Dark Energy & General Relativity
« *Some theoretical thoughts* »

Jean-Philippe UZAN

Cosmological models

Theoretical physics → Cosmology ← Astrophysics

Principles
Local law of nature

Phenomena

Extrapolations → models ← Constraints

Gravity=GR
Matter=SU(3)XSU(2)xU(1)

Compatible with local physics
Save the apperances

Its construction relies on 4 hypothesis

1. Theory of gravity [General relativity]
2. Matter [Standard model fields + CDM + Λ]
3. Symmetry hypothesis [Copernican Principle]
4. Global structure [Topology of space is trivial]

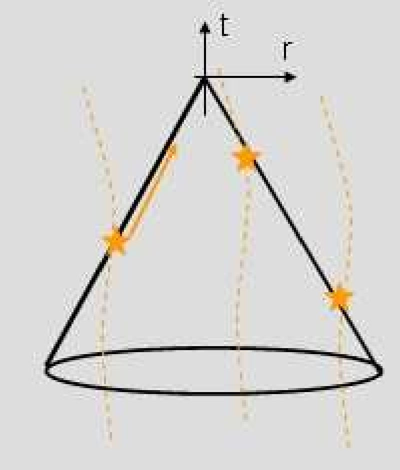
(1–2) → New physics with simple cosmological solution

(3–4) → Standard physics with more involved solution

In agreement with all the data.

Implications of the Copernican principle

Independently of any theory (**H1, H3**), the Copernican principle implies that the geometry of the universe reduces to *a(t).*



Consequences:

- $1 + z = \frac{E_{rec}}{E_{em}} \overset{\text{H2}}{=} \frac{a_0}{a(t)}$

- $a(t) = a_0 \left[1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots \right]$

  so that

$$H^2(z)/H_0^2 = 1 + (q_0 + 1)z + \mathcal{O}(z^2)$$

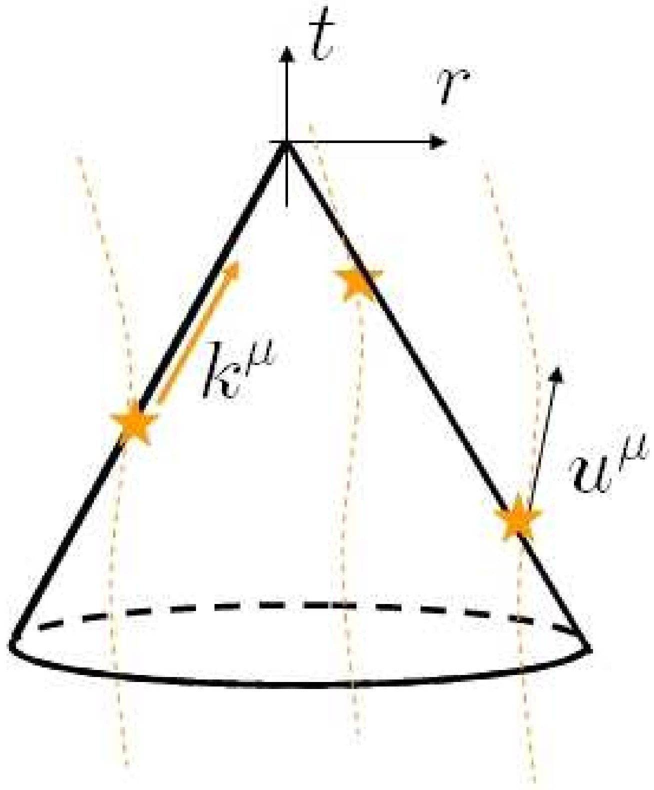
$$q_0 = \Omega_{m0}/2$$

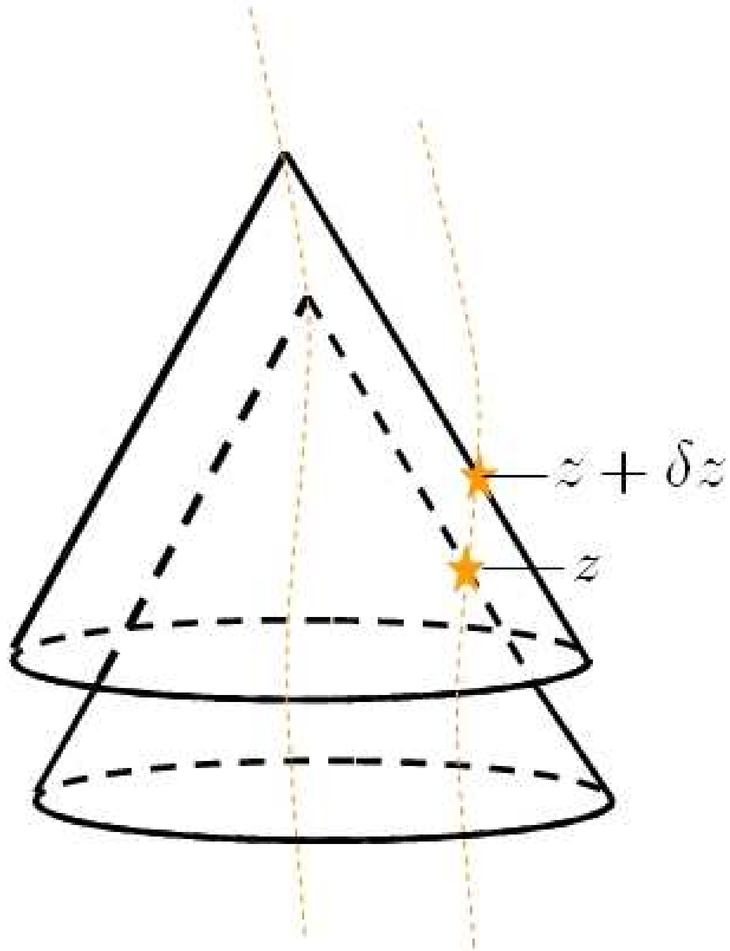
- **Hubble diagram** gives
  - H_0 at small z
  - q_0

