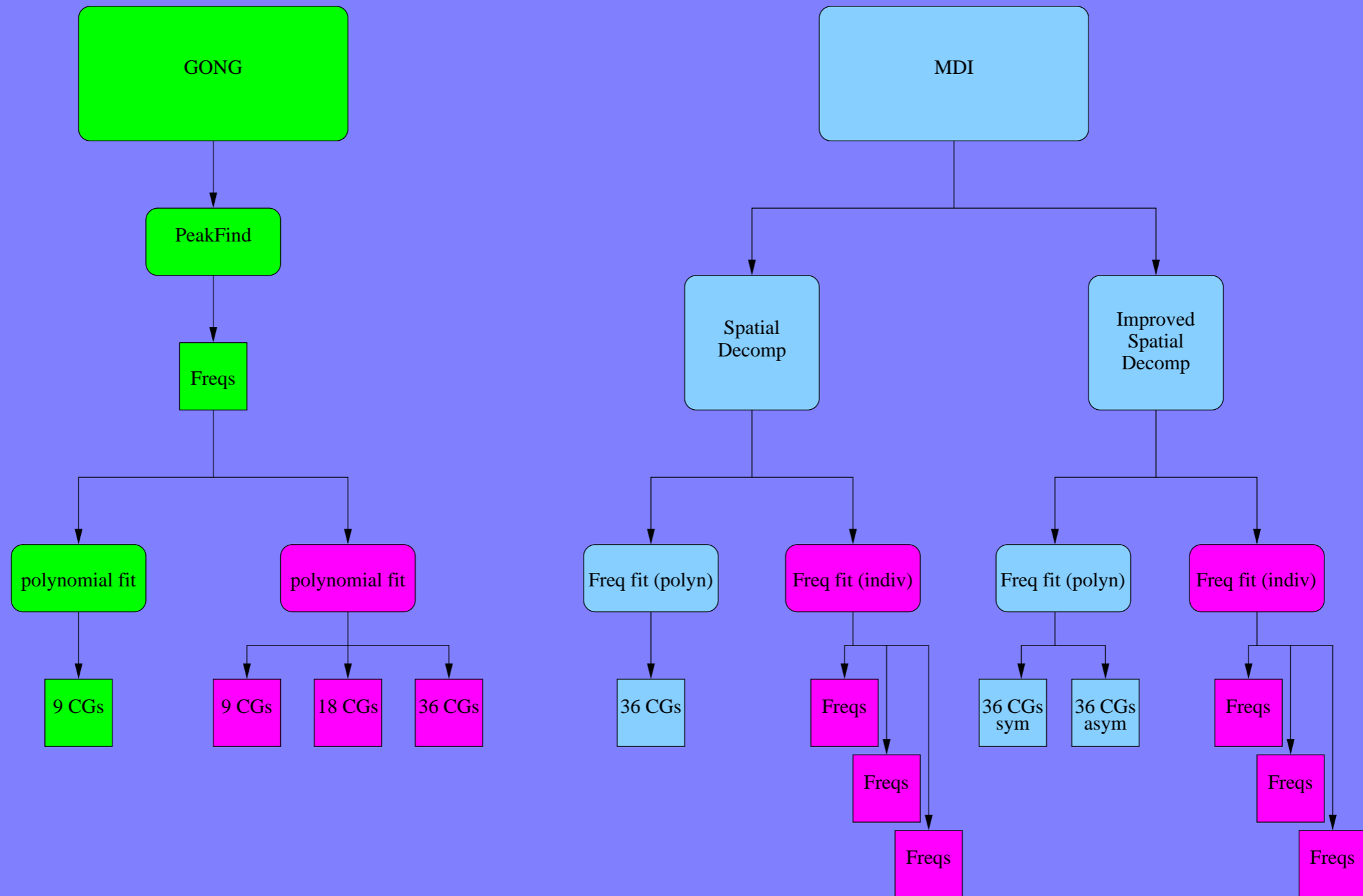
---

---

f-mode –  $64 \times 72d - \ell = 120, (2), 300$

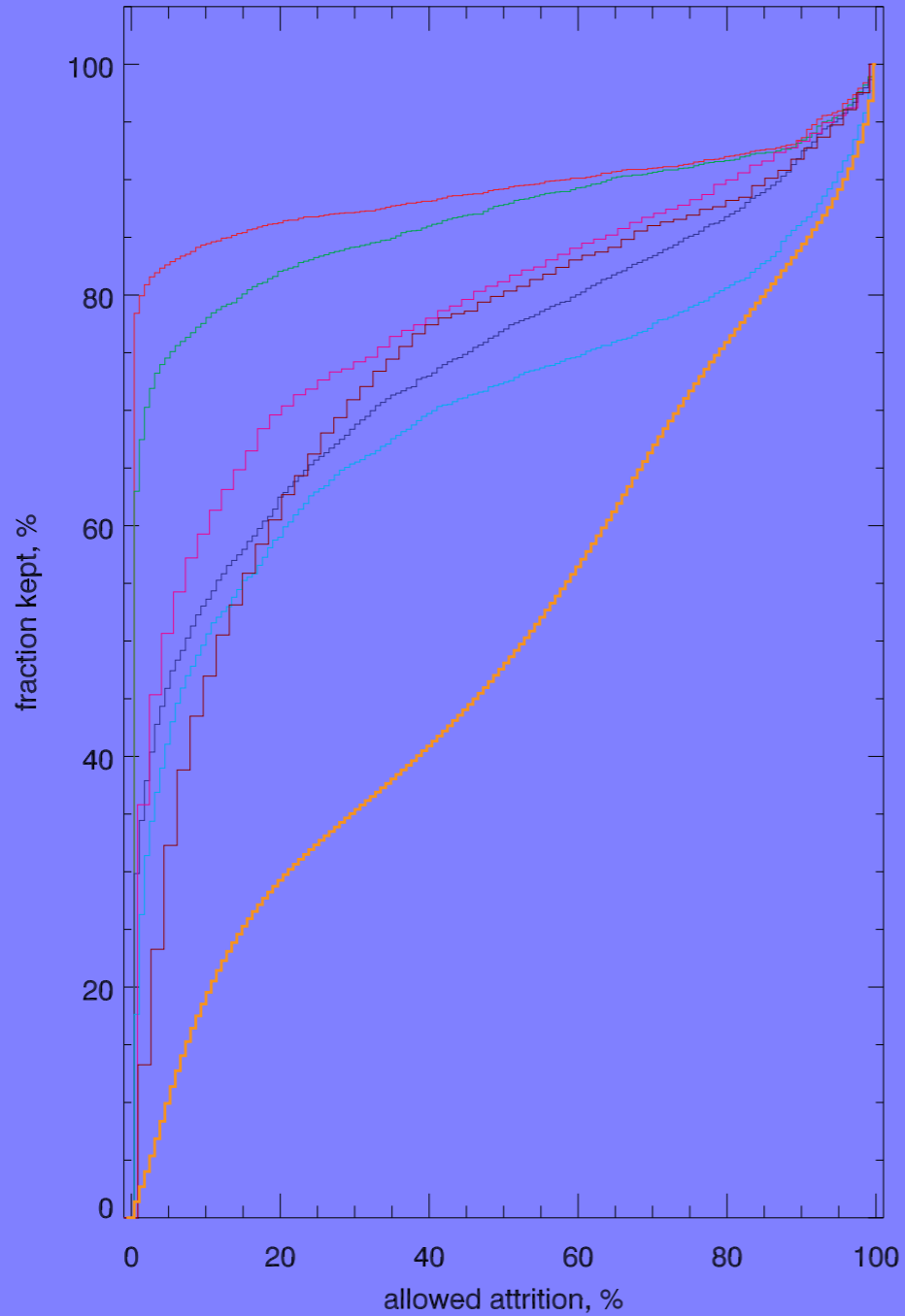


