Clusters – Systematics Example: Sunyaev-Zel'dovich Cluster Counts

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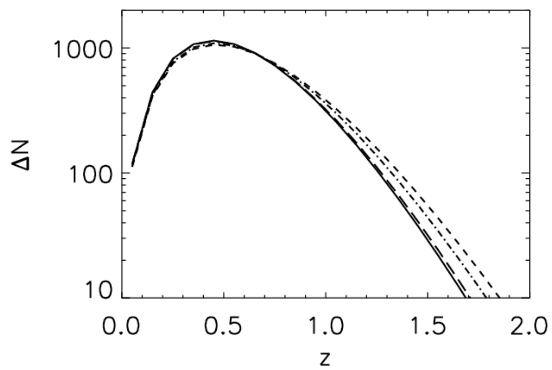
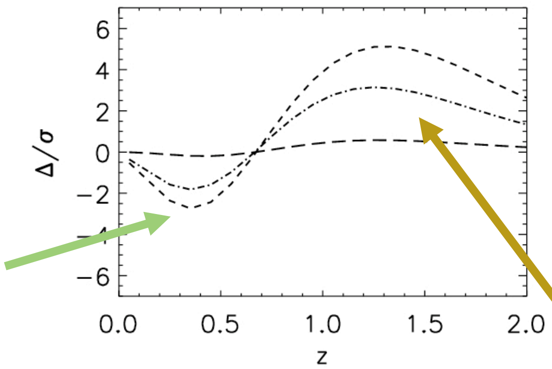
Predicting Cluster Number Counts

$$\Delta N(z) = \Delta\Omega \int_{z-\Delta z/2}^{z+\Delta z/2} dz \frac{d^2V}{d\Omega dz} \int_{M_{\rm lim}}^{\infty} \frac{dn}{dM}\, dM$$

- Survey sky coverage
- Redshift bins
- Volume element
- Limiting mass of survey (redshift dependent)
- Cosmology dependence driven by volume element and mass function

Cosmology Dependence of Number Counts

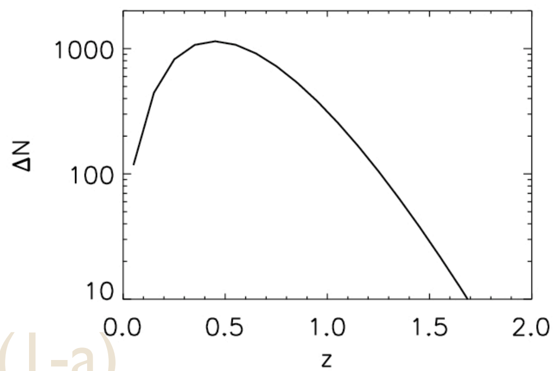




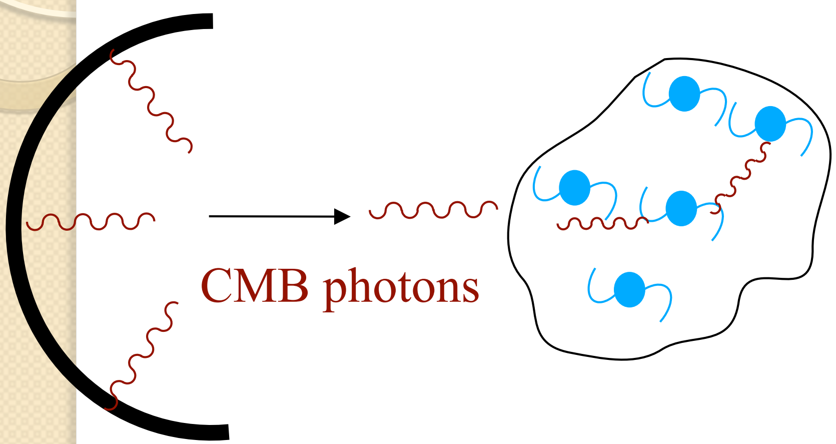
change in volume

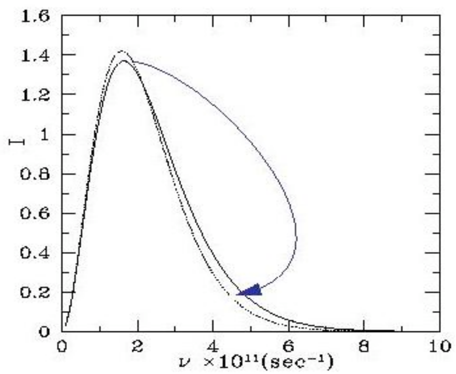
change in growth factor

- concordance cosmology: $\Omega_m = 0.3$; $\sigma_8 = 0.78$; n=1, h=0.72; w=-1, $\Delta\Omega$ = 4.000 deg^2 $M_{lim} = 1.7\times10^{14} h^{-1} M_\odot$
- $\Omega_m = 0.4$
- $\sigma_8 = 0.85$
- w = - 0.8
- w = - 0.7
- w = -1+0.2(1-a)



Sunyaev-Zel'dovich Effect





- Compton Scattering: $e^- + \gamma \rightarrow e^- + \gamma$
- Conservation of overall number of photons
- Decrease in flux or temperature in Rayleigh - Jeans part of the spectrum
- Decrement independent of redshift. Cosmic diming $\sim(1+z)^{-4}$ is balanced by larger density of photons $\sim(1+z)^4$ which are (inverse) Compton scattered.