Supernovae data (1998+) show

$q_0 < 0$ ⟷ The expansion is now **accelerating**

Testing the Copernican principle





Testing the Copernican principle

An interesting observable is the time drift of the redshift

FL $$\dot{z} = H_0(1+z) - H(z)$$ cf Sandage1962

Typical order of magnitude (z~4)

$$\delta z \sim -5 \times 10^{-10} \quad \text{on} \quad \delta t \sim 10\,\mathrm{yr}$$

CODEX project for ELT

Measure de H(z)
Variance
Constante

JPU, Bernardeau, Mellier, arXiv:0711.1950

LTB $$\dot{z} = H_0(1+z) - H(z) + \frac{1}{\sqrt{3}}\sigma(z)$$

JPU, Clarkson, Ellis

Allow to fully reconstruct the geometry of an under-dense region around us

If CP holds, then we need new *physical* degrees of freedom

Dark sector called by the observations

Galaxie rotation curves

Taken as a proof of the existence of dark matter
MOND alternative: modification of Newton law in low acceleration

Acceleration of the universe

SNIa
Conclusion depends only on the validity of the Copernican principle

IF CP holds THEN necessity to extend our reference theory

« Dark energy »

Various ways to achieve this.

Gravitation = any long range force that cannot be screened

Reference theory

Standard theory of gravitation: General relativity / No freedom!

Based on 2 hypothesis:

- all matter fields are minimally coupled to a *single* metric tensor.
  This implies the weak equivalence principle

$$S_{matter}(\psi, g_{\mu\nu})$$

- Einstein Hilbert action

$$S_{grav} = \frac{c^3}{16\pi G}\int\sqrt{-g_*}\,R_*\,d^4x$$

Both metrics coincide

$$g_{\mu\nu} = g^*_{\mu\nu}$$

Very well tested in many systems.

	Measurement	Scale	Ω_m
1	peculiar velocities: relative rms	20 kpc $\lesssim r \lesssim$ 1 Mpc	$0.20e^{\pm 0.4}$
2	redshift space anisotropy	10 Mpc $\lesssim r \lesssim$ 30 Mpc	0.30 ± 0.08
3	mean relative velocities	10 Mpc $\lesssim r \lesssim$ 30 Mpc	$0.30^{+0.17}_{-0.07}$
4	numerical action solutions	$r \sim$ 1 Mpc	0.15 ± 0.08
5	virgocentric flow	$r \sim$ 20 Mpc	$0.20^{+0.22}_{-0.15}$
6	weak lensing: galaxy-mass	100 kpc $\lesssim r \lesssim$ 1 Mpc	$0.20^{+0.06}_{-0.05}$
7	mass-mass	300 kpc $\lesssim r \lesssim$ 3 Mpc	0.31 ± 0.08

[Peebles, astro-ph/0410284]

Models

Λ

Quintessence

TeVeS

Multigravity

K-essence

DGP

Chameleon

Extended quintessence

Tachyon

f(R)

Quintom

Cardassian

AWE

Cosmon

Chaplygin gaz

Universality classes of extensions

Ordinary matter ↔ $g_{\mu\nu}$ ↔ ϕ

Ex : quintessence, ...

$S_{\rm de}[{\rm de}; g_{\mu\nu}]$

Variation of constants

Poisson equation

Ordinary matter ↔ $g_{\mu\nu}$ ↔ ϕ

Ex : scalar-tensor, TeVeS ....

$S_\varphi[\varphi; g_{\mu\nu}]$

$S_m[{\rm mat}; g_{\mu\nu}] \to S_m[{\rm mat}_i; A_i^2(\varphi) g_{\mu\nu}]$.

$S_{\rm em}[A_\mu; g_{\mu\nu}] \to S_{\rm em}[A_\mu, a_\mu; g_{\mu\nu}]$.

Ordinary matter ↔ $g_{\mu\nu}$

A_μ ↕ a_μ

Ex : axion-photon mixing

Distance duality

Variation of constants

Poisson equation

Ordinary matter ↔ $\tilde{g}^{(i)}_{\mu\nu}$

Ex : brane induced gravity multigravity,...

Always need NEW fields

[JPU, Aghanim, Mellier, PRD 05]
[JPU, GRG 2007]

Extensions

Any of these extensions requires new-degrees of freedom

we always have new matter fields

distinction matter/gravity is a Newtonian notion

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<u>*Matter:*</u> amount imposed by initial conditions

This matter dominates matter content and triggers acceleration (**dark energy**)

This matter clusters and generates potential wells (**dark matter**)

<u>*Gravity:*</u> ordinary matter « generates » an effective dark matter halo

« induces » a effective dark energy fluid

We would like to determine

the nature of these degrees of freedom

the nature of their couplings

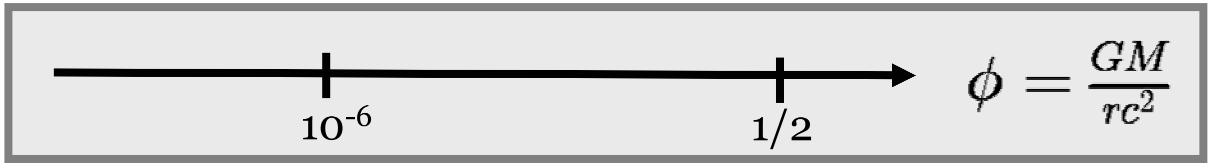
If they are light and if they couple to ordinary matter

responsible for a long range interaction

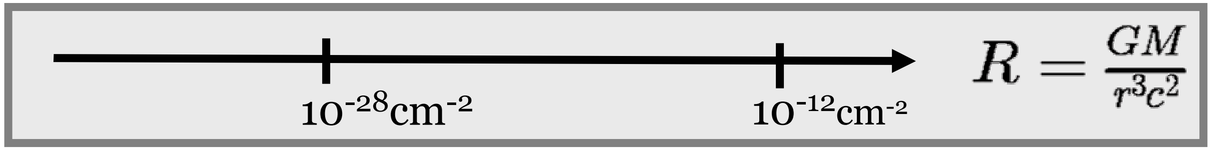
Most models contain ΛCDM as a continuous limit.