# The Data

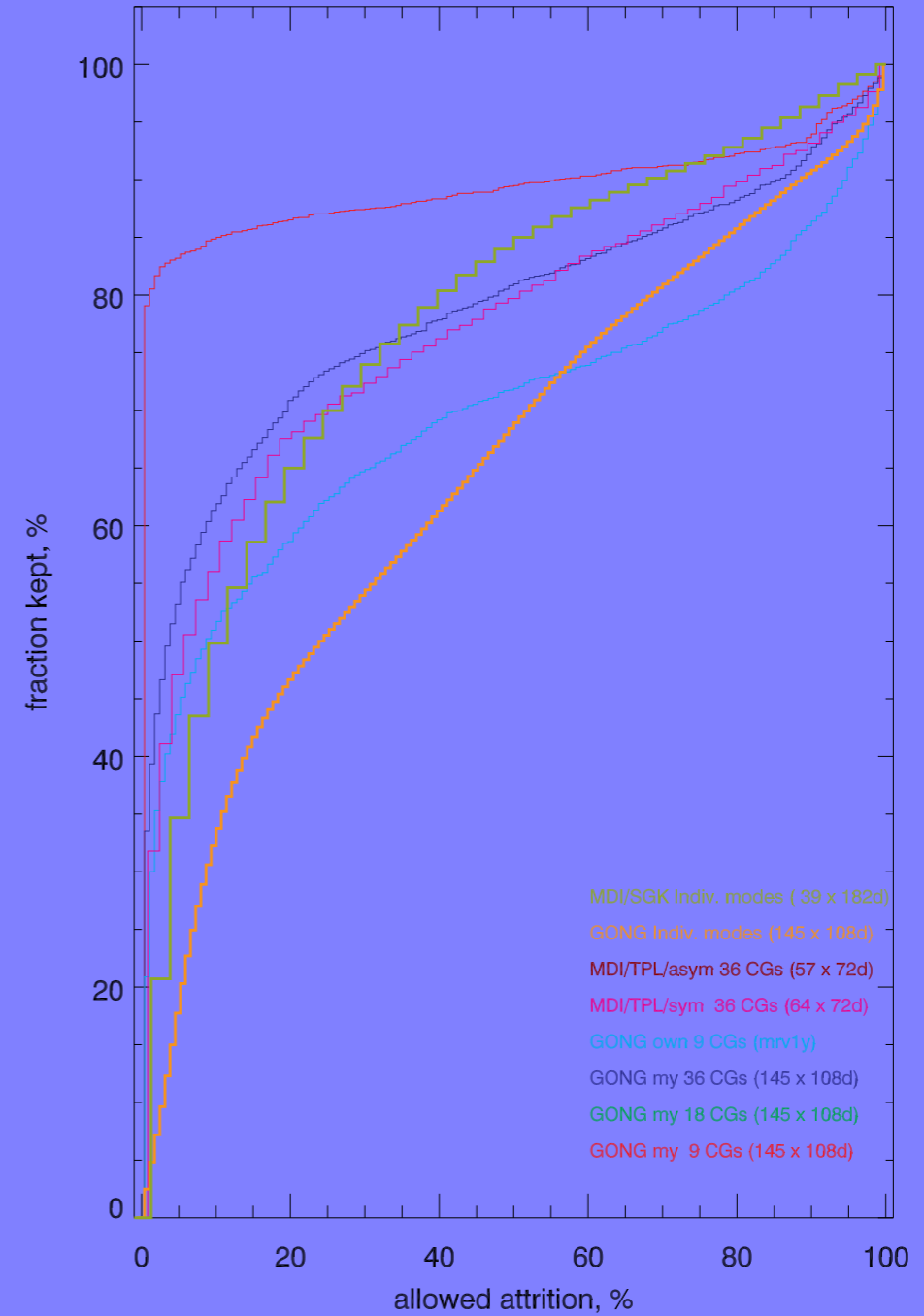


# Attrition – Summary

Cumulative Histogram -- Attrition in