Sunyaev-Zel'dovich Flux Decrement

$$S_\nu \propto T_{\rm CMB} f(\nu) \int dl\, n_e T_e$$

- frequency dependence with zero at 217 GHz
- integrated gas pressure along line of sight
- integrated over whole cluster:

$$S_\nu = \int S_\nu(\theta)\, d\Omega$$

Limiting Mass from Limiting Flux

$$S_\nu \propto T_{\rm CMB} d_{\rm A}^{-2} \langle T_{\rm e} \rangle M_{\rm vir} f_{\rm gas}$$

- to work out mass require mass - temperature relation
- Assume clusters are virialized spherically collapsed objects

$$\frac{M_{\rm vir}}{10^{15} h^{-1} M_\odot} = \left(\frac{\langle T_e \rangle_n}{T_*} \right)^{3/2} \left(\Delta_c E(z)^2 \right)^{-1/2} \left[1 + (1+3w) \frac{\Omega_{de}(z)}{\Delta_c} \right]^{-3/2}$$

Normalization to simulations or observations: $T_* = 1.6$

From spherical collapse including dark energy (w)

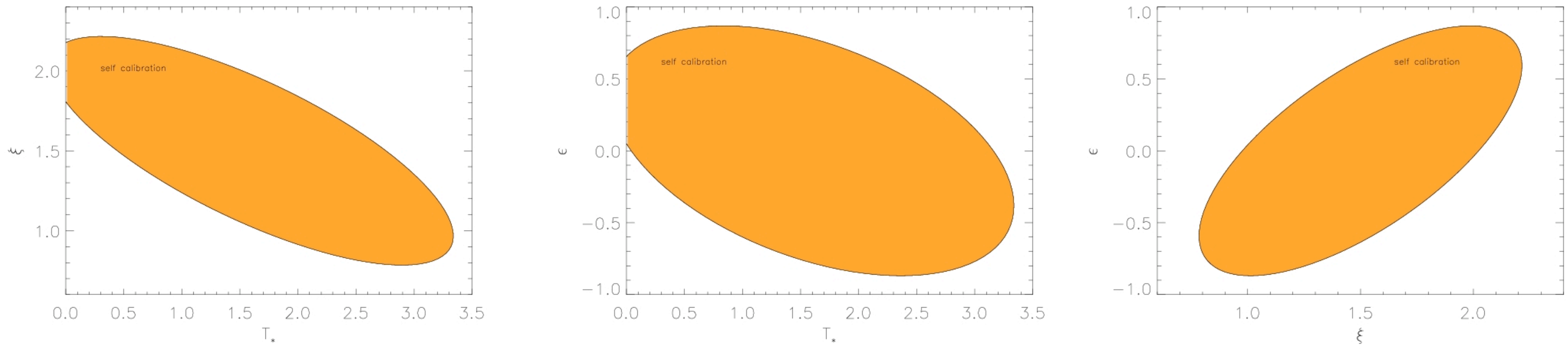
Uncertainty in Mass Temperature Relation

$$\langle T_e \rangle_n \propto T_* \left(\Delta_c E(z)^2\right)^{1/3} (1+z)^{\epsilon-1} \left(\frac{M_{vir}}{10^{15} h^{-1} M_\odot}\right)^{1/\xi}$$

- Normalization introduced in previous talk
- ε=1 standard; models deviation from complete virialization for early clusters (ε<1) and ongoing mergers at early redshifts or other forms of redshift dependent heat injection (ε>1)
- ξ=3/2 standard; models heat input since small clusters are more likely to get heated (ξ>3/2); observations and simulations suggest (1.48< ξ < 1.98)

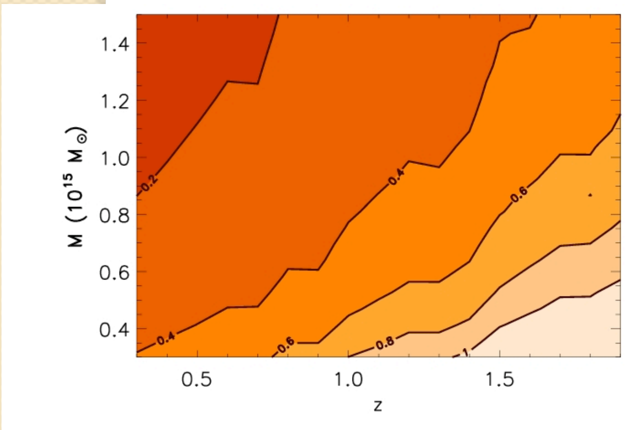
Self-Calibration I: Use cluster counts to callibrate uncertainy in mass - temperature relation