In which regime

Usually, we distinguish *weak-strong field* regimes



Corrective terms in the action have to be compared to *R*:



Also discussed in distinguishing *large-small distances*

Static configuation:

these limits are related because main dependance is (M,r)

acceleration may also be the best parameter (e.g. rotation curves)

Cosmology:

background level: R increases with z

perturbation: always in weak field

but at late time, we can have high curvature corrections

Parameter space

Tests of general relativity on astrophysical scales are needed
- galaxy rotation curves: low acceleration
- acceleration: low curvature

Solar system:

$$\frac{R}{\phi^3} = \frac{c^4}{G^2 M_\odot^2}$$

Cosmology:

$$R = 3H_0^2\{\Omega_m(1+z)^3 + 4\Omega_\Lambda\}$$

Dark energy:

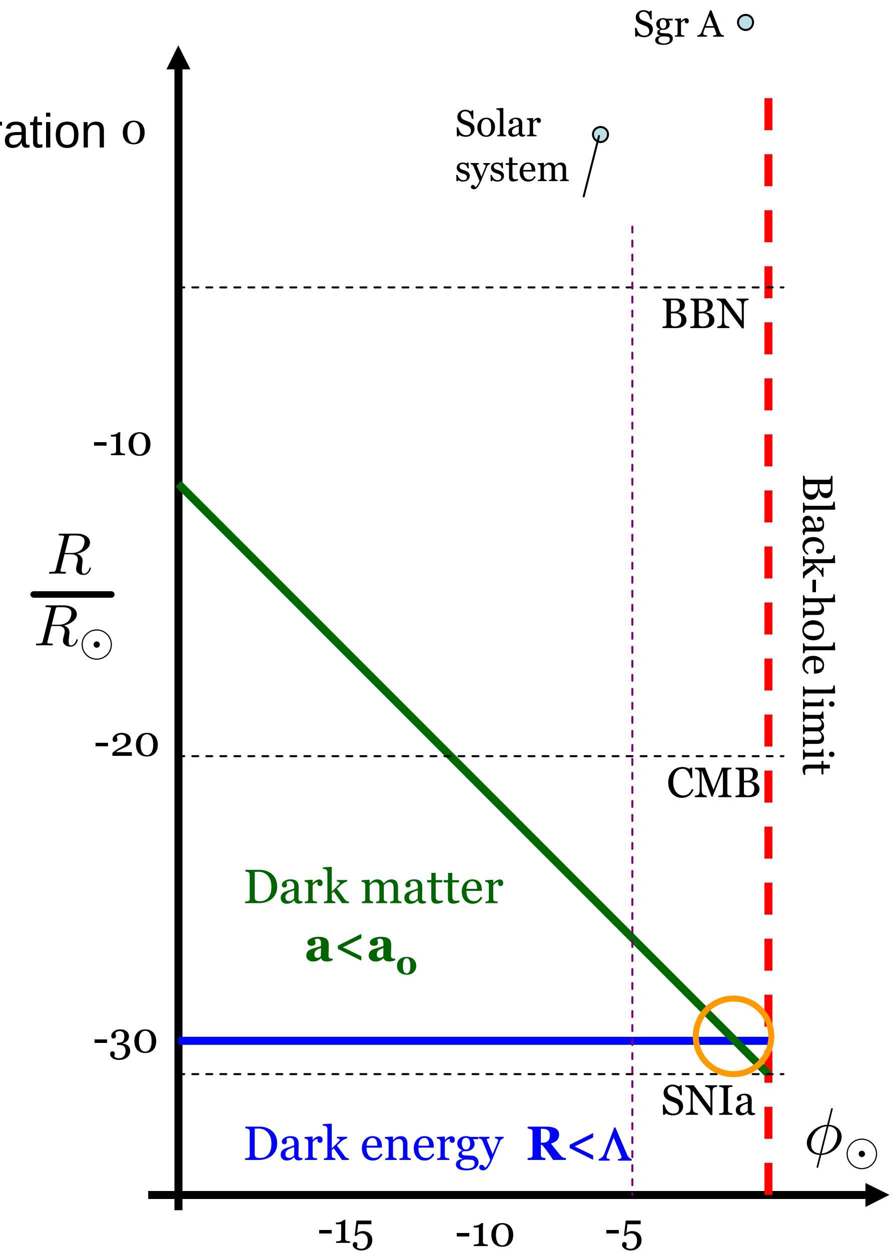
$$R < R_\Lambda = 12H_0^2\Omega_\Lambda$$

Dark matter:

$$a < a_0 \sim 10^{-8}\mathrm{cm.s}^{-2}$$

$$a^2 = \phi R < a_0^2$$

[Psaltis, 0806.1531]



Solar system

Metric theories are usually tested in the PPN formalism

$$ds^2 = (-1 + 2U + 2(\beta - \gamma)U^2)dt^2 + (1 + 2\gamma U)dr^2 + r^2 d\Omega^2$$

Light deflection

$$\Delta\theta = 2(1+\gamma)\frac{GM}{bc^2}$$

Perihelion shift of Mercury

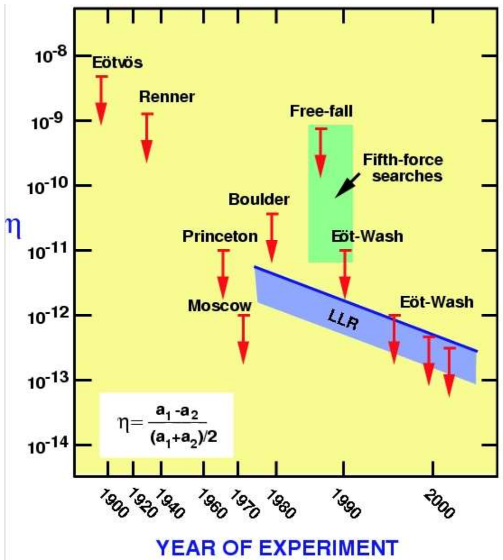
$$\Delta\varphi = \frac{2\pi GM}{a(1-e^2)}(2 + 2\gamma - \beta)$$