Tables ( $l \leq 300$ )

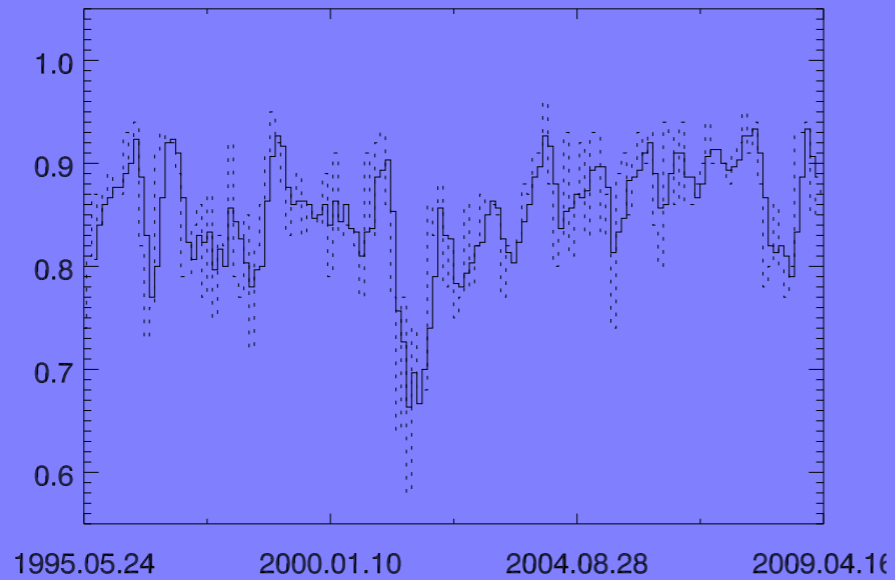


Cumulative Histogram -- Attrition in Tables ( $l \leq 100$ )