- In addition to cosmological parameters fit for cluster parameters $T_* ; \xi ; \varepsilon$

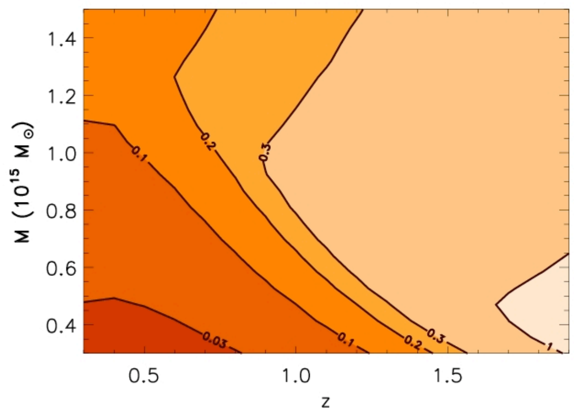


Weak Lensing Callibration of Mass - SZ Observable Relation with DES

- Simple estimate: 15 background galaxies/ sq. arcmin
- Distribution: $dn/dz = \exp(-z/z_c)$; $z_c = 0.5$

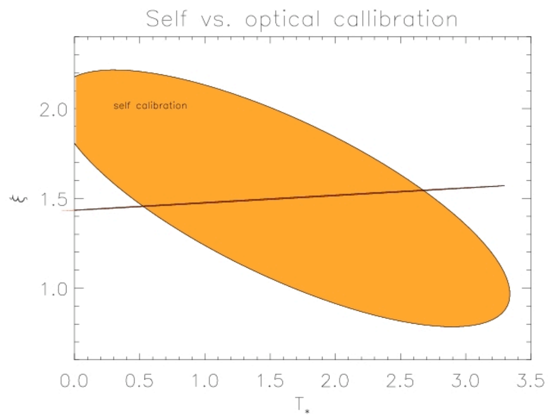


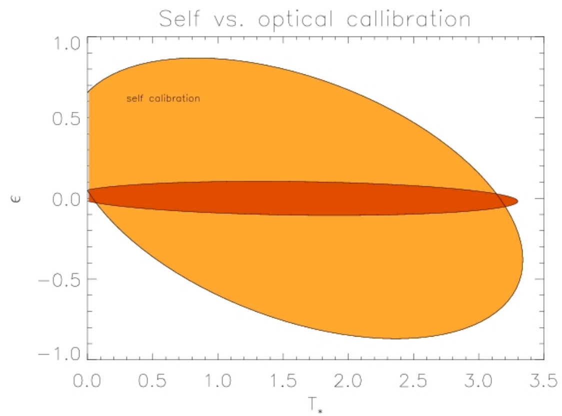
Projected errors on single cluster

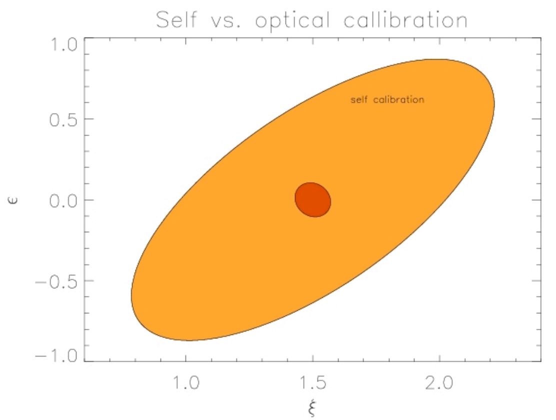


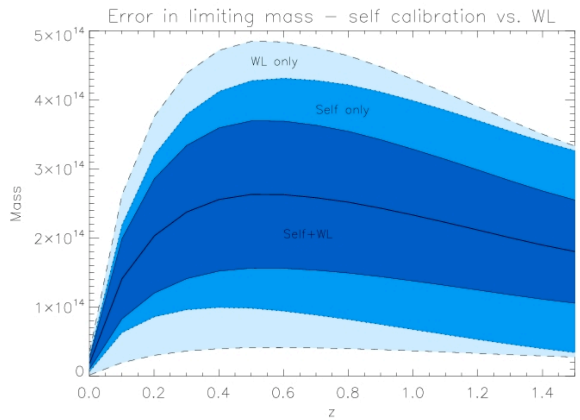
Fractional errors on cluster mass after stacking in redshift bins $\Delta z = 0.1$ and $\Delta M = 10^{14} M_\odot$