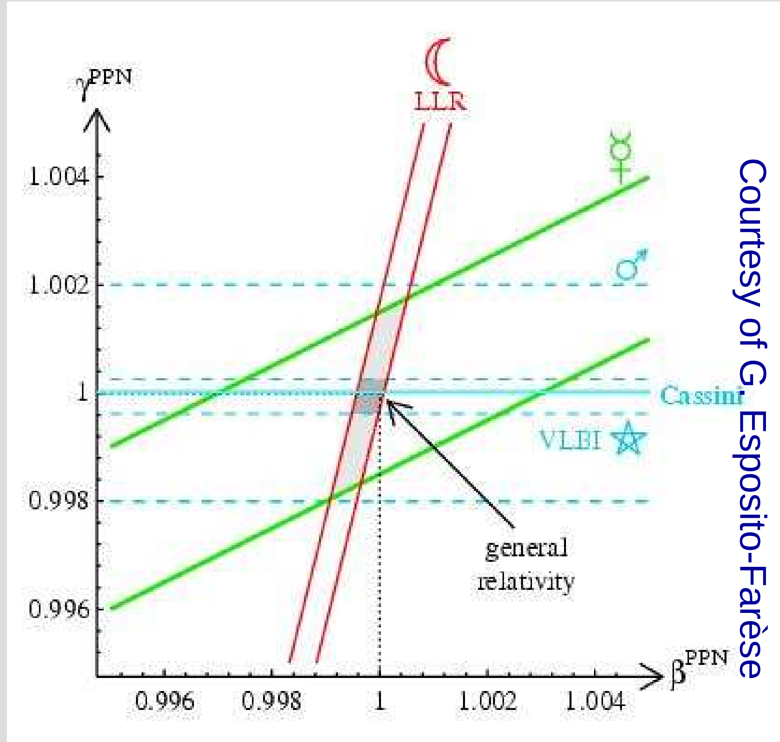
Nordtvedt effect

$$\delta r \sim 13.1(4\beta - \gamma - 3)\cos(\omega_0 - \omega_s)t \quad (\mathrm{m})$$

Shapiro time delay

$$\delta t \propto (1+\gamma)$$





$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

$$|2\gamma - \beta - 1| < 3 \times 10^{-3}$$

Modifying GR

The number of modifications are numerous.

I restrict to field theory.

We can require the following constraints:

- Well defined **mathematically**
  - *full Hamiltonian should be bounded by below*
    - *-no ghost ($E_{kinetic}$ >0)*
    - *-No tachyon (m^2>0)*
  - *Cauchy problem well-posed*

- In agreement with existing **experimental** data
  - *Solar system & binary pulsar tests*
  - *Lensing by « dark matter » - rotation curve*
  - *Large scale structure – CMB – BBN - …*

- Not pure fit of the data!

Design

The regimes in which we need to modify GR to explain DE and DM are different.

DM case: *we need a force ~ 1/r*

a priori easy:
- consider $V(\varphi) = -2a^2e^{-b\varphi}$ [Not bounded from below]
- static configuration: $\Delta\varphi = V'(\varphi)$ and thus $\varphi = (2/b)\ln(abr)$

But:
The constant (2/b) has to be identified with $M^{1/2}$ **!!**

[see PRD76 (2007) 124012]

DE case:

Coincidence problem
ST: 2 free functions that can be determine to reproduce
H(z) and $D_+(z)$.

	bgd	bgd + Newt. pert.	bgd + Newt. pert. + Solar syst.
DGP vs quintessence	Y	N	N
DGP vs scalar-tensor	Y	?	N

First example: higher-order gravity...

At quadratic order

$$S_g = \frac{c^3}{16\pi G}\int\left(R + \alpha C^2_{\mu\nu\rho\sigma} + \beta R^2 + \gamma GB\right)\sqrt{-g}\,d^4x$$

- $GB = R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$ does not contribute to the field eqs.

- $\alpha C^2_{\mu\nu\rho\sigma}$ theory contains a ghost [Stelle, PRD**16** (1977) 953]

$$\frac{1}{p^2+\alpha p^4} = \frac{1}{p^2} - \frac{1}{p^2+\alpha^{-1}}$$

$\frac{1}{p^2}$ → massless graviton

$\frac{1}{p^2+\alpha^{-1}}$ → massive degrees of freedom with $m^2=1/\alpha$

"−" → **carries negative energy**

$\alpha<0$: it is also a tachyon.

- βR^2 equivalent to positive energy massive scalar d.o.f

...and beyond

These considerations can be extended to $f(R,R_{\mu\nu},R_{\mu\nu\alpha\beta})$

[Hindawi et al., PRD**53** (1996) 5597]

Generically contains massive spin-2 ghosts but for f(R)

These models involve generically higher-order terms of the variables.

the Hamiltonian is then generically non-bounded by below

[Ostrogradsky, 1850]
[Woodard, 0601672]

Argument does not apply for an infinite number of derivative

non-local theories may avoide these arguments

Only allowed models of this class are *f(R)*.

Reconstruction of the cosmological dynamics [see Amendola, Dunsby talks]

Why not...

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-\,g}\,R$$

$$+\,S_M(\psi; f(g_{\mu\nu}, R^{\pi}_{\mu\nu\rho}, \ldots))$$

Identical to GR in vacuum

Can reproduce MOND and late time acceleration.
But: instable within matter

$f(R)$ and scalar-tensor theories

We consider the theory $S_g = \int f(R)\sqrt{-g}\, d^4x$

Introducing a Lagrange parameter to rewrite it as

$$S_g = \int \{ f(\phi) + (R-\phi) f'(\phi) \} \sqrt{-g}\, d^4x$$

The field equation for ϕ reads $(R-\phi) f''(\phi) = 0$

The field equations of the 2 theories are identical.