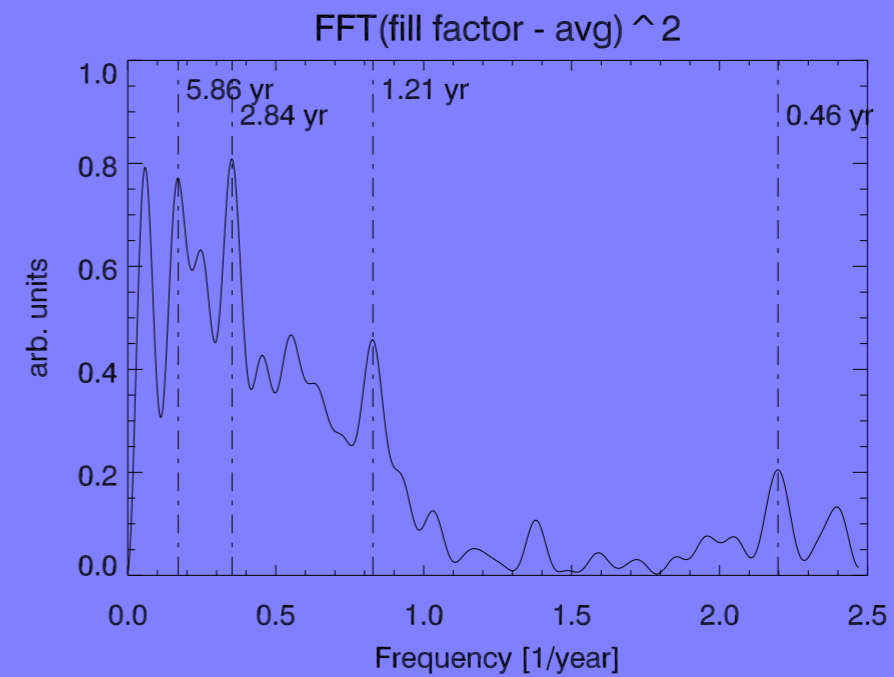
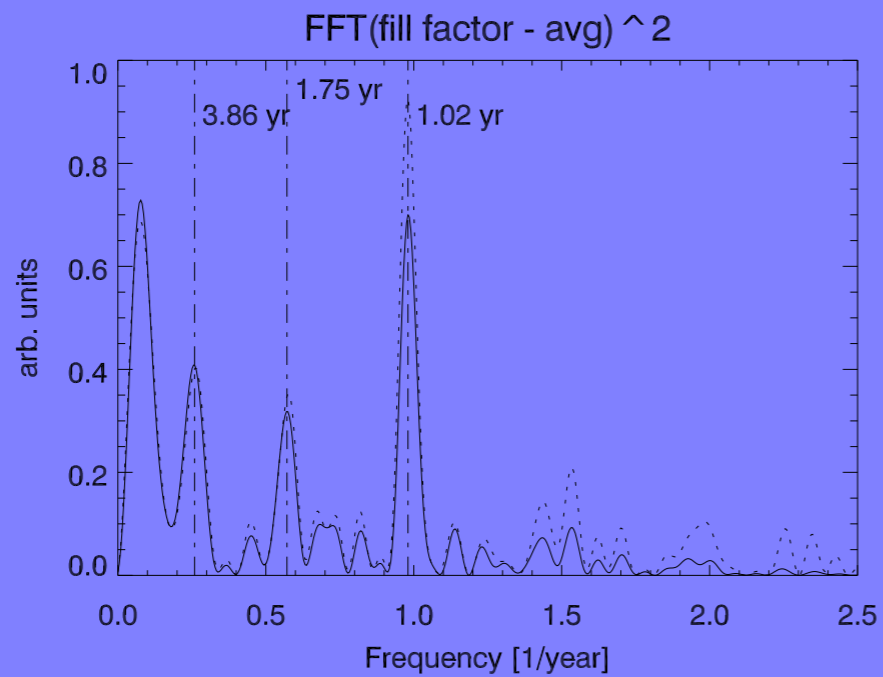
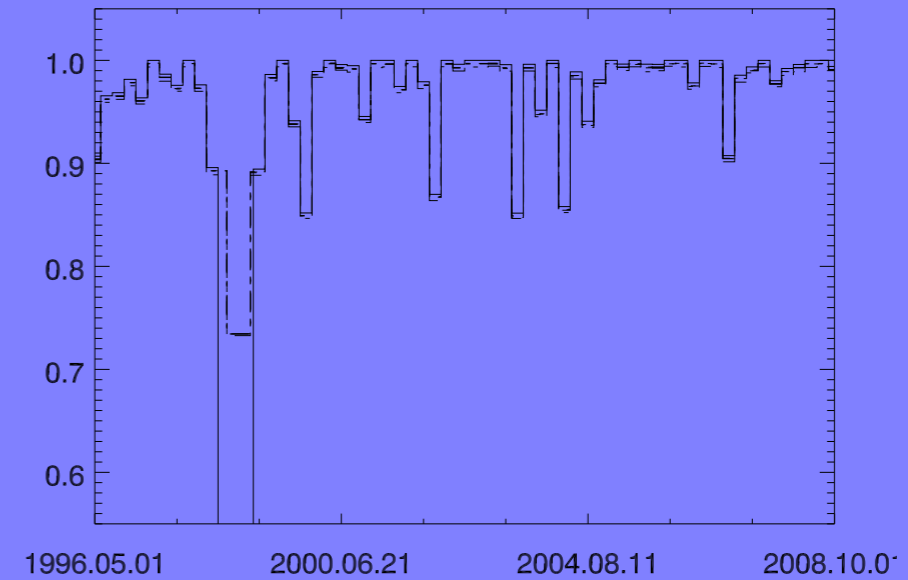