Weak Lensing Calibration II









Self-Calibration à la Lima & Hu

- Use free form mass-observable relation

$$M_{obs} = A(z)e^{p(z)}$$

- fix A(z) and p(z) in redshift bins of width $\Delta z = 0.1$
- assign likelihood for observed mass for a true mass $p(M_{obs} \mid M)$ with a bias and a scatter included

$$p(M_{obs}|M) = \frac{1}{\sqrt{2\pi\sigma^2_{\ln M}}} \exp\left[-x^2(M_{obs})\right]$$

$$x(M_{obs}) = \frac{\ln M_{obs} - \ln M - \ln M_{bias}}{\sigma_{\ln M}}$$

Lima & Hu Calibration II

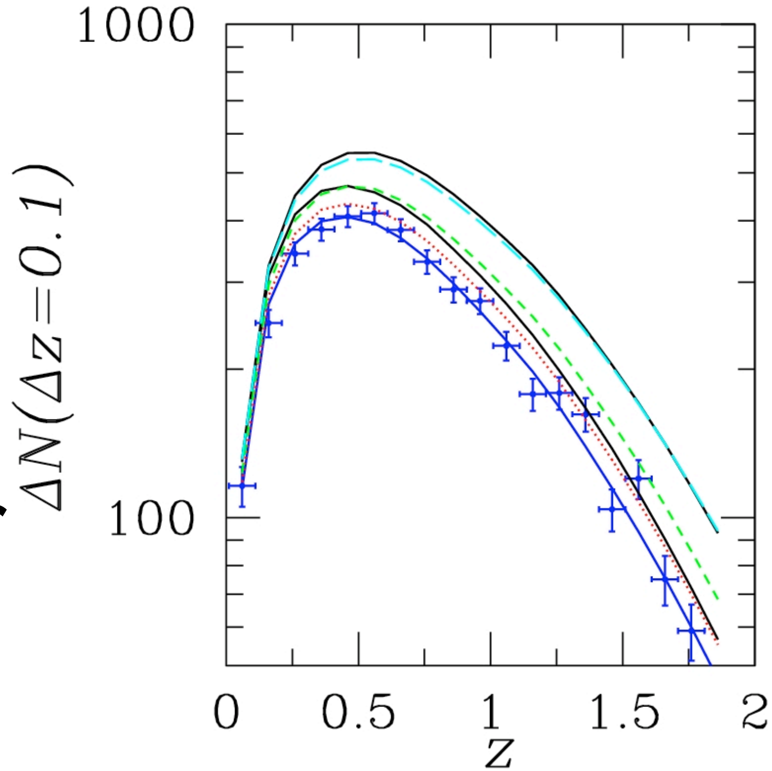
$$n_i = \int_{M^i_{obs}}^{M^{i+1}_{obs}} \frac{dM_{obs}}{M_{obs}} \int \frac{dM}{M} \frac{dn}{d\ln M} p(M_{obs}|M)$$

- Exploit shape of mass function to calibrate for bias and scatter in constant mass bins
- Further use clustering of clusters (cross-correlated to other probes ?)
- Result: scatter in mass-observable relation is not the problem: Increases number of clusters, hence better statistics

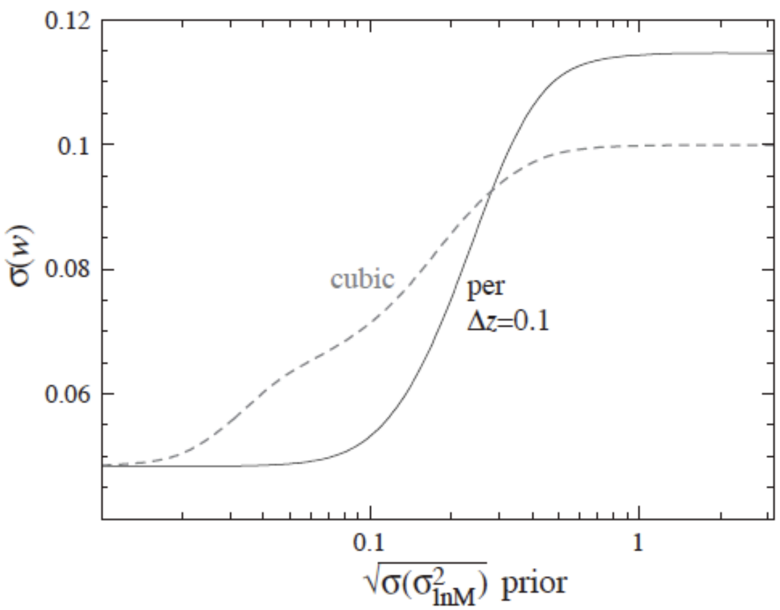
Simple Scatter Analysis

$$\frac{dN}{dz} = \Delta\Omega \frac{dV}{dz d\Omega}(z) \int_0^\infty \phi(M,z) \frac{dn}{dM} dM \quad \phi(M,z) = \frac{1}{2}\left\{ erf\left[\frac{M - M_{lim}(z)}{\delta M_{lim}(z)}\right] + 1 \right\}$$

- dashed and dotted lines δ=20%, 30%, 40%
- did not marginalize over scatter - need prior