The theory is thus equivalent to the ST:

$$S_g = \int \{ f'(\phi) R - (\phi f'(\phi) - f'(\phi)) \} \sqrt{-g}\, d^4x$$

<u>Einstein frame:</u> $\varphi = \frac{\sqrt{3}}{2} \ln f'(\phi) \quad V(\varphi) = \frac{\phi f'(\phi) - f(\phi)}{4 f'^2(\phi)} \quad A(\varphi) = e^{\phi/\sqrt{3}} \quad g^*_{\mu\nu} = A^2 g_{\mu\nu}$

<u>Generalisation:</u>

$$f(R, \nabla^2 R, \ldots, (\nabla^2)^n R)$$

[Teyssandier, Tourrenc, JMP **24** (1983) 2793]
[Wands, CQG **11** (1994) 269]...

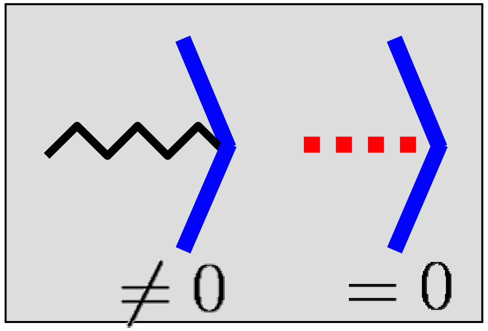
spin 2

spin 0

$$S = \frac{c^3}{16\pi G}\int\sqrt{-g}\{R - 2(\partial_\mu\phi)^2 - V(\phi)\} + S_m\{\text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}\}$$

Maxwell electromagnetism is conformally invariant in d=4

$$S_{em} = \frac{1}{4}\int\sqrt{-\tilde{g}}\,\tilde{g}^{ab}\tilde{g}^{cd}F_{ac}F_{bd}\mathrm{d}^dx$$
$$= \frac{1}{4}\int\sqrt{-g}\,g^{ab}g^{cd}F_{ac}F_{bd}A^{d-4}(\phi)\mathrm{d}^dx$$



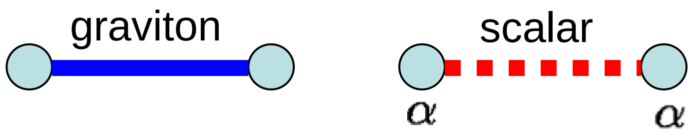
Light deflection is given as in GR

$$\delta\theta = \frac{4GM}{bc^2}$$

What is the difference?

The difference with GR comes from the fact that massive matter feels the scalar field

$$G_{\rm cav} = G(1+\alpha^2)$$



$$\alpha = \mathrm{d}\ln A/\mathrm{d}\phi$$

Motion of massive bodies determines $G_{cav}M$ **not** GM.

Thus, in terms of observable quantities, light deflection is given by

$$\delta\theta = \frac{4G_{\rm N}M}{(1+\alpha^2)bc^2} \leq \frac{4GM}{bc^2}$$

which means

$$M_{\rm lens} \leq M_{\rm rot}$$

Disformal coupling

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi$$

Bekenstein, gr-qc/9211017

Bekenstein, Sanders, 9311062

Preferred direction
(radial for spherical system)

It was extended by Bekenstein (TeVeS theory…)

$$\tilde{g}_{\mu\nu} = A^2(\varphi) g_{\mu\nu} + B(\varphi) V_\mu V_\nu$$

Dynamical unit timelike vector

This is at the basis of the construction of TeVeS theories and many other bimetric theories:

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\,\{R - 2f(\partial_\mu\varphi\partial^\mu\varphi)\} + S_M(\psi, \tilde{g}_{\mu\nu})$$

« k-essence » can extract
MOND behaviour or acceleration

Matter coupled to ϕ

Necessary
for lensing

Problems

Gravitational waves and bimetric

In models involving 2 metrics (scalar-tensor, TeVeS,...), gravitons and standard matter are coupled to different metrics.

In GR:

photons and gravitons are massless and follow geodesics of the same spacetime

$$\delta T_{\gamma g} = T_\gamma - T_g = 0$$

In bi-metric:

photons and gravitons follow geodesics of two spacetimes

(not in scalar-tensor theories)

$$\delta T_{\gamma g} \neq 0$$

Example:

TeVeS model. Observable=SN1987a

$$\delta T_{\gamma g} = -5.3\,\mathrm{days}$$

Kahya & Woodard, 0705.0153

Extensions

We can also allow for non-minimal couplings

$$S_M = \sum_i S_{M_i}(\psi_i; A_i^2(\phi) g_{\mu\nu})$$

String inspired models

$$S = \int d^4x \sqrt{-g} \Big(B_g(\phi) R - B_\phi(\phi)(\partial\phi)^2 - \tfrac{1}{4} B_F(\phi) F^2 - V(\phi) - \sum_a m_a(\phi) \sqrt{-g_{\mu\nu} dx^\mu dx^\nu} \Big)$$

Damour, Polyakov (1994)

This generally implies variation of the fundamental constants.