# Fill factors, GONG & MDI

GONG monthly fill factor & 108-day-long tables fill factc



MDI fill factor (72-day long time series)



# Mean Rotation Rate – 12.6 yr – MDI

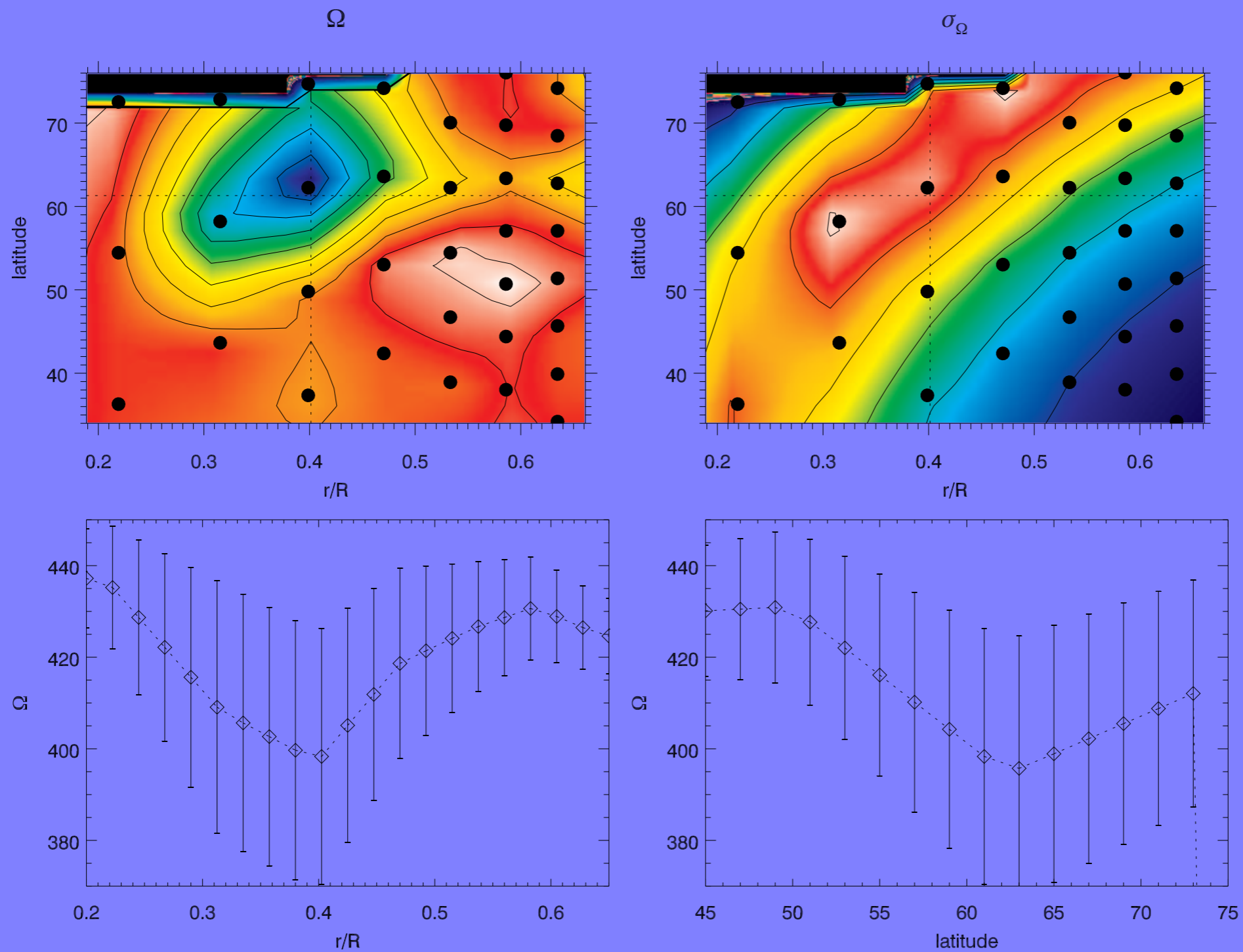
---

---

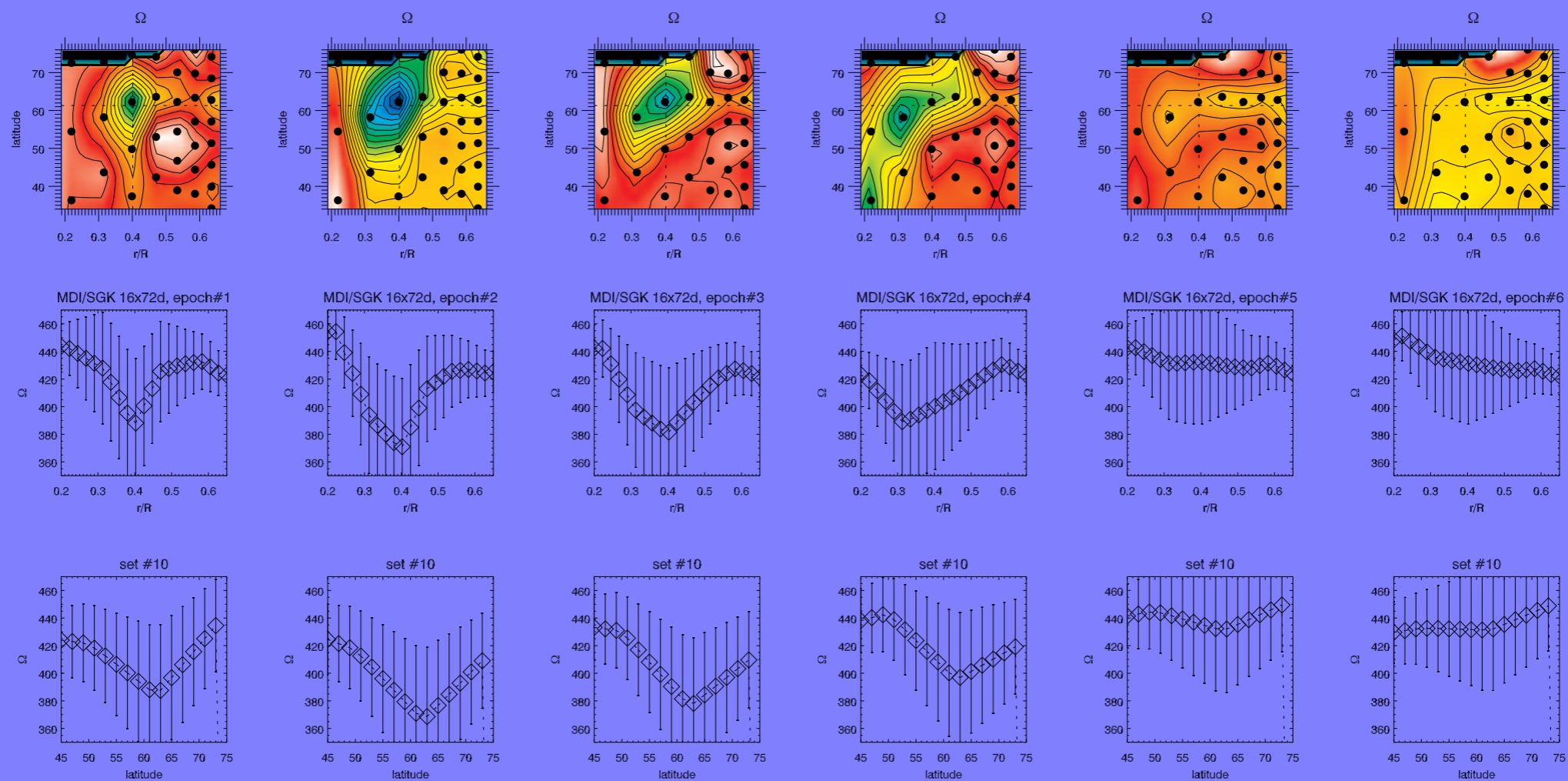
$\Omega(r, \theta)$ , 64 × 72d, change with trade-off