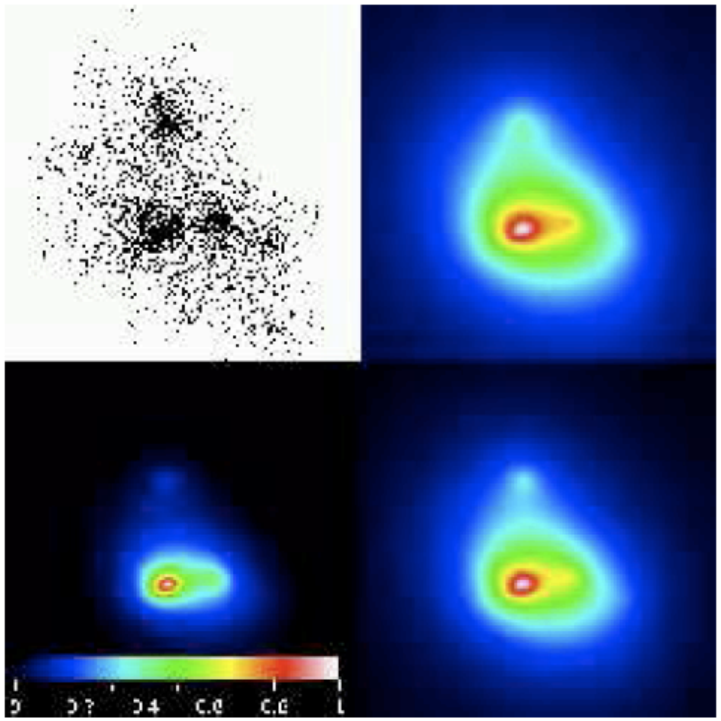


Lima & Hu Self-Calibration III



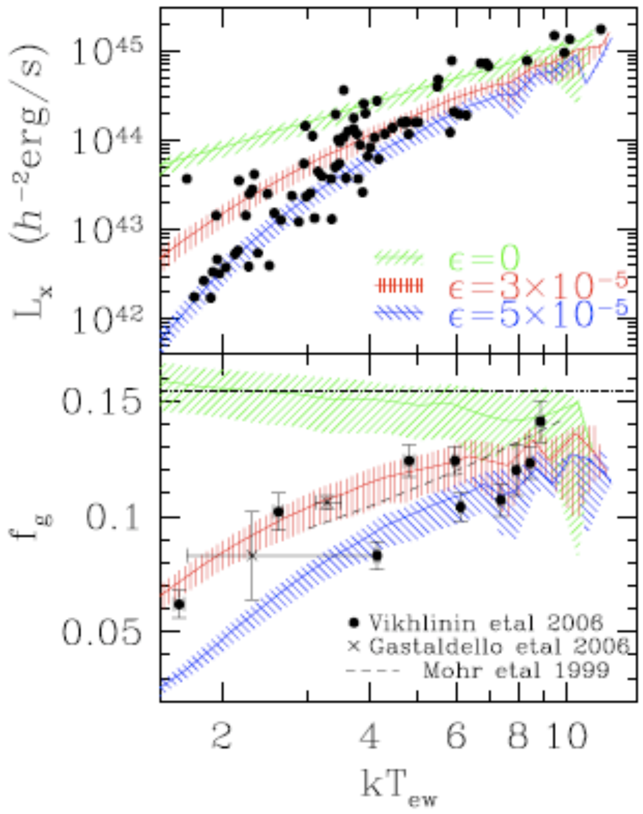
- However: UNCERTAINTY IN SCATTER is problem
- Problem - mass - observable nuissance parameters are degenerate with cosmology (not included in the Lima & Hu free form fit)
- Prior on uncertainty in scatter required ?
- How to use lensing mass estimates in this approach ?

Gas in CDM Simulations



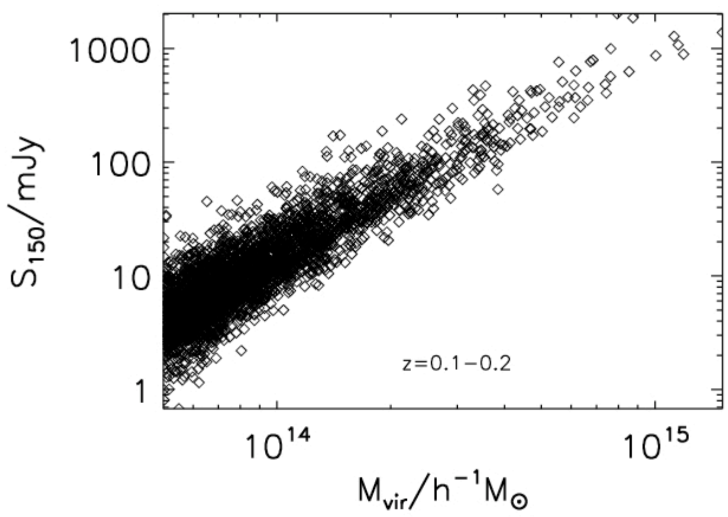
- Ostriker, Babul, Bode: gas follows dm potential as a polytropic fluid
- Included feedback parameters, calibrated to x-ray data

Calibration to x-ray data



- Note small difference for high mass clusters (feedback energy small compared to potential)
- universal f_{gas} lowered by 10% (stars)