As well as universality of free-fall

$B(\phi) F_{\mu\nu} F^{\mu\nu}$ electromagnetic self energy contributes to the mass

mass of hadrons are proportional to $\Lambda_{\rm QCD} \propto B_F^{-1/2}(\phi)\, e^{-8\pi^2 b_3^{-1} B_F(\phi)} \Lambda_s$

Order of magnitude $\eta_{12} \simeq M_p f_{\rm ext} \partial_\phi |\ln(m_1/m_2)|$

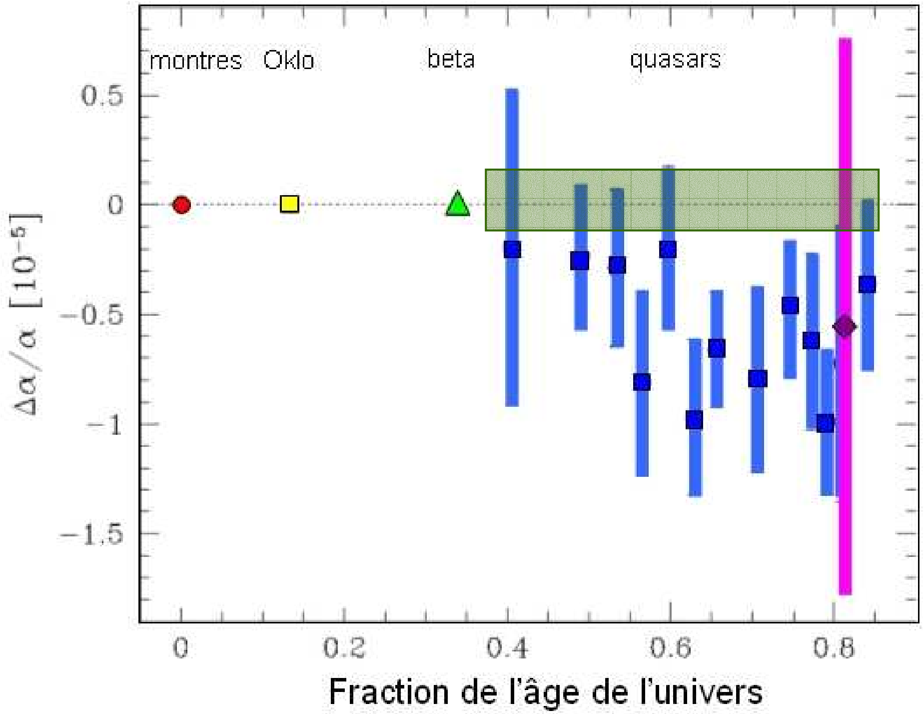
[JPU, RMP **75** (2003) 425; astro-ph/0409424]

Constants (local position invariance)

Many tests concerning various constants (α, μ, G mainly).

Tests on different time scales:

local	(z=0)	atomic clocks, Solar System
geophysical	(z=0.1..0.4)	Oklo, meteorites
astrophysical	(z=0.2-3.5)	quasars
cosmological	($z=10^3$, 10^8)	CMB, BBN.



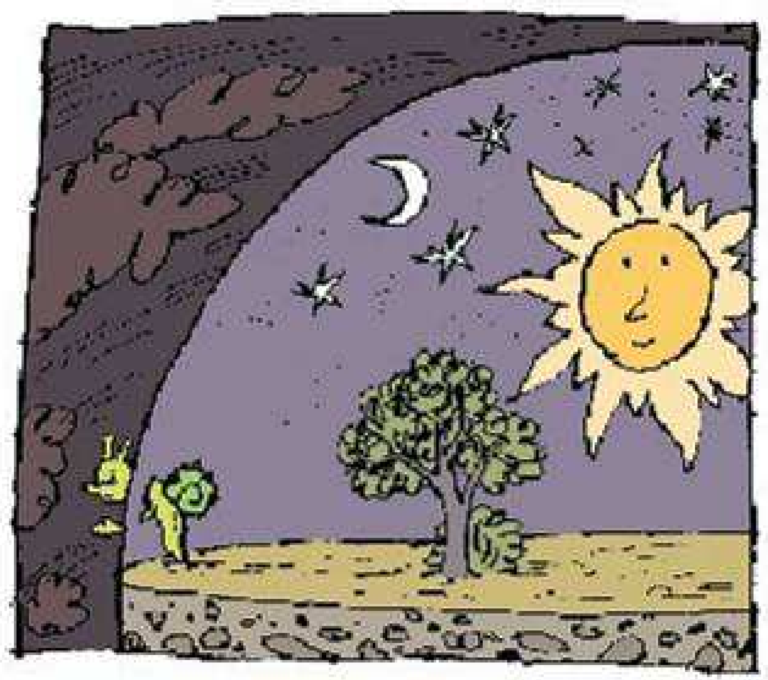
General investigation of the link of these constraints and gravity theories

[JPU, RMP **75** (2003) 425; astro-ph/0409424]

Cosmological effects

How do these modifications influence the cosmology ?

Community seems to reach a state of thermal equilibrium of
How to test deviation from ΛCDM.



Times they are a changin'.

On sub-Hubble scales, the gravitational potential and density contrast are related by

$$\Delta\Phi = 4\pi G\rho a^2\delta$$

Galaxy catalogs (SDSS,2dF...)

measurement of $\xi(r)$ up to $500h^{-1}$Mpc

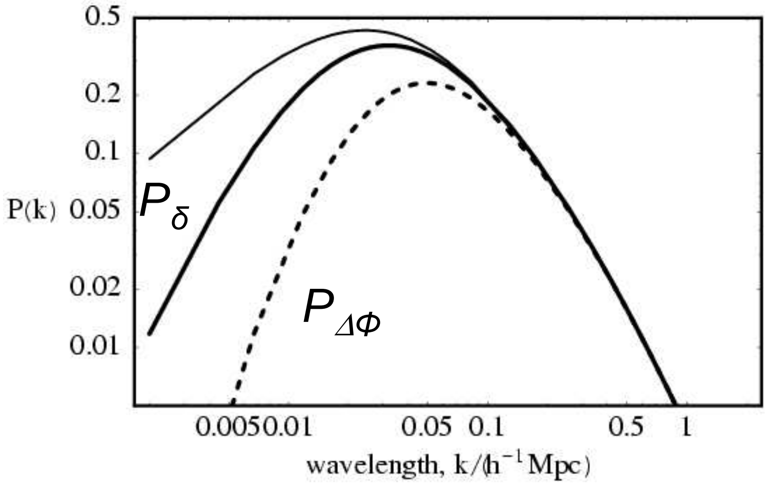
Weak lensing

will be measured up to $100h^{-1}$Mpc

Toy model: 4D-5D gravity

perturbations freeze on large scales (idem as Λ)

power spectra of Φ and δ are not identical



[JPU & Bernardeau, PRD **64** (2000)]