# Dip at $r/R = 0.4, \theta = 63^\circ$ – MDI $64 \times 72d$

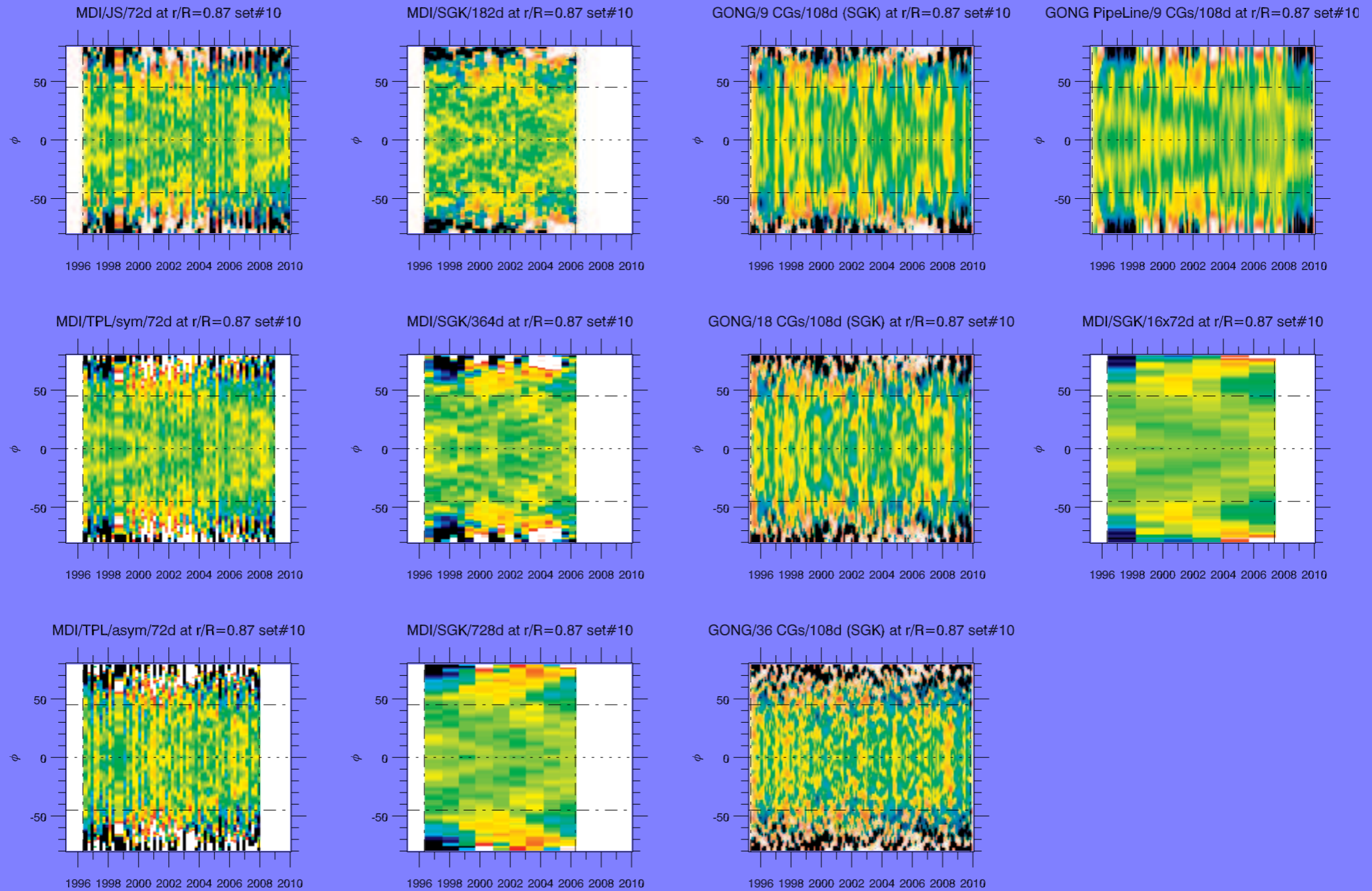


# Dip at $r/R = 0.4, \theta = 63^\circ$ – MDI $16 \times 72d$