Measure Mass Observable Relation in Simulation

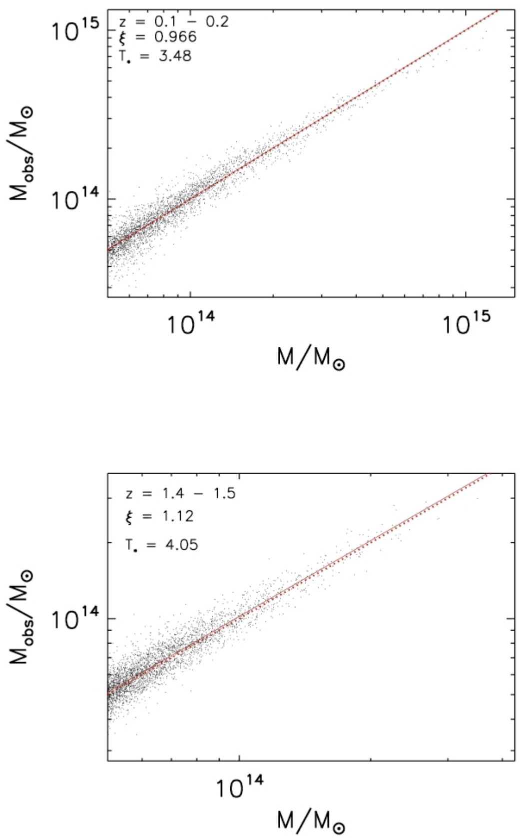


- Large scatter at low masses, but SPT limit at about $2\times10^{14}M_{\odot}$
- Source of scatter: projection effects
- Similar for larger redshift

Possible Sources of Scatter

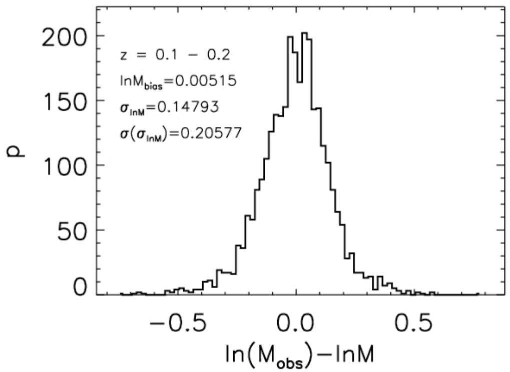
- concentration
- substructure - orientation - morphology
- kinetic vs. thermal energy; rotational energy
- feedback
- shock heating from mergers
- scatter in M/L ratio, at the moment fixed to 10%
- explored scatter in polytropic index (probably weak effect)

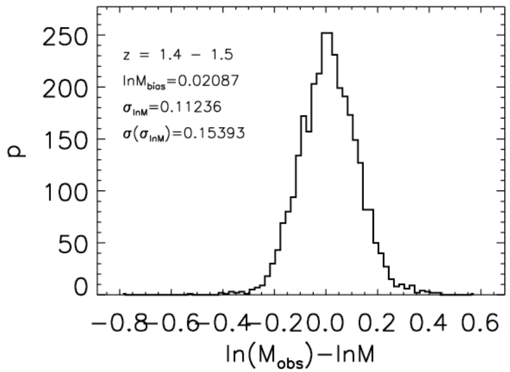
Fitting the Mass - Observable Relation



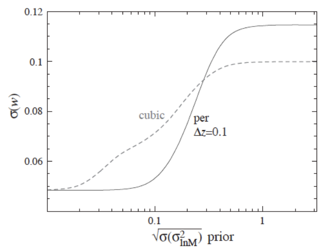
- Note fitted slopes much less than 1.5 !
- Reason: f_{gas} depends on mass as well; not usually considered
- If f_{gas} slope is folded in get about 1.6-1.7 as slope !

Scatter Statistics from Simulation





- Measured scatter of 15%-20%
- Sample of >500 clusters in low redshift bin
- Non-Gaussian distribution
- To measure uncertainty need different realizations. Indications are: 15% level