Origin of the rigidity

In the linear regime, the growth of density perturbation is then dictated by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_{\rm mat}\delta = 0$$

This implies a *rigidity* between the growth rate and the expansion history

Bertschinger, astro-ph/0604485,
JPU, astro-ph/0605313

It can be considered as an equation for *H(a)*

Chiba & Takahashi, astro-ph/0703347

$$(H^2)' + 2\left(\frac{3}{a} + \frac{\delta''}{\delta'}\right)H^2 = 3\frac{\Omega_0 H_0^2 \delta}{a^5 \delta'}$$

$$\frac{H^2}{H_0^2} = 3\Omega_{m0}\frac{(1+z)^2}{\delta'(z)^2}\int_z \frac{\delta}{1+z}(-\,\delta')\mathrm{d}z$$

H(a) from the background (geometry) and growth of perturbation have to agree.

"Post ΛCDM"

Restricting to low-*z* and sub-Hubble regime

$$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta^2 + (1-2\Psi)\gamma_{ij}dx^i dx^j]$$

Background

$$H^2/H_0^2 = \Omega_m^0(1+z)^3 + (1-\Omega_m^0-\Omega_\Lambda^0)(1+z)^2 + \boxed{\Omega_{\rm de}(z)}$$

Sub-Hubble perturbations

$$\Delta(\Phi-\Psi) = \boxed{\pi_{\rm de}}$$

$$-k^2\Phi = 4\pi G_N \boxed{F(k,H)} \rho a^2 \delta + \boxed{\Delta_{\rm de}}$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi + \boxed{S_{\rm de}}$$

JPU, astro-ph/0605313

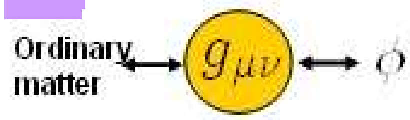
ΛCDM

$$(F, \pi_{\rm de}, \Delta_{\rm de}, S_{\rm de}) = (1,0,0,0)$$

Links with the classification

$$(\mathcal{S}_{\rm de}, \Delta_{\rm de}, F_{\rm m}, \Pi_{\rm de}) = (0, \Delta_{\rm de}, 1, \Pi_{\rm de})$$

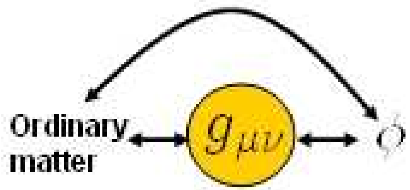
A



Ex: quintessence, ...

ST: $$(\mathcal{S}_{\rm de}, \Delta_{\rm de}, F_{\rm m}, \Pi_{\rm de}) \simeq \left(0, 0, \frac{G_{\rm cav}}{G_{\rm cav0}}, \frac{F_\varphi^2}{F + 2F_\varphi^2}\Delta\Phi\right)$$

C



Ex: scalar-tensor, ...

Variation of constants

Poisson equation

Distance duality

Ex: axion-photon mixing

B

$$(\mathcal{S}_{\rm de}, \delta_{\rm de}, F_{\rm m}, \Pi_{\rm de}) = (0, 0, 1, 0)$$

Poisson equation

Ex: brane induced gravity
multigravity,...

D

DGP: $$(\mathcal{S}_{\rm de}, \Delta_{\rm de}, F_{\rm m}, \Pi_{\rm de}) = \left(0, 0, 1 + \frac{1}{3\beta}, \frac{8\pi G}{3\beta}\rho_{\rm m} a^2 \delta_{\rm m}\right)$$

See JPU, GRG (2007)

Data

DATA	OBSERVABLE
Weak lensing	$\kappa \propto \Delta(\Phi + \Psi)$
Galaxy map	$\delta_g = b\,\delta$
Velocity field	$\theta = \beta\,\delta$
Integrated Sachs-Wolfe	$\Theta_{SW} \propto \dot{\Phi} + \dot{\Psi}$

Various correlations of these variables have been considered

[see talk by M. Kunz]

Conclusions

GR well tested in the Solar system but there is still place for modifications

Many extensions have been condered.

Field theory extentions are constrained
- Hamiltonian bounded from below (n ghost – tachyon)
- Cauchy problem
- need to go beyond a pure fit of the data

String inspired models
- generally leads to scalar-tensor theories [compactification] but usually induce variation of constants.
- brane models usually inlude massive gravitons.

Non-local models may avoid the general theorems.

Tests require combination of Solar system/strong-field/cosmology.

Cosmology does not reduce to large scale structure

Cosmological features of ST theories

Close to GR today

assume light scalar field

Can be attracted toward GR during the cosmological evolution.

[Damour, Nordtvedt]

Dilaton can also be a quintessence field

[JPU, PRD 1999]