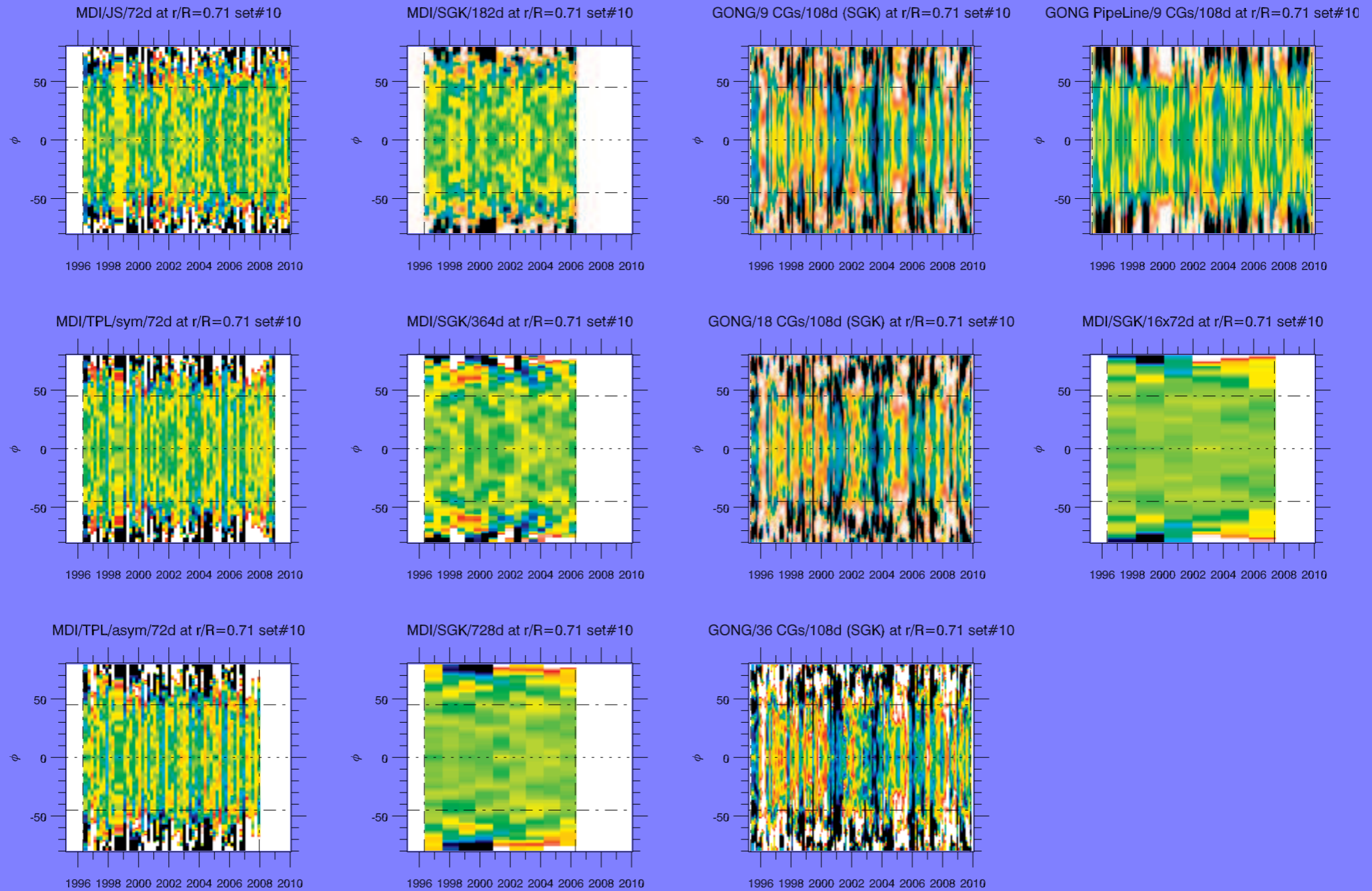




# Rotation Rate Changes – $r/R = 0.87$



# Rotation Rate Changes – $r/R = 0.71$



# The End

---

---

[back to thumbnails](#)