Non-Gaussianity of Scatter

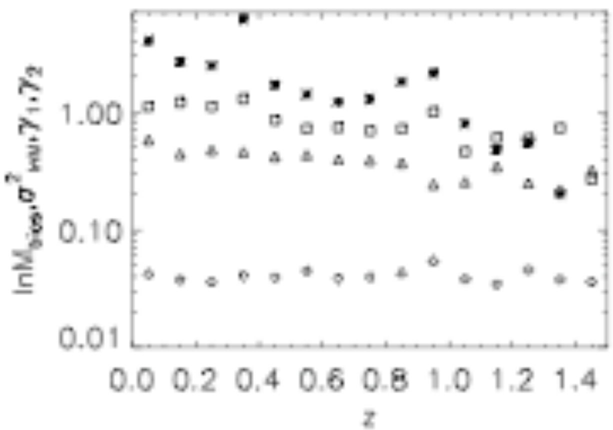
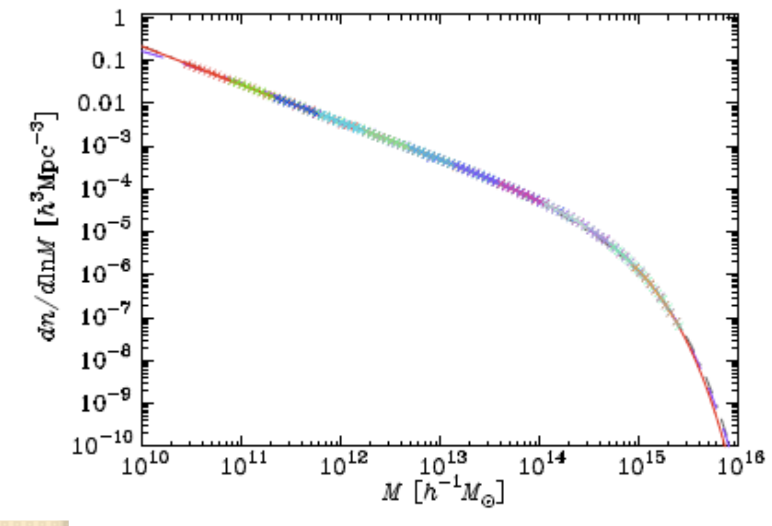


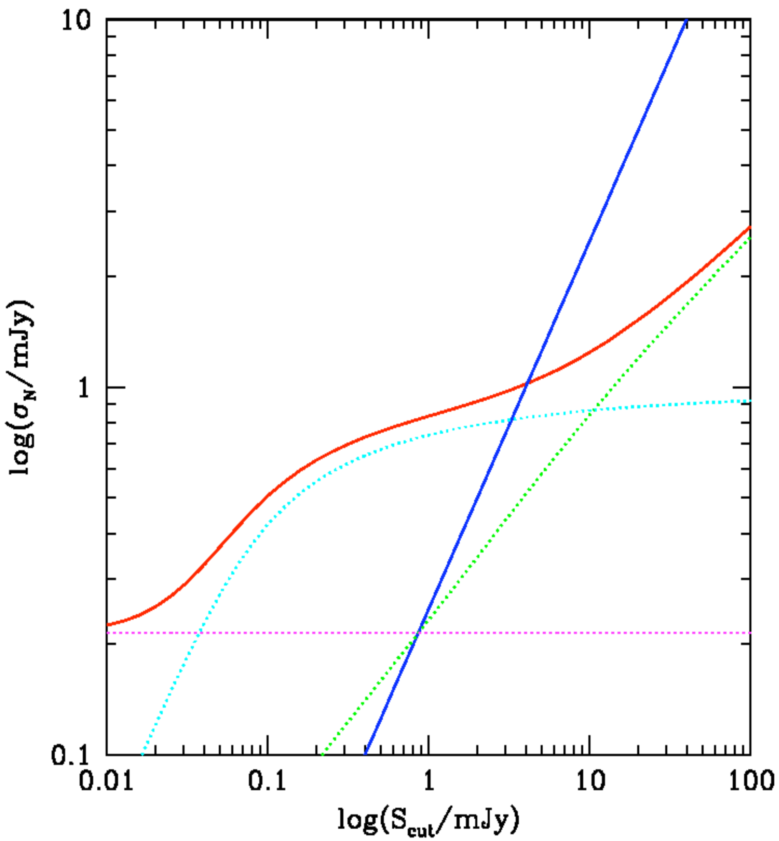
FIG. 15.— Statistical coefficients of the distribution of the mass - observable relation in different redshift bins. The mean (or bias) $\ln M^{\rm bias}$ as triangles, the variance $\sigma^2_{\ln M}$ as diamonds, the skewness as squares and the kurtosis as asterisks.

Uncertainty in theoretical mass function



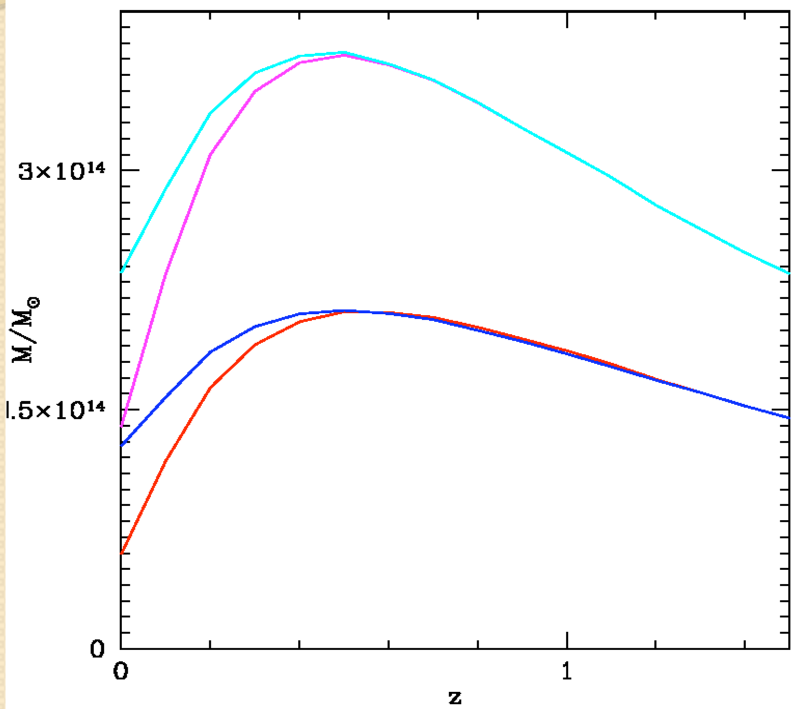
- Warren et al. 2005 high precision mass function
- Uncertainty in fitting parameters does **NOT** degrade ability to constrain dark energy
- Question remains how universal mass function is (see previous talk)