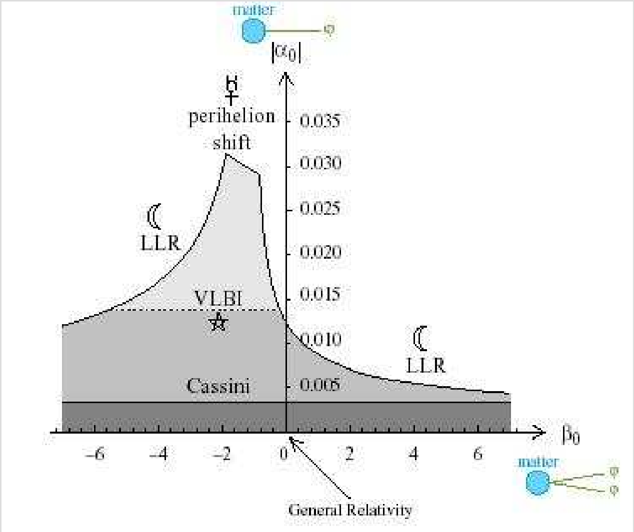
Equation of state today

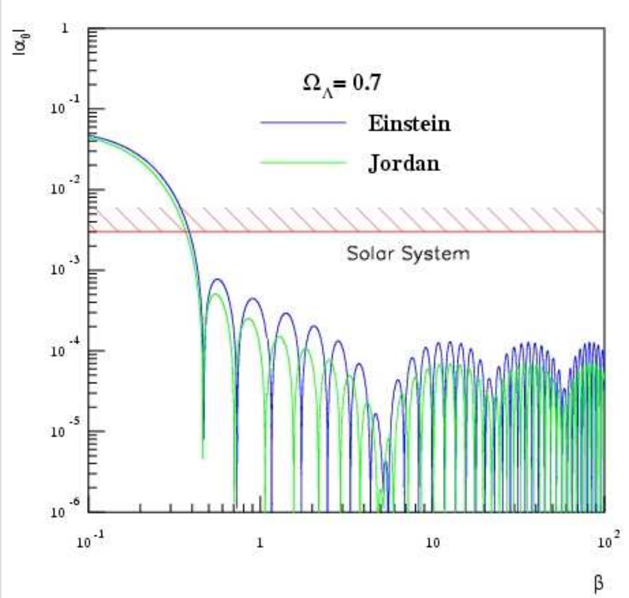
$$3\Omega_{de0}(w_0 + 1) \simeq 2(1 - \beta_0)\phi_0'^2 - 2\alpha_0\phi_0''$$

[Martin, Schimd, JPU, 0510208]

Cosmological predictions computable (BBN, CMB, WL,...]

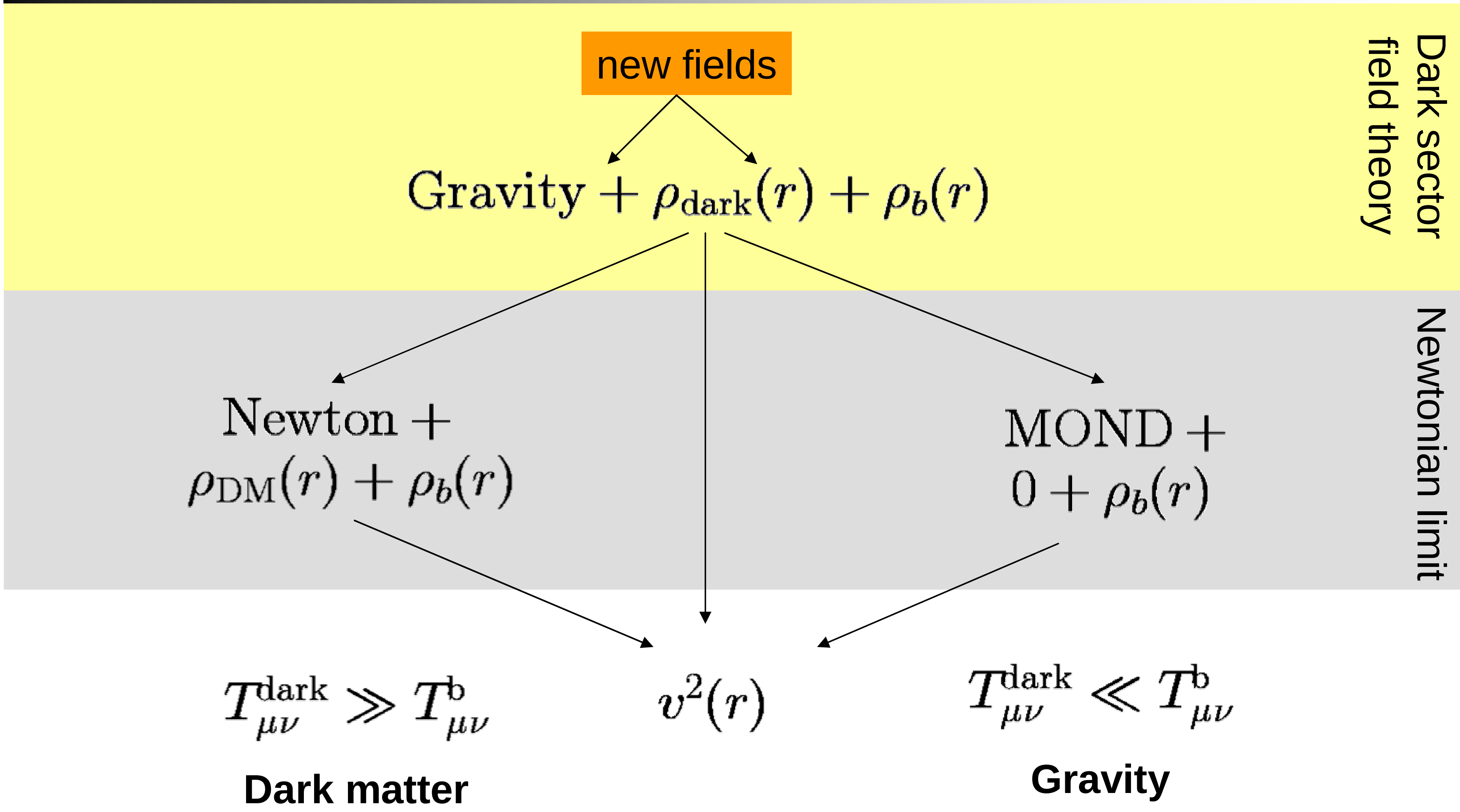
[Schimd et al., 2005; Riazuelo JPU, 2000, Coc et al., 2005]





[Coc et al, 0601299]

Example


new fields
Gravity + $\rho_{\rm dark}(r) + \rho_b(r)$
Dark sector field theory
Newton + $\rho_{\rm DM}(r) + \rho_b(r)$
MOND + $0 + \rho_b(r)$
Newtonian limit
$T^{\rm dark}_{\mu\nu} \gg T^{\rm b}_{\mu\nu}$
$v^2(r)$
$T^{\rm dark}_{\mu\nu} \ll T^{\rm b}_{\mu\nu}$
Dark matter
Gravity