The really nasty stuff - point sources



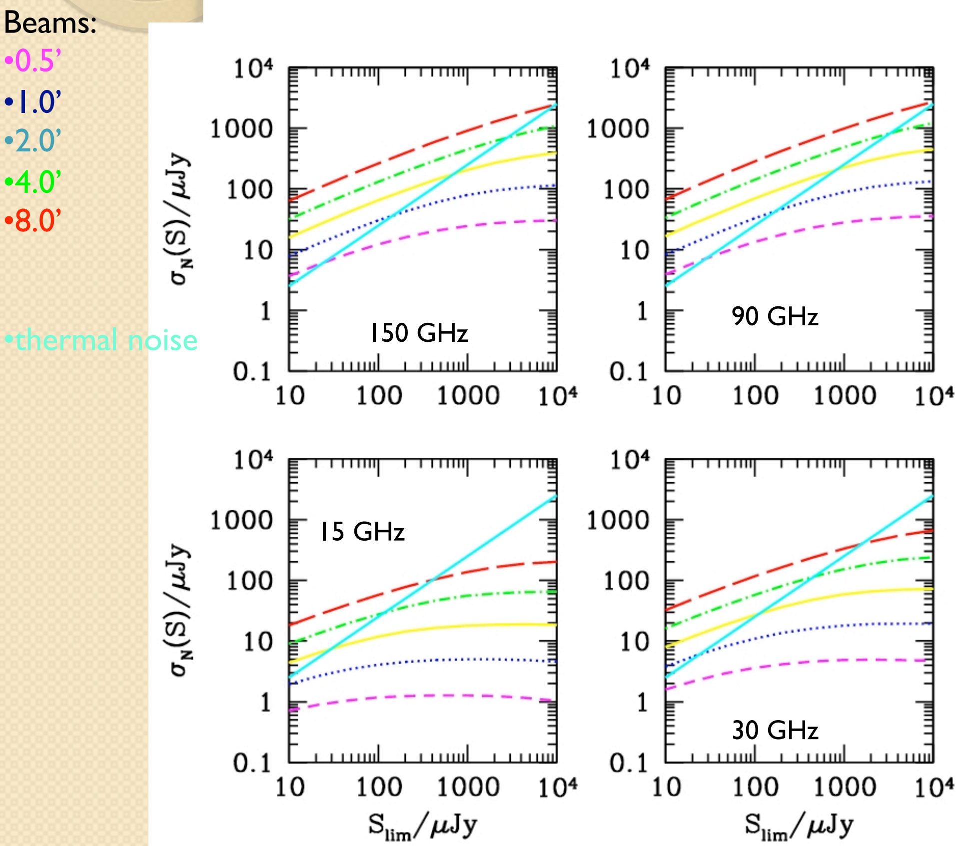
- What happens if point sources are not resolved or just single frequency
- f=90GHz; θ=4'
- Thermal
- IR-point sources
- Radio point sources
- Primary CMB
- Sum of confusion noise in quadrature

Mass Limit including Point Source Confusion



- thermal only
- thermal + CMB
- thermal+PS
- thermal + PS + CMB
- Multifrequency or high resolution follow up

Cluster-Cluster confusion



- confusion noise below S_{lim} (4σ thermal noise)
- Planck: 150GHz, 36mJy, 8'
- SPT: 150GHz, 1.6mJy, 1'
- ACT: depth of 150μJy?
- Observer's rule of thumb: not more than 1 cluster per 60 beam areas
- Note: used only Poisson noise - no power spectrum!
- Much better done by Holder et al.
- Ask Bjorn

Conclusion

- Clusters of Galaxies are a very sensitive probe of the growth of structure in the Universe
- Mass - Observable Relation for SZ effect has uncertainties, but they seem to be controllable (see also simulations by Motl and Nagai)
- WL calibration of mass - observable relation has large potential
- conservative lower limit on uncertainty in scatter at 15-20%, is this the limit ?
- To get better understanding on systematics go directly from maps
- No knowledge of exact coefficients/form of scaling relation and its redshift dependence is required (well it is to forecast numbers, but not in the analysis)
- Methods can also be applied to other cluster probes: optical and x